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Technology Gap, Efficiency, and a Stochastic Metafrontier Function

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Abstract

This paper considers a stochastic metafrontier function to investigate the technical efficiencies of firms in different groups that may not have the same technology. A decomposition of output is presented involving the technology gap and technical efficiency ratios for firms in a group relative to the best practice in the industry.

Key word:

JEL classification: C5; D24

1. Introduction

The metaproduction function was first introduced by Hayami (1969) and Hayami and Ruttan (1970, 1971). As stated by Hayami and Ruttan (1971, p. 82), "The metaproduction function can be regarded as the envelope of commonly conceived neoclassical production functions." In their discussion of agricultural productivity across various countries, Ruttan et al. (1978, p. 46) state, "We now define the metaproduction function as the envelope of the production points of the most efficient countries." The concept of a metaproduction function is theoretically attractive because it is based on the simple hypothesis that all producers in different groups (countries, regions, etc.) have potential access to the same technology. Following the seminal work of Hayami and Ruttan (1970), Mundlak and Hellinghausen (1982) and Lau and Yotopoulos (1989) employed the approach to compare agricultural productivity across countries.

Some econometric advantages of applying the metaproduction function are discussed by Lau and Yotopoulos (1989), but the lack of comparable data and the presence of inherent differences across groups are the major limitations of the ap-

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proach. Boskin and Lau (1992) used a new framework for the analysis of productivity and technical progress, based on direct econometric estimation of the aggregate metaproduction function.

The concept of a *stochastic metafrontier* function, used in this paper, operationalises the standard metaproduction function approach. The stochastic metafrontier model has an error term that comprises a symmetric random error and a non-negative technical inefficiency term, as in the stochastic frontier production function model, originally proposed by Aigner, Lovell, and Schmidt (1977) and Meeusen and van den Broeck (1977). However, a stochastic metafrontier function may not envelop the separate production frontiers for the different groups involved. It is possible to use *non-stochastic* approaches to construct metafrontier functions.

A stochastic metafrontier model was adopted by Gunaratne and Leung (2001) and Sharma and Leung (2000) in studies of the efficiency of aquaculture farms in several countries. Sharma and Leung (2000) used the Battese and Coelli (1995) model for the technical inefficiency effects in the stochastic metafrontier function in their empirical analysis of data on carp pond culture in South-Asian countries.

2. Stochastic Metafrontier Model

Suppose that the inputs and outputs for firms in a given industry are such that stochastic frontier production function models are defined for different groups within the industry. Our analysis assumes that there are several well-defined *groups* for the same industry, such as different regions within a country, different types of ownership or ethnic groups involved in production, but not for different industries within the same country or among countries. Suppose that, for the jth group, there are sample data on N_j firms that produce one output from the various inputs and the stochastic frontier model for this group is defined by

$$Y_{ij} = f(x_{ij},\beta)\, e^{V_{ij}-U_{ij}}, \quad i = 1, 2, \ldots, N_j, \tag{1}$$

where Y_{ij} denotes the output for the ith firm in the jth group; x_{ij} denotes a vector of functions of the inputs used by the ith firm in the jth group; the $V_{ij}s$ are assumed to be identically and independently distributed as $N(0,\sigma_v^2)$ -random variables, independent of the $U_{ij}s,$ which are defined by the truncation (at zero) of the $N(\mu_{ij},\sigma^2)$-distributions, where the μ_{ij}s are defined by some appropriate inefficiency model, e.g., one of the Battese and Coelli (1992, 1995) models. For simplicity, the subscript j is omitted hereafter, so that the model for the jth group is given by

$$Y_i = f(x_i,\beta)\ e^{V_i-U_i} \equiv e^{x_i\beta+V_i-U_i}\ . \tag{2}$$

The expression of equation (2) assumes that the exponent of the frontier production function is linear in the parameter vector β, so that x_i is a vector of functions of (logarithms of) the inputs for the ith firm involved. The Cobb-Douglas or translog production functions are convenient for the presentation of the decomposition to be presented in Section 3 below.

The stochastic metafrontier model for firms in all groups of the industry is expressed by

$$Y_i = f(x_i, \beta^*) e^{V_i^* - U_i^*} \equiv e^{x_i \beta^* + V_i^* - U_i^*}, \quad i = 1, 2, ..., N, \tag{3}$$

where $N = \sum_{j=1}^{R} N_j$ is the total number of sample firms in all R groups and the assumptions for the V_i^*s and the U_i^*s are analogous to those for the V_is and the U_is, respectively.

If the assumptions for the stochastic frontiers for the different groups associated with equations (1) and (2) are reasonable for given sample data, then the corresponding assumptions associated with the stochastic metafrontier model of equation (3) may not be appropriate (e.g., the V_i^*s may not be identically distributed over all groups). The parameters of a given frontier for a group are estimated using data from firms *in that group*. The parameters of the metafrontier model are estimated using data from firms *in all groups* (in the combined data set).

The metafrontier of equation (3) is considered to be an envelope function of the stochastic frontiers of the different groups such that it is defined by all observations in the different groups in a way that is consistent with the specifications of a stochastic frontier model. Observations on individual firms in the different groups may be greater than the deterministic component of the stochastic frontier model, but deviations from the stochastic frontier outputs are due to inefficiency of the firms in the different groups. The stochastic frontiers for the different groups and that of the metafrontier would generally be assumed to be of the same functional form (e.g., Cobb-Douglas or translog), but there are no problems of aggregation as with the relationship between firm and industry functions.

In the discussion below, the parameters β (for the frontier for the jth group) and β^* (for the metafrontier) are assumed known. The productivity and technical efficiency of firms in the jth group can be investigated using either the frontier for the jth group or the metafrontier.

The maximum-likelihood estimates of the parameters of the stochastic metafrontier model (3) do not necessarily result in the estimated function being an *envelope* of the estimated production frontiers for the different groups. That is, the maximisation of the likelihood function for all observations *does not guarantee* that the estimated metafrontier envelops the estimated frontiers for the different groups. Thus, for some groups, the metafrontier function could have values less than the corresponding frontier for a given group. However, it is possible to constrain the estimation of the metafrontier, such that it is an envelope of observations for efficient firms in all groups. A constrained mathematical programming algorithm, such as in data envelopment analysis (DEA) could be used in the estimation of a metafrontier. However, such non-stochastic frontier methods do not adequately account for the presence of traditional random errors and assume that all deviations from the "frontier" are due to inefficiency. We are working on a possible method of imposing such restrictions in the context of a *stochastic metafrontier* model, but this cannot be included in this paper because it is in the development stage. Clearly, this is an important issue that requires further research. The comparison of technical efficiencies of firms in different groups is a common problem that has been of concern to many researchers.

3. Technology Gap and Efficiency Levels

The observed output for the *i*th firm in the *j*th group can be expressed by $Y_i = e^{x_i \beta + V_i - U_i}$ or $Y_i = e^{x_i \beta^* + V_i^* - U_i^*}$, as specified by equations (2) or (3), respectively, from which it follows that the relationship $x_i \beta + V_i - U_i = x_i \beta^* + V_i^* - U_i^*$ is satisfied.

It is expected that the deterministic values $x_i\beta$ and $x_i \beta^*$ satisfy the inequality $x_i \beta \le x_i \beta^*$ because $x_i \beta^*$ is from the metafrontier. If the metafrontier were estimated to be an envelope function for efficient firms, then the relationship would be satisfied by the estimated functions.

This relationship can be rewritten as

$$1 = \frac{e^{x_i\beta}}{e^{x_i\beta^*}} \cdot \frac{e^{V_i}}{e^{V_i^*}} \cdot \frac{e^{-U_i}}{e^{-U_i^*}} . \tag{4}$$

The three ratios on the right-hand side of this equation are called the *technology gap ratio* (TGR), the *random error ratio* (RER), and the *technical efficiency ratio* (TER), i.e.,

$$TGR_i \equiv \frac{e^{x_i \beta}}{e^{x_i \beta^*}} \equiv e^{-x_i(\beta^* - \beta)}, \; RER_i \equiv \frac{e^{V_i}}{e^{V_i^*}} \equiv e^{V_i - V_i^*}, \text{ and } \; TER_i \equiv \frac{e^{-U_i}}{e^{-U_i^*}} \equiv \frac{TE_i}{TE_i^*} .$$

The technology gap ratio indicates the technology gap for the given group according to currently available technology for firms in that group, relative to the technology available in the whole industry. This ratio and the technical efficiencies (and, hence, the technical efficiency ratio) can be estimated for each individual firm. The technical efficiency of firm *i* relative to the frontier for its group $TE_i \equiv e^{-U_i}$ can be estimated by $\hat{TE} \equiv E(e^{-U_i} | E_i \equiv V_i - U_i)$. The technical efficiency of firm *i* can be estimated relative to the metafrontier by $\hat{TE}_i^* \equiv E(e^{-U_i^*} | E_i^* \equiv V_i^* - U_i^*)$. Clearly, the identity $E_i^* \equiv E_i - x_i(\beta^* - \beta)$ is satisfied.

Consider the technical efficiency ratio $TER_i \equiv TE_i / TE_i^* \equiv e^{-U_i} / e^{-U_i^*}$. Generally this ratio is expected to be greater than or equal to unity. Because U_i and U_i^* are random variables, there is a non-zero probability that the ratio TER_i is less than unity. Now, $TE_i \le TE_i^*$ if and only if $e^{-U_i} \le e^{-U_i^*}$, or $U_i^* - U_i \le 0$. But $U_i^* - U_i$ $\equiv x_i(\beta^* - \beta) + V_i^* - V_i$. The probability that U_i^* is no greater than U_i is

$$P(U_i^* - U_i \le 0) = P(V_i^* - V_i \le -x_i(\beta^* - \beta)) = \Phi(-x(\beta^* - \beta \Big/ \sqrt{\sigma_v^2 + \sigma_{v^*}^2}) ,$$

if V_i and V_i^* are independent normal random errors, where $\Phi(.)$ represents the distribution function for the standard normal distribution. Clearly, the greater $x_i\beta^*$ exceeds $x_i\beta$, the smaller the probability that U_i^* is less than U_i .

Further, it can be shown that

$$E\left(Y_i \middle| x_i = \bar{x} \right) = e^{\overline{x\beta}} E\left(e^{V_i}\right) E\left(e^{-U_i}\right) = e^{\bar{x}\beta} e^{\frac{1}{2}\sigma_v^2} \left\{ e^{-\mu + \frac{1}{2}\sigma^2} \Phi\left(\frac{\mu}{\sigma} - \sigma\right) \Big/ \Phi\left(\frac{\mu}{\sigma}\right) \right\} .$$

Another identity relationship, based on the expected output under the particular group frontier and the metafrontier, is derived as follows:

$$\frac{e^{\bar{x}\beta}}{e^{\bar{x}\beta^*}} \cdot \frac{e^{\frac{1}{2}\sigma_v^2}}{e^{\frac{1}{2}\sigma_{v^*}^2}} \cdot \frac{E\left(e^{-U_i}\right)}{E\left(e^{-U_i^*}\right)} , \tag{5}$$

where $TGR \equiv \dfrac{e^{\bar{x}\beta}}{e^{\bar{x}\beta^*}} = e^{-\bar{x}(\beta^* - \beta)}$ is the mean technology gap ratio;

$RER \equiv \dfrac{e^{\frac{1}{2}\sigma_v^2}}{e^{\frac{1}{2}\sigma_{v^*}^2}}$ is the mean random error ratio; and

$TER \equiv \dfrac{E\left(e^{-U_i}\right)}{E\left(e^{-U_i^*}\right)}$ is the mean technical efficiency ratio.

Clearly, from equation (5), only two of these ratios need to be independently estimated in any empirical application.

4. Conclusions

With the main objective of providing comparable technical efficiency scores for firms across groups, the technical efficiencies of firms can be estimated using a stochastic metafrontier model. In addition, we present a more transparent analysis of the technology gap of different groups and their efficiency levels by using a decomposition result.

The mean technology gap, random error, and technical efficiency ratios give additional explanation compared with the analysis based only on stochastic frontier functions for the different groups. The technology gap ratio plays an important part in explaining the ability of the firms in one group to compete with other firms from different groups within the industry. This ratio provides an estimate of the technology gap between the groups and the industry as a whole.

The analysis of technical efficiency using a stochastic metafrontier model also gives a better overview of the comparability of technical efficiency scores across groups. How technical inefficiency changes over time is obviously associated with the model that is assumed for the inefficiency effects. Empirical analyses with alternative stochastic frontier models are clearly desirable. Battese, Rao, and Walujadi (2001) apply the methods of this paper to investigate the technology gap and technical efficiencies of firms in the garment industry in different regions of Indonesia. Further theoretical research is desirable for the estimation of stochastic metafrontier models.

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