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Technological Diffusion in the Ramsey Model

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Abstract

This paper introduces technological diffusion in the neoclassical growth model. A log-linearized approximation of the model is solved analytically. Due to the presence of two negative eigenvalues, the model's dynamics are richer than the dynamics of the basic neoclassical growth model.

Key words: convergence; diffusion of technology; neoclassical growth

JEL classification: O4

1. Introduction

In the theory of economic growth, there exist two main sources of convergence: diminishing returns to capital and technological diffusion. Diminishing returns to capital apply in neoclassical growth models [see, for example, Barro and Sala-i-Martin (1995), chapters 1 and 2]. The present paper combines the neoclassical growth framework with the diffusion of technology from the world's leading country. An important observation is that the given framework can be solved analytically in its log-linear approximation. As expected, the presence of technological diffusion leads to a richer convergence pattern in the neoclassical growth model.

The literature on technology diffusion goes back to Nelson and Phelps (1966), who develop a simple model in which the speed of technological diffusion depends positively on the technological gap between leading and following countries. Using the idea of Nelson and Phelps, Findlay (1978a) proposes a formal model in which the speed of technological transfer between advanced and backward regions is treated. Barro and Sala-i-Martin (1997) provide microeconomic foundations for the Nelson and Phelps model. Barro and Sala-i-Martin's framework exhibits convergence because imitations are assumed to be cheaper than original innovations in technologically leading economies. Barro and Sala-i-Martin's contribution justifies the differential equation for technology in the present model.

Other studies examining technological diffusion include Baranson (1970),

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Mansfield (1975), Findlay (1978b), and Wang and Blomström (1992). Baranson (1970) discusses the role of international firms in the cross-country technological transfer; his study focuses on the transfer logistics and the conflicts of interest in licensing versus investment decisions. Mansfield (1975) addresses the importance of resource costs in the technological transfer and briefly examines the case of the U.S.-U.S.S.R. transfer of technology. In general, the benefits of technological transfer include factor-cost reductions. The costs of transfer involve design and adaptation costs. Findlay (1978b) proposes a simple framework in which the optimal technological transfer is achieved at a point which maximizes the difference between the benefits and the costs. Wang and Blomström (1992) analyze a model of international technology transfer via foreign direct investment. In their paper, the technology transfer is discussed as an endogenous phenomenon which results from the strategic interaction between subsidiaries of multinational corporations and host country firms.

The present paper introduces technological diffusion in an *exogenous* growth model. Exogenous growth models exhibit a convergence pattern, while the simplest endogenous growth model (the AK model) does not exhibit convergence. Therefore, the dynamics of an exogenous growth model with technological diffusion are richer than the dynamics of the simple endogenous growth model with technological diffusion. Thus we focus on an exogenous rather than endogenous framework.

The model's dynamics involve two negative eigenvalues. Elsewhere [Duczynski (2002)] I show that an open-economy model with two types of capital and different adjustment costs for investment exhibits a similar pattern. I make the same observation in Duczynski (2003), where I study an open-economy model with one type of capital, technological diffusion, and adjustment costs. It is plausible that these models with two negative eigenvalues provide a better description of reality than standard models with one negative eigenvalue. The evolution of capital, output, and consumption in the real world may be much more complicated than the predicted behavior in standard growth models. Of course, a rigorous empirical justification of models with two negative eigenvalues is left for future research.

2. The Model

This section considers an extension of the Ramsey-Cass-Koopmans neoclassical growth model [for the model, see, for example, Barro and Sala-i-Martin (1995), chapter 2)]. Let the aggregate production be given by the Cobb-Douglas production function

$$Y = K^{\alpha}(AL)^{1-\alpha}, \tag{1}$$

where Y is output, K is capital, A is a technological parameter, L is labor, and α is the capital share satisfying $0 < \alpha < 1$. Labor is assumed to grow at a constant rate, n. The evolution of the technological parameter is given by

$$x \equiv \frac{\dot{A}}{A} = g + \lambda \, \frac{\tau \, A_L - A}{A} = g + \lambda \, (1/\alpha - 1) \, . \tag{2}$$

where A_L is the level of technology in the world's technological leader, $a = A/(\tau \, A_L) \leq 1$, and g, λ, and τ are positive parameters. The technological dynamics are assumed to be influenced by two sets of forces. The term g in (2) reflects domestic forces of technological innovations (e.g., domestic R&D). Similar to the standard neoclassical growth model, this term is given exogenously. The term $\lambda \, (\tau \, A_L - A)/A$ relates to the diffusion of technology from the leading country. [See Nelson and Phelps (1966) for a similar expression. The present term differs from Nelson and Phelps in parameter τ and in abstracting from the role of human capital in technological diffusion. Nelson and Phelps claim that λ should depend positively on human-capital intensity.] Thus, the speed of technological diffusion is faster the further the level of technology is below its long-run target, $\tau \, A_L$. The speed of technological transfer is proportionate to λ, which is an exogenously given parameter. Parameter τ is assumed to be less than 1, which captures the fact that the economy is converging to a lower steady state than the level of the leading country.

There exist two principal reasons why the transfer of technology depends positively on the technological gap. First, technologically backward economies have a large number of products which can be imitated. Second, a lower quality of existing products in backward economies leads to a higher potential for technological jumps on the quality ladder. The equation of motion for A_L is assumed to take the form:

$$\frac{\dot{A}_L}{A_L} = g \, . \tag{3}$$

Thus, for simplicity, the technological change in the leading country is assumed to be the same as the long-run technological growth rate of the converging country. Assuming different growth rates would probably substantially complicate the analysis. Because of the absence of externalities, the decentralized outcome in the present model coincides with the solution to the planner's problem. The planner's problem is to find

$$\max_{C} \int_0^{\infty} \frac{(C/L)^{1-\theta} - 1}{1-\theta} e^{(n-\rho)t} dt \, ,$$

subject to

$$\dot{K} = K^{\alpha} \, (AL)^{1-\alpha} - C - \delta \, K \, , \tag{4}$$

where C is aggregate consumption, θ is the inverse elasticity of intertemporal substitution, ρ is the rate of time preference, and δ is the depreciation rate of capital. The present-value Hamiltonian for this problem is

$$\mathcal{H} = \frac{(C/L)^{1-\theta} - 1}{1-\theta} e^{(n-\rho)t} + \mu \left[K^{\alpha} (AL)^{1-\alpha} - C - \delta K \right],$$

where μ is a co-state variable indicating the marginal shadow value of K. The first-order conditions are

$$C^{-\theta} L^{\theta-1} e^{(n-\rho)t} = \mu, \tag{5}$$

$$\dot{\mu} = -\frac{\partial \mathcal{H}}{\partial K} = -\mu \alpha K^{\alpha-1} (AL)^{1-\alpha} + \delta \mu. \tag{6}$$

From this it follows that

$$\frac{\dot{C}}{C} = \frac{\alpha K^{\alpha-1} (AL)^{1-\alpha} - \delta - \rho}{\theta} + n . \tag{7}$$

The transversality condition is

$$\lim_{t \to \infty} \mu K = 0. \tag{8}$$

It is useful to rewrite the equations of motion in terms of variables per effective worker, $\tilde{k} = K/(AL)$, and $\tilde{c} = C/(AL)$:

$$\dot{\tilde{k}} = \tilde{k}^{\alpha} - \tilde{c} - (\delta + n + x)\tilde{k}, \tag{9}$$

$$\dot{\tilde{c}} = \frac{\alpha \tilde{k}^{\alpha-1} - \delta - \rho}{\theta} \tilde{c} - x\tilde{c}. \tag{10}$$

In a steady state, $a^* = 1$, $x^* = g$, and

$$\alpha \tilde{k}^{*\,\alpha-1} = \delta + \rho + \theta g, \tag{11}$$

$$\frac{\tilde{c}^*}{\tilde{k}^*} = \tilde{k}^{*\,\alpha-1} - (\delta + n + g), \tag{12}$$

where asterisks denote steady-state values. The equations of motion for $\tilde{k}$, $\tilde{c}$, and a can be log-linearized around the steady state:

$$\begin{pmatrix} d\ln(\tilde{k}/\tilde{k}^*)/dt \\ d\ln(\tilde{c}/\tilde{c}^*)/dt \\ d\ln a/dt \end{pmatrix} = \begin{pmatrix} \mathcal{A} & \mathcal{B} & \lambda \\ \mathcal{C} & 0 & \lambda \\ 0 & 0 & -\lambda \end{pmatrix} \begin{pmatrix} \ln(\tilde{k}/\tilde{k}^*) \\ \ln(\tilde{c}/\tilde{c}^*) \\ \ln a \end{pmatrix}, \tag{13}$$

where

$$\mathcal{A} = \alpha \tilde{k}^{*\,\alpha-1} - (\delta + n + g) = \rho - n + \theta g - g > 0, \tag{14}$$

$$\mathcal{B} = \delta + n + g - \tilde{k}^{*\,\alpha-1} = \delta + n + g - \frac{\delta + \rho + \theta g}{\alpha} < 0, \tag{15}$$

$$\mathcal{C} = \frac{(\alpha - 1)(\delta + \rho + \theta g)}{\theta} < 0. \tag{16}$$

The positive sign of $\mathcal{A}$ follows from the transversality condition. The negative sign of $\mathcal{B}$ results from (12). The Jacobian matrix has two negative eigenvalues: $-\lambda$ and $-\omega$, where

$$\omega = \frac{\sqrt{\mathcal{A}^2 + 4\mathcal{B}\mathcal{C}} - \mathcal{A}}{2}. \tag{17}$$

The positive eigenvalue of the Jacobian matrix is excluded from the analysis to ensure the validity of the transversality condition. The negative eigenvalues are analogous to the convergence coefficient in standard growth models. The first negative eigenvalue corresponds to the speed of technological diffusion. The second negative eigenvalue is identical with the eigenvalue for the basic Ramsey model [see Barro and Sala-i-Martin (1995), chapter 2]. Each eigenvalue corresponds to one source of convergence. Thus the solution to the log-linearized system takes the form

$$\ln(\tilde{k}/\tilde{k}^*) = v_{\lambda\tilde{k}} e^{-\lambda t} + v_{\omega\tilde{k}} e^{-\omega t}, \tag{18}$$

$$\ln(\tilde{c}/\tilde{c}^*) = v_{\lambda\tilde{c}} e^{-\lambda t} + v_{\omega\tilde{c}} e^{-\omega t}, \tag{19}$$

$$\ln a = v_{\lambda a} e^{-\lambda t} + v_{\omega a} e^{-\omega t}. \tag{20}$$

Coefficients v are given by initial conditions for $\tilde{k}$ and a and by components of eigenvectors corresponding to eigenvalues $-\lambda$ and $-\omega$:

$$v_{\lambda\tilde{k}} = -\frac{\lambda \ln a(0)}{\mathcal{C} + \lambda \dfrac{\mathcal{A} + \lambda - \mathcal{C}}{\lambda - \mathcal{B}}}, \tag{21}$$

$$v_{\lambda\tilde{c}} = -\frac{\lambda \ln a(0)}{\dfrac{\mathcal{C}(\lambda - \mathcal{B})}{\mathcal{A} + \lambda - \mathcal{C}} + \lambda}, \tag{22}$$

$$v_{\lambda a} = \ln a(0), \tag{23}$$

$$v_{\omega\tilde{k}} = \ln\left[\tilde{k}(0)/\tilde{k}^*\right] + \frac{\lambda \ln a(0)}{\mathcal{C} + \lambda \dfrac{\mathcal{A} + \lambda - \mathcal{C}}{\lambda - \mathcal{B}}}, \tag{24}$$

$$v_{\omega\tilde{c}} = -\frac{\mathcal{C}}{\omega}\left\{\ln\left[\tilde{k}(0)/\tilde{k}^*\right] + \frac{\lambda \ln a(0)}{\mathcal{C} + \lambda \dfrac{\mathcal{A} + \lambda - \mathcal{C}}{\lambda - \mathcal{B}}}\right\}, \tag{25}$$

$$v_{\omega a} = 0. \tag{26}$$

The policy function is

$$\ln(\tilde{c}/\tilde{c}^*) = \frac{\mathcal{C} - \lambda - \mathcal{A} + \mathcal{C}(\mathcal{B} - \lambda)/\omega}{\lambda^2 + \lambda\mathcal{A} - \mathcal{B}\mathcal{C}} \lambda \ln a - \frac{\mathcal{C}}{\omega} \ln(\tilde{k}/\tilde{k}^*). \tag{27}$$

Thus, consumption per effective worker depends positively on capital per effective worker. Output per capita, y, satisfies

$$y = A\tilde{k}^{\alpha}. \tag{28}$$

The steady-state output per capita, y^*, satisfies

$$y^* = A_L \tau \tilde{k}^{*\alpha}. \tag{29}$$

The equation of motion for y is

$$\ln(y/y^*) = \left[\ln a(0) + \alpha v_{\lambda\tilde{k}}\right] e^{-\lambda t} + \alpha v_{\omega\tilde{k}} e^{-\omega t}. \tag{30}$$

The behavior of output (relative to its steady-state level) may be non-monotonic over time. This situation applies, for example, if $g = 0.02$, $\delta = 0.05$, $\rho = 0.02$, $\theta = 2$, $\alpha = 0.3$, $\lambda = 0.01$, $a(0) = 0.5$, $n = 0.01$ and $\tilde{k}(0)/\tilde{k}^* = 1.2$. In this calibration, $\omega = 0.09$. Output decreases initially and increases asymptotically. This is a significant difference from the standard Ramsey model, where the behavior of output is monotonic. In the present calibration, the initial fall in output is connected with the fact that capital is initially high relative to the initial level of technology. It is optimal to dissave (there are low returns on capital), and this effect is strong if the capital share is sufficiently low. In this setup, the Ramsey effect of diminishing returns works initially in the opposite direction than the effect of technological diffusion.

The steady-state growth rate of output per capita is g. If an economy is below its steady state, it should grow faster than at the rate of g. The speed of convergence for output, β, reflects the tendency of the economy to grow faster the further its per capita output is below the steady state. Consequently, the speed of convergence is defined by

$$\beta \ln\left(\frac{y}{y^*}\right) = g - \dot{y}/y = \lambda \ln a - \alpha \dot{\tilde{k}}/\tilde{k}. \tag{31}$$

Thus,

$$\beta = \frac{\lambda \ln a + \alpha \lambda v_{\lambda\tilde{k}} e^{-\lambda t} + \omega \alpha v_{\omega\tilde{k}} e^{-\omega t}}{\ln a + \alpha \ln(\tilde{k}/\tilde{k}^*)}. \tag{32}$$

After substitution for $\ln a$ and $\ln(\tilde{k}/\tilde{k}^*)$, we obtain

$$\beta = \frac{\lambda (v_{\lambda a} + \alpha v_{\lambda \tilde{k}}) e^{-\lambda t} + \omega \alpha v_{\omega \tilde{k}} e^{-\omega t}}{(v_{\lambda a} + \alpha v_{\lambda \tilde{k}}) e^{-\lambda t} + \alpha v_{\omega \tilde{k}} e^{-\omega t}}. \tag{33}$$

Thus the speed of convergence is a weighted average of ω and λ. Asymptotically, β tends to the minimum of ω and λ. The initial value of β depends on initial values of two state variables (capital and technology). The value of β cannot be expressed in terms of only one state variable (such as the output level).

3. Conclusion

The Ramsey growth model is a good approximation of the real world; nevertheless, it describes the economic reality incompletely. It abstracts from technological diffusion. Similarly, models with pure technological diffusion cannot completely describe reality; among other things, they typically do not consider the convergence effect arising from diminishing returns to capital. The present paper combines these two important phenomena of the real world. One of the main merits of the paper is in showing that the Ramsey model with technological diffusion is tractable. The model can be solved analytically in its log-linear approximation, and its dynamics are richer than the dynamics of the standard Ramsey model.

The model belongs to the class of models with two negative eigenvalues. In these models, the speed of convergence is a weighted average of the negative eigenvalues. Systems with two negative eigenvalues appear to be better approximations of the real world than systems with one negative eigenvalue.

An interesting aspect of the present model is the possibility of non-monotonic behavior of output relative its steady-state level if the initial level of capital is high and the level of technology is low. This feature constitutes a significant difference from the standard Ramsey model.

The present model tries to demonstrate that the time evolution of capital, output, and consumption in the real world may be a complicated process; more complicated than predicted by existing economic models. On the other hand, the time evolution of technology is assumed to be very simple—it is exogenously given. At this point, the model departs from the economic reality. In the real world, the evolution of technology is definitely a complex phenomenon. Endogenizing the evolution of technology in the present setup is left for future research.

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