International Journal of Business and Economics, 2009, Vol. 8, No. 1, 1-6

Constant Best-Response Functions: Interpreting Cournot

Zivan Forshner
Department of Economics, University of Haifa, Israel

Oz Shy*
Research Department, Federal Reserve Bank of Boston, U.S.A.

Abstract

Following Amir and Grilo (1999), we characterize a class of demand functions that generate constant quantity best-response functions. We examine implications of constant best-response functions for the invariance of equilibrium outcomes with respect to the assumed market structure of quantity games. We argue that, unlike the class of linear demand functions, this class of demand functions supports the pure interpretation of Cournot conjectures.

Key words: constant best-response functions; Cournot equilibrium; Cournot conjectures; Stackelberg equilibrium

JEL classification: C72; D43

1. Introduction

Perhaps the major difficulty with empirical research related to oligopoly theory is that equilibrium market outcomes tend to be highly sensitive to changing the order of decision making by firms in an industry and/or changing the equilibrium concept (i.e., market structure). A natural question to ask is whether there exist demand functions in which the resulting market equilibrium outcomes are invariant with respect to the assumed market structure.

Amir and Grilo (1999, p. 9) demonstrated the existence of a class of demand functions for which the resulting best-response functions are constant in quantity oligopoly games. The desired invariance to the impact of the order of decisions in quantity oligopoly games on market outcomes immediately follows.

From a theoretical point of view, the Cournot market structure may have two interpretations associated with two different assumptions concerning firms' expectations. The widely-used (weak) interpretation is that a Cournot equilibrium is

Received May 15, 2009, revised May 17, 2009, accepted May 18, 2009.
*Correspondence to: Research Department, Federal Reserve Bank Boston, 600 Atlantic Avenue, Boston, MA 02210, U.S.A. E-mail: Oz.Shy@bos.frb.org. We thank Editor Jong-Shin Wei for most valuable suggestions and comments on earlier drafts.

a Nash equilibrium (Nash, 1950) in the sense that each firm expects the other firm to maintain a constant output level. Clearly, these expectations are consistent only in equilibrium, at least for linear demand functions. However, a stronger interpretation of Cournot equilibrium is to assume that firms expect rival firms to have constant best-response functions. In contrast with linear demand functions, the class of demand functions identified in this paper is consistent with the strong interpretation of Cournot conjectures. In fact, strategic games may have several interpretations; see for example Osborne and Rubinstein (1994, Section 2.1.2).

The paper is organized as follows. Section 2 identifies the class of demand functions yielding constant best-response functions. Section 3 demonstrates that the realization of output levels is invariant with respect to changing from Cournot to Stackelberg quantity-setting market structures. Section 4 concludes with a general discussion and demonstrates how, in contrast with linear demand functions, this class of demand functions supports the pure interpretation of Cournot conjectures.

2. Demand Functions Yielding Constant Best-Response Functions

Consider a market for a homogeneous product with an aggregate downward sloping inverse demand function denoted by $p(Q)$, where Q denotes aggregate quantity demanded and p the market price. We assume that $p(Q)$ is twice continuously differentiable with respect to Q.

There are N firms indexed by $i = 1, \ldots, N$ ($N \geq 2$) which can costlessly produce this product. Let q_i ($q_i \geq 0$) denote the output of firm i, so $Q = \sum_{i=1}^{N} q_i$. Also, define $q = (q_1, \ldots, q_N)$ as the N-dimensional vector of firms' output levels and $q_{(i)} = (q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_N)$ as the ($N-1$)-dimensional vector of output levels of all firms except firm i. Finally, let $\pi_i(q) = p(\sum_{j=1}^{N} q_j) q_i$ denote the profit function of firm i, $i = 1, \ldots, N$.

For a given $q_{(i)}$, the set of maximizers is given by:

$$\psi(q_{(i)}) = \left\{ q_i \in \Re_+ : p\left(q_i + \sum_{j \neq i} q_j\right) q_i \geq p\left(x + \sum_{j \neq i} q_j\right) x \text{ for all } x \in [0, \infty) \right\}.$$

We assume that $\psi(q_{(i)}) \neq \text{empty}$ for all $q_{(i)} \in \Re_+^{N-1}$. We call ψ the best-response correspondence. A best-response function $R_i(q_{(i)})$ is a selection out of $\psi(q_{(i)})$; i.e., $R_i(q_{(i)}) : \Re_+^{N-1} \to \psi(q_{(i)})$. A best-response function is said to be constant if there exists a constant $k_i \geq 0$ for which $R_i(q_{(i)}) = k_i$ for all $q_{(i)}$.

Following Amir and Grilo (1999, p. 9), we formalize the existence of this class of functions in the following proposition.

Proposition 1. There exists a class of downward-sloping market demand functions for which the corresponding best-response functions are constant. Formally, this class of demand functions is given by:

$$p(Q) = e^{-Q/\beta} = e^{-\frac{1}{\beta}\sum_{i=1}^{N} q_i} \tag{1}$$

for any $\beta > 0$.

Proof. We derive the best-response function of firm i, which chooses q_i that solves:

$$\max_{q_i} \pi_i(q_i, q_{(i)}) = q_i e^{-\frac{1}{\beta}(q_i + \sum_{j \neq i} q_j)}.$$

The first-order condition is given by:

$$\frac{\partial}{\partial q_i} \pi_i(q_i, q_{(i)}) = \frac{(\beta - q_i)}{\beta} e^{-\frac{1}{\beta}\sum_{j=1}^{N} q_j} = 0.$$

Hence, $q_i = \beta$ is an extremum point. Since the local second-order condition satisfies:

$$\left.\frac{\partial^2}{\partial (q_i)^2} \pi_i(q_i, q_{(i)})\right|_{q_i = \beta} = \frac{(\beta - 2\beta)}{\beta^2} e^{-\frac{1}{\beta}\sum_{j=1}^{N} q_j} < 0,$$

and upon checking the graph of π_i (as a function of q_i) we see this local maximum happens to be the global maximum over $(0, \infty)$ at $q_i = \beta$. Observe that:

$$\pi_i(q_1, \ldots, q_{i-1}, \beta, q_{i+1}, \ldots, q_N) = \beta e^{-\frac{1}{\beta}(\beta + \sum_{j \neq i} q_j)} > 0.$$

Finally, since $\pi_i(0, q_{(i)}) = 0$, π_i attains a global maximum over $[0, \infty)$ at $q_i = \beta$. It follows that $\psi(q_{(i)}) \neq \text{empty}$, and in fact $\psi(q_{(i)}) = \{\beta\}$. Therefore, the best-response correspondence is a function.

We have shown that producing output level β constitutes a dominant strategy of any firm.

3. Equilibrium Oligopoly Market Outcomes

We now solve for the equilibrium market outcomes under Cournot and Stackelberg market structures.

A Cournot-Nash equilibrium (Cournot, 1838) is an outcome $q^c \in \Re_+^N$ such that $R_i(q_{(i)}^c) = q_i^c$ for all $i = 1, \ldots, N$. Clearly, in this environment, the unique Nash-Cournot equilibrium is $q_i^c = \beta$ and the aggregate output level is $Q = N\beta$.

A Stackelberg equilibrium in which firm 1 is a leader (Stackelberg, 1934) is the outcome $q^S \in \Re_+^N$, which is obtained by solving:

$$\begin{aligned} &\max_{q_1} \pi_1(q_1, q_{(1)}) \\ &\text{s.t. } q_j = R_j(q_{(j)}) \quad \text{for all} \quad j \geq 2. \end{aligned} \tag{2}$$

That is, the leader solves for the followers' best-response functions and chooses its output level accordingly. Followers behave in a Cournot fashion by simply following their best-response functions.

Note that once we know that $q_i = \beta$ is the unique dominant strategy of every firm i, it immediately follows that equilibria in any of the following three scenarios yield identical output decisions. (a) The leader moves first, while the other $N-1$ firms move simultaneously as followers. (b) All N firms move sequentially and one at a time. (c) All N firms move simultaneously. Hence, the order of output determination does not influence output level decisions. Thus, the order-invariance result derived for this specific market demand function constitutes the major strength of the present model. Moreover, for this reason, we deliberately refrained from defining the Stackelberg market structure as a specific (ad hoc) extensive-form game. This is because our result is more general in the sense that it may apply to sequential games which cannot be represented by a conventional tree but still have some normal-form representations. We can now state the following proposition.

Proposition 2. The realization of output levels is invariant with respect to the choice of Cournot and Stackelberg market structures.

It is interesting to point out that for this particular inverse-demand function, each firm can consider itself to be a Stackelberg leader in the sense that it solves the maximization problem (2), while firms' conjectures maintain mutual consistency. This invalidates a common perception that, in a Stackelberg market structure when there are two firms, there is no equilibrium when both firms view themselves as leaders (e.g., Kreps, p. 330). Moreover, our result is related to the endogenous-timing papers, such as Hamilton and Slutsky (1990) and Amir and Grilo (1999), which obtain in a more general setting the possibility that both firms will choose their output level at the same time (i.e., a simultaneous game). Here, we demonstrate the possibility of having a complete invariance with respect to the order of moves in the absence of production costs. In the presence of production costs, it is clear that for sufficiently-high production costs at least one firm will choose not to produce in which case the order of moves may affect market outcome.

4. Interpreting Cournot: A Discussion

From a theoretical point of view, we would like to point out the following.

(a) Cournot (1927) never suggested the widely used class of linear demand functions as an example for his model. The class of linear demand functions belongs to a modern interpretation of his model. In fact, it seems as if Cournot left it vague which class of demand functions is consistent with his model.

(b) Unlike the class of linear demand functions, the class of demand functions identified in this note is consistent with the strong interpretation of Cournot conjectures in the sense that firms correctly believe that rival firms will not deviate from their output level regardless of their actions. This interpretation does not hold for the class of linear demand functions.

(c) One cannot argue that the widely-used class of linear demand functions is more general than the class identified in this note. In fact, as we argue below, we believe that the present class of demand functions better suits econometric modeling.

(d) It is indeed possible that Cournot himself thought about constant best-response functions. Our interpretation of Cournot is based on Cournot's system of two equations given in Cournot (1927, Ch. 7, p. 66), which can be written using our notation and for $N = 2$ as:

$$\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} = 0 \Rightarrow p(\beta + q_2) + \frac{dp(Q)}{dQ}\frac{\partial Q}{\partial q_1}\beta \equiv 0 \text{, for all } q_2 \tag{3}$$

$$\frac{\partial \pi_2(q_1, q_2)}{\partial q_2} = 0 \Rightarrow p(q_1 + \beta) + \frac{dp(Q)}{dQ}\frac{\partial Q}{\partial q_2}\beta \equiv 0 \text{, for all } q_1 \tag{4}$$

Equation (3) reflects what firm 2 conjectures about firm 1, namely that its optimization always yields $q_1 = \beta$ regardless of the level of output set by firm 2. Thus, firm 2 conjectures that $q_1 = \beta$ is a dominant action of firm 1. Similarly, (4) reflects what firm 1 conjectures about firm 2, namely that its optimization always yields $q_2 = \beta$ regardless of the level of output set by firm 1. Thus, firm 1 conjectures that $q_2 = \beta$ is a dominant action of firm 2.

Therefore, our interpretation of Cournot's conjecture is that (3) and (4) hold as identities independently of each other. In contrast, the common interpretation is to require that this system of equations holds in equilibrium, in which case the Cournot conjecture about rival firms holding their output constant is inconsistent with firms' having downward-sloping best-response functions.

Finally, various authors (e.g., Bergstrom and Varian, 1985) have attempted to facilitate empirical research in oligopoly markets using simple Cournot market structures. The class of demand functions characterized in this note has significant implications for empirical research. This is because econometric models based on a demand function in the class identified by (1) can have strong predictive power since the market outcome is invariant with respect to the assumed market structure. This means that, in these markets, applied economists do not have to search across sequential dynamic games in order to find a particular ad hoc order of moves which best fits the data.

References

Amir, R. and I. Grilo, (1999), "Stackelberg versus Cournot Equilibrium," *Games and Economic Behavior*, 26, 1-21.

Bergstrom, T. and H. Varian, (1985), "When Are Nash Equilibria Independent of the Distribution of Agents' Characteristics?" *Review of Economics Studies*, 52, 715-718.

Cournot, A., (1927), *Researches into the Mathematical Principles of the Theory of Wealth*, Translated by Nathaniel T. Bacon, New York: Macmillan.

Hamilton, J. and S. Slutsky, (1990), "Endogenous Timing in Duopoly Games: Stackelberg or Cournot Equilibria," *Games and Economic Behavior*, 2, 29-46.

Kreps, D., (1990), *A Course in Microeconomic Theory*, Princeton: Princeton University Press.

Nash, J., (1950), "Equilibrium Points in N-Person Games," *Proceedings of the National Academy of Sciences of the United States of America*, 36, 48-49.

Osborne, M. and A. Rubinstein, (1994), *A Course in Game Theory*, Cambridge, MA: The MIT Press.

von Stackelberg, H., (1934), *Marktform und Gleichgewicht*, Vienna: Springer-Verlag. English Translation: *The Theory of the Market Economy*, Oxford: Oxford University Press.