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Consumer and Trade Prices in General Equilibrium with Imperfect Competition

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Abstract

A two-sector model of imperfect competition with intermediate goods is analyzed. An objective demand function is constructed and equilibrium studied through simulation. The results indicate that trade prices may exceed consumer prices and that collusion between firms may benefit both firms and consumers and result in intermediate goods trading at less than marginal cost.

Key words: prices; equilibrium; oligopoly

JEL classification: D5; L1

1. Introduction

Firms conventionally charge lower prices for "trade" sales--sales of their products as intermediate goods to other firms--than they do for sales to final consumers. The explanation for this phenomenon is found in imperfect competition, for without it such price discrimination could not be supported. However, the literature on general equilibrium with imperfect competition developed since Negishi (1961) has been restricted to models without intermediate goods. As noted by Hart (1985), this has been due to the difficulty of constructing consistent objective demand functions that incorporate intermediate demand. Consequently, there has been no formal analysis of trade prices, their relation to consumer prices and their economic effects.

This paper introduces a method for constructing objective demand and employs this to numerically analyze a two-sector general equilibrium model of imperfect competition with intermediate goods. The simulations investigate three equilibrium concepts that differ in the possibilities for price discrimination and collusion. The results show that there are ranges of parameter values for which price discrimination between final and intermediate consumers is harmful to both the (aggregate) con-

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sumer and the firms. Trade prices for intermediate goods may be higher than final consumer prices. Collusion between the firms over input prices benefits the consumer, and intermediate goods may be sold below marginal cost. Without collusion, there is excessive substitution of labour for intermediate goods.

2. The Model and Objective Demand

The analysis is developed in the context of a two-good economy but the ideas can be easily generalized. There are two firms, labelled 1 and 2, and each produces a single good using labour and the product of the other firm. Two different prices are distinguished: intermediate or "trade" prices p_1, p_2 and final consumer prices q_1 and q_2. It is assumed that the firms are price takers when acting on the demand side of a market and only have monopoly power when on the supply side. The demand for final consumption goods and the supply of labour are derived from the actions of a single, aggregate, utility-maximizing consumer.

The utility of the single consumer is represented by the function

$$U = U(X^{1C}, X^{2C}, L)\,, \tag{1}$$

and the consumer's budget constraint is

$$\pi + wL = q_1 X^{1C} + q_2 X^{2C}\,, \tag{2}$$

where $\pi = \pi^1 + \pi^2$ is profit income (π^j is the profit of firm j), X^{iC} is the consumption of good i, w is the wage rate, and L is the total labour supply. From (1) and (2), utility maximization results in consumption demands of the general form

$$X^{iC} = X^{iC}(q_1,\, q_2,\, w,\, \pi) \geq 0, \quad i = 1, 2\,. \tag{3}$$

The cost function of firm j is given by

$$C^j = C^j(p_i,\, w,\, X^j), \quad i, j = 1, 2, \quad i \neq j\,, \tag{4}$$

where X^j is the total production of firm j. Using Shephard's lemma, the intermediate good demand facing firm j, X^{jF}, conditional on the output of firm i, is given by

$$X^{jF} = \frac{\partial C^i(p_j,\, w,\, X^i)}{\partial p_j} \equiv C_1^i(p_j,\, w,\, X^i), \quad i, j = 1, 2, \quad i \neq j\,. \tag{5}$$

Equation (5) demonstrates the difficulty of defining intermediate demand since X^{jF} is conditional on X^i, which can only be determined by the equilibrium of the system and cannot be employed directly in the description of firm i's objective function.

To construct demand functions that overcome this circularity, it is first observed that the system must be consistent with demand equal to production. This implies

$$X^{j} = X^{jC}(q_1, q_2, w, \pi) + C_1^{i}(p_j, w, X^{i}), \quad i, j = 1, 2, \quad i \neq j. \tag{6}$$

Substituting for X^{i},

$$X^{j} = X^{jC}(q_1, q_2, w, \pi) + C_1^{i}\left(p_j, w, X^{iC}(q_1, q_2, w, \pi) + C_1^{j}(p_i, w, X^{j})\right), \tag{7}$$
$$i, j = 1, 2, \quad i \neq j.$$

If this equation can be solved for X^{j}, the solution will be of the form

$$X^{j} = X^{j}(q_1, q_2, p_1, p_2, w, \pi), \quad j = 1, 2. \tag{8}$$

Equation (8) represents the derived objective aggregate demand facing firm j incorporating the effects of input demand from firm i. Using (3) and (8), objective intermediate demand can be defined as

$$X^{jF} = X^{j}(q_1, q_2, p_1, p_2, w, \pi) - X^{jC}(q_1, q_2, w, \pi), \quad j = 1, 2. \tag{9}$$

Lemma 1 now justifies the construction above under the following assumptions:

Assumption 1: For all $q_1, q_2, w > 0, \quad X^{jC}(q_1, q_2, w, \pi) \geq 0, \quad j = 1, 2$.

Assumption 2: $C_1^{j}(p_i, w, X^{j}) \geq 0, \quad i, j = 1, 2, \quad i \neq j$.

Lemma 1: *There exists a non-negative solution to* (7) *if* $1 - C_{10}^{1} C_{10}^{2} > 0$, *where*

$$C_{10}^{i} \equiv \partial C_1^{i}(p_j, w, X^{i}) / \partial X^{1} \geq 0, \; i, j = 1, 2, \quad i \neq j.$$

Proof: Write (7) in the form

$$g^{j}(X^{j}) = X^{jC}(q_1, q_2, w, \pi) + C_1^{i}\left(p_j, w, X^{iC}(q_1, q_2, w, \pi) + C_1^{j}(p_i, w, X^{j})\right). \tag{10}$$

It is clear that the solution occurs when $X^{j} - g^{j}(X^{j}) = 0$. Now let $X^{j} = 0$. From Assumptions 1 and 2, $0 - g^{j}(0) \leq 0$; if it is zero then a non-negative solution has been found. Now take the case that $0 - g^{j}(0) < 0$. When $1 - C_{10}^{1} C_{10}^{2} > 0$, $g^{j}(X^{j})$ is a contraction mapping and thus has a unique fixed point. This fixed point must be positive and is continuously dependent on the parameters.

As C_{10}^{i} represents marginal intermediate input use, the restriction $1 - C_{10}^{1} C_{10}^{2} > 0$ is connected to the notion that the system is productive in the sense of being able to produce positive net outputs of both goods. The construction of demand, and the proof of the existence of a positive solution via the contraction mapping theorem, can clearly be extended to more general settings.

Using the derived demand, the model is completed by specifying the firms' profit functions

$$\pi^j = q_j X^{jC} + p_j[X^j - X^{jC}] - C^j, \quad j = 1, 2. \tag{11}$$

The structure of the model allows for several equilibrium concepts, and the simulations consider three of these. The first assumes that there can be no price discrimination between intermediate and final consumers so the firms simultaneously choose prices taking the wage rate as given and the equilibrium concept is a Nash equilibrium. The second equilibrium involves price discrimination: the producers set prices and then, at the second stage, simultaneously set consumer prices given the known producer prices. Equilibrium is defined as the perfect equilibrium of this game. It is therefore constructed using backward induction. The final equilibrium concept permits price discrimination but also has an element of collusion. The firms compete in their choice of final prices conditional upon previously selected trade prices, but trade prices are selected collusively to maximize joint profits given the behavior of final prices at the second stage. The second stage of the game is solved as a Nash equilibrium. The motivation behind this form of equilibrium is that collusion over final prices is generally prevented by legislation but over trade prices is less well regulated.

3. Simulation Specification

The simulations capture the influence of two general factors: the elasticity of substitution between labour and produced goods as inputs and the complementarity/substitutability relation on final goods markets. To allow the degree of substitutability to vary in production, the cost function for firm i is chosen to be of the C.E.S. form

$$\begin{aligned} C^i(p_j,\, w,\, X^i) &= K_i\left[(m_i)^{\frac{1}{1-\rho_i}} p_j^{\frac{\rho_i}{\rho_i-1}} + (1-m_i)^{\frac{1}{1-\rho_i}} w^{\frac{\rho_i}{\rho_i-1}}\right]^{\frac{\rho_i-1}{\rho_i}} X^i \\ &\equiv C^i(p_j,\, w)X^i, \end{aligned} \tag{12}$$

where ρ_i and m_i are the parameters that define the underlying production function and K_i is a scaling constant. When $\rho_i \to 1$, the technology is linear with an infinite elasticity of substitution. It is Cobb-Douglas, with unit elasticity, when $\rho_i \to 0$, and Leontief as $\rho_i \to -\infty$. The assumed form of the utility function is

$$U = \sum_{i=1}^{2}\left[\alpha_i X^{iC} - \frac{\beta_i [X^{iC}]^2}{2}\right] + \delta X^{1C} X^{2C} - L, \tag{13}$$

where $\alpha_i, \beta_i, i = 1, 2$ are positive constants. The two goods are gross substitutes if $\delta < 0$ and gross complements if $\delta > 0$. Solving as in (7), the aggregate demands are found to be

$$X^i = \left[\frac{a_i + c^j a_j}{1 - c^i c^j}\right] - \left[\frac{d_i - c^j b_j}{1 - c^i c^j}\right] q_i + \left[\frac{d_i + c^j d_j}{1 - c^i c^j}\right] q_j, \quad i, j = 1, 2, \quad i \neq j, \qquad (14)$$

where $a_i = \alpha_i \delta + \alpha_i \beta_j / (\beta_i \beta_j - \delta^2)$, $b_i = \beta_j / [\beta_i \beta_j - \delta^2] w$, and $d_i = -\delta / [\beta_i \beta_j - \delta^2] w$. Note that these demand functions are nonlinear in trade prices via the terms c^1 and c^2.

In the single-stage game, profit is defined using the cost function (12) and the demand function (14). The first-order condition is derived and then solved by a grid-search. In both two-stage games, the linearity in q of (14) allows the first-order conditions for the second stage to be solved explicitly to give q as a function of p. This solution is substituted back into profit, and the implied first-order conditions for p are solved by a grid search.

4. Results

Nine tables of results are given. The first three relate to the no-discrimination case and the remainder to the equilibria with discrimination. In each case the tables are distinguished by the value of δ. For each value of δ, equilibrium prices and quantities are given for a range of values of ρ from 0.99, representing an elasticity of substitution close to infinity, to –10000, which is almost a zero elasticity of substitution. The remaining parameter values, which are constant throughout, are as follows:

$$\alpha_1 = \alpha_2 = 2000, \ \beta_1 = \beta_2 = 0.8, \ m_1 = m_2 = 0.5, \ K_1 = K_2 = 0.5, \text{ and } w = 1.$$

Tables 1-3 present the results for the single price equilibrium. X^C, C^F, and X represent respectively final, intermediate, and total consumption of each good. As the equilibria are symmetric, the prices and quantities are the same for both goods. L is the total labour use and profit is that of a single firm.

Table 1. $\delta = -0.4$ **(Gross Substitutes), No Discrimination**

ρ	p	c	X^C	X^F	X	L	Profit	Utility
0.99	667.3	1.007	1110.5	0	1110.5	2237	740000	2959967
0.6	667.7	1.587	1110.2	1.5E-4	1110.2	3524	739579	2958234
-1	1118.6	296.6	734.5	254.7	989.2	17037	813093	2273575
-10	1288.3	602.0	593.1	518.8	1111.9	1989	763074	1948246
-10000	1333.7	667.3	555.2	555.2	1110.4	1111	739981	1849925

Table 2. $\delta = 0$, No Discrimination

ρ	p	c	X^C	X^F	X	L	Profit	Utility
0.99	1000.6	1.007	1249.2	0	1249.2	2516	1248742	3745983
0.6	1000.8	1.587	1249.0	6.3E-5	1249.0	3965	1248017	3744034
-1	1376.4	362.9	779.5	269.3	1048.8	19979	1062914	2611925
-10	1470.3	686.9	662.1	579.1	1241.3	2248	972398	2295524
-10000	1500.3	750.6	624.6	624.5	1249.2	1249	936499	2185125

Table 3. $\delta = 0.4$ (Gross Complements), No Discrimination

ρ	p	c	X^C	X^F	X	L	Profit	Utility
0.99	1200.5	1.007	1998.7	0	1998.7	4025	2397487	6392974
0.6	1200.7	1.587	1998.2	6.3E-5	1998.2	6344	2396127	6389455
-1	1491.4	392.4	1271.5	438.6	1710.1	33876	1879377	4405439
-10	1542.9	720.8	1142.7	999.5	2142.3	3896	1761201	4044752
-10000	1565.5	783.2	1086.2	1086.1	2172.3	2173	91699437	3870851

Reviewing Tables 1-3, it can be seen that the equilibrium price rises as the elasticity of substitution falls, due to aggregate demand becoming less elastic, and that at high elasticities of substitution no intermediate input is used. Labour use is the highest for the Cobb-Douglas case of $\rho \to 0$, with production taking place at extreme points on the isoquants. For δ equal to 0 and 0.4, the firms prefer a high elasticity of substitution because it prevents exploitation by their rival, despite the reduction it causes in their own market power. In contrast, for $\delta = -0.4$, profit first falls then rises and then falls again. The period of rising profit coincides with a rapidly rising price and substitution of intermediate input for labour. Utility is highest in all cases when the elasticity of substitution is high, the consumer benefiting from the firms being unable to exploit each other's demand for intermediate inputs.

Table 4. $\delta = -0.4$ (Gross Substitutes), Price Discrimination

ρ	p	c	q	X^C	X^F	X	L	Profit	Utility
0.99	16.4	1.007	667.3	1110.5	0	1110.5	2237	739994	2959976
0.6	163.0	1.587	667.7	1110.2	5.1E-3	1110.2	3522	739566	2958271
-1	5429.7	1394.5	1776.0	186.7	63.35	250.0	9337	326875	915801
-10	3075.5	1435.8	1645.7	295.2	258.2	553.4	1072	485359	1845224
-10000	2956.2	1478.5	1652.3	289.7	289.6	579.4	580	478404	1914093

Table 5. $\delta = 0$, Price Discrimination

ρ	p	c	q	X^C	X^F	X	L	Profit	Utility
0.99	16.5	1.007	1000.5	1249.4	0	1249.4	2516	1248741	3746225
0.6	143.5	1.587	1000.8	1249.0	8.0E-3	1249.0	3961	1248018	3744070
-1	7790.1	1991.9	1797.9	252.5	85.45	338.0	15084	446503	1245470
-10	3775.4	1762.4	1601.1	498.6	436.0	934.6	1844	797346	3037723
-10000	3599.3	1800.0	1600.2	499.8	499.7	1000.2	1000	799216	3197698

Table 6. $\delta = 0.4$ **(Gross Complements), Price Discrimination**

ρ	p	c	q	X^C	X^F	X	L	Profit	Utility
0.99	16.4	1.007	1200.4	1999.0	0	1999.0	4026	2397584	6393557
0.6	128.0	1.587	1200.6	1998.4	0.02	1998.4	6337	2396196	6389896
-1	1174.5	311.0	1686.4	784.0	264.5	1048.5	57340	1293412	3696933
-10	4521.7	2110.5	1488.3	1279.2	1118.7	2398.0	4809	1901447	7286843
-10000	4257.1	2128.9	1484.1	1289.7	1289.5	2579.2	2581	1912762	7653282

The results for the equilibrium with price discrimination, but no collusion, are given in Tables 4-6. Trade prices first rise and then fall. It can be seen that trade prices are not always lower than final prices. The strategic importance of the trade price eliminates the incentive to charge a low price in order to receive a lower-priced input in return. Profits are markedly reduced by discrimination and the strategies of the firms have a "beggar thy neighbour" effect. The consumer is obviously better off with extremes of substitutability since this reduces the firms' market power. Labour use around $\rho \to 0$ is always substantially higher in the price discrimination case. This reflects the high levels of producer prices forcing labour to be substituted for intermediate inputs.

Table 7. $\delta = -0.4$ **(Gross Substitutes), Collusion**

ρ	p	c	q	X^C	X^F	X	L	Profit	Utility
0.99	1.01	1.004	667.3	1110.5	1110.5	222.1	2221	740000	2962222
0.6	1.20	1.089	667.4	1110.5	715.9	1826.4	2258	740016	3300465
-1	1586.1	416.7	996.6	836.2	288.1	1124.3	22951	821848	2957422
-10	1047.7	489.7	1000.6	832.9	728.6	1561.4	2742	831962	3317299
-10000	1000.4	500.7	1000.4	833.0	832.8	1665.8	1667	832500	3331666

Table 8. $\delta = 0$ **, Collusion**

ρ	p	c	q	X^C	X^F	X	L	Profit	Utility
All values	1.0	1.0	100.5	1249.4	1249.4	2498.8	2498.8	1248750	3746301

Table 9. $\delta = 0.4$ **(Gross Complements), collusion**

ρ	p	c	q	X^C	X^F	X	L	Profit	Utility
0.99	0.9995	0.9997	1200.4	1999.0	2101.5	4100.5	3998.0	2397601	9672384
0.6	0.9779	0.9888	1200.4	1999.0	2114.5	4113.5	3999.3	2397601	9681681
-1	0.9024	0.9506	1200.39	1999.0	2107.2	4106.2	4003.4	2397604	9676451
-10	0.6585	0.8276	1200.38	1999.0	2084.8	4083.9	4014.2	2397614	9660269
-10000	0.0033	0.5016	1200.33	1999.2	2001.2	4000.3	4000.1	2397666	9596273

The final three tables report the results for the equilibrium with both price discrimination and collusion. The effect of introducing collusion has quite a dramatic effect on the results. In Table 7, with the goods as gross substitutes, trade prices start low and then "leap" to a higher level of price due to a discontinuity at $\rho = -0.05043$. This arises from the two conflicting aims of preferring a low-priced input but at the same time competing on the final goods market. Profits are far higher with collusion than for the other two cases; the expected conclusion. However, what is most surprising is that utility is also higher, so that collusion is in the interest of both the firms and the consumer. This can be explained by the more effective use of intermediate inputs in production and the move away from labour intensive techniques. The explanation for Table 8 lies in the fact that the firms are not interacting at all on the final goods market, and the collusive setting of trade prices is then aimed solely at aiding their independent profit maximization at the final stage. A trade price of 1 indicates that the collusion results in marginal cost pricing of intermediate inputs, hence the only inefficiency in this case is arising through the monopoly pricing on the final market. The final table, involving gross complements and collusion, shows that the effect of the complementarity on the final goods market is to reduce the trade price below marginal cost. As in the previous two tables, collusion is beneficial for both the firms and the consumer.

5. Conclusions

The paper has constructed and analysed a two-sector model of imperfect competition with intermediate goods. The dependence of the results upon the equilibrium concept emphasizes that the form of market organization is of critical importance for the equilibrium that emerges. The small scale of the model has made the equilibrium values highly sensitive to the value of the elasticity of substitution. This is particularly true of the price discrimination model, but it clearly shows that price discrimination may be mutually harmful. Collusion has been seen to be beneficial both to the firms and to the consumer and to lead to prices that fall below marginal cost. Finally, the excessive substitution of labour for produced input reduces the total output of society and represents a welfare loss additional to that typically identified in the analysis of monopoly.

References

Hart, O. D., (1985), "Imperfect Competition in General Equilibrium: An Overview of Recent Work," in *Frontiers of Economics*, K.J. Arrow and S. Honkapohja, eds., Oxford: Basil Blackwell.

Negishi, T., (1961), "Monopolistic Competition and General Equilibrium," *Review of Economic Studies*, 28, 196-201.