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A Note on Environmental R&D under Time-Consistent Emission Tax

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1. Motivation

In a recent publication in *Journal of Economic Behavior and Organization*, Poyago-Theotoky (2007) developed a three-stage game model and also derived theoretical findings and important policy implications for environmental R&D under a time-consistent emission tax. Among the conclusions presented in that paper, it was stated that with inefficient environmental R&D technology and small environmental damage, cooperative environmental R&D engenders larger environmental R&D efforts and greater social welfare than non-cooperative environmental R&D does. This note describes that the results of Poyago-Theotoky's (2007, 2010) works are still robust in a relaxed wider parameter range of the environmental damage coefficient. Furthermore, we provide the generalized sufficient condition of damage coefficient to guarantee an interior solution for R&D in an extended framework.

The characteristics and time structure of the model are as follows. For details of the model and the solution procedure, see Poyago-Theotoky (2007).

(i) Two homogeneous firms (i and j) are engaging in quantity competition.
(ii) Linear demand is $p(q_i, q_j) = a - (q_i + q_j)$, ($i, j = 1, 2$; $i \neq j$).

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(iii) They have the same cost structure of $C_i(q_i) = cq_i$.
(iv) They have the same emissions-reducing technology.
(v) Spillover effects ($\beta \in [0, 1]$) occur from the rival firm's environmental R&D.
(vi) The emission function is $e_i(q_i, z_i) = q_i - z_i - \beta z_j$.
(vii) R&D expenditures are $(\gamma/2)z_i^2$.
(viii) The environmental damage is $D(E) = (d/2)E^2$, where $E \equiv e_i + e_j$.
(ix) The regulator is unable to pre-commit to an emission tax rate.

Timeline is as follows:
Stage 1: Firms determine environmental R&D efforts (abatement levels).
Stage 2: The regulator determines the emission tax rate.
Stage 3: Firms determine production levels.

Furthermore, Poyago-Theotoky (2010) provides a corrigendum, showing that the negative emission tax (emission subsidy) can improve the market inefficiency caused by Cournot duopolists. These works, especially welfare comparison under two regimes of cooperative R&D and non-cooperative R&D, constitute significant contributions for competition policy.

However, an unnecessarily strict assumption related to the environmental damage coefficient has remained. The purpose of this note is to point out that oversight. In fact, Poyago-Theotoky (2010) provides the correction: "... To guarantee an interior solution for R&D and a positive emission tax, we will assume that $d > 1/2$ ...," where d captures environmental damage. Instead, it should be read as "… To guarantee an interior solution for R&D, we will assume that $d > 1/2$ ...."

2. Investigation and Result

The reason of our note is described below. Indeed the assumption $d > 1/2$ guarantees an interior solution for R&D, but it is too strict. From equations (4) and (9) in Poyago-Theotoky (2007), the equilibrium values of environmental R&D efforts in two R&D regimes (non-cooperative R&D and cooperative R&D) are, respectively:

$$z_{nc} = \frac{[(1+d)(2d-1) + d(1+\beta)]A}{2\gamma(1+d)^2 + d(1+\beta)[3(3+\beta) + d(7+\beta)]},$$

$$z_{erc} = \frac{(1+\beta)[(1+d)(2d-1) + 2d]A}{2\gamma(1+d)^2 + 4d(3+2d)(1+\beta)^2},$$

where γ and $A(\equiv a - c)$ are both positive parameters. As shown in footnotes 10 and 13 in Poyago-Theotoky (2007), the assumption related to the environmental damage coefficient ($d > 1/2$) is sufficient to ensure that z_{nc} and z_{erc} are both positive.

However, careful investigation reveals that even if $d > (-1 + \sqrt{3})/2$, then z_{nc} and z_{erc} are simultaneously positive irrespective of the value of spillover parameter $\beta(\in [0, 1])$. That analytical process is as follows. First, we focus on verifying the sign of each numerator of z_{nc} and z_{erc} because the sign of each denominator is always positive. The polynomial in square brackets in each numerator is a quadric expression in d . Now, with respect to those quadratic polynomials, we define:

$$d_{nc} \equiv \{d \mid (1+d)(2d-1)+d(1+\beta)=0,\, d>0\},$$
$$d_{erc} \equiv \{d \mid (1+d)(2d-1)+2d=0,\, d>0\}.$$

It is straightforward to verify that if $d > d_{erc} = (-3+\sqrt{17})/4$, then $z_{erc} > 0$. On the other hand, the value of d_{nc} is strictly decreasing in $\beta \in [0,1]$. Hence, if $d > d_{nc}\big|_{\beta=0} = (-1+\sqrt{3})/2 \,(> d_{erc})$, then $z_{nc} > 0$ and $z_{erc} > 0$ for all $\beta \in [0,1]$. Therefore, we should relax the assumption of $d > 1/2$ to the following correct assumption.

Assumption (symmetric duopoly): $d > (-1+\sqrt{3})/2$.

The results of Poyago-Theotoky (2007, 2010) remain robust in this relaxed wider parameter range. Accordingly, this relaxation of damage parameter indicates that previous arguments are still valid in a regulatory environment of the lower environmental damage.

3. An Extension

Our note is necessary for related future research. As an example, let us consider the case of product differentiation between the duopolists' products. Presuming that the inverse demand function is $p_i(q_i, q_j) = a - (q_i + \theta q_j)$, ($i,\ j = 1, 2$; $i \neq j$). The degree of product differentiation is captured by $\theta \in [0,1]$. When $\theta = 1$ ($\theta = 0$), the products are perfect substitutes (independent). This linear demand is used by Spence (1976) and Dixit (1979). The rest of the model is identical. Then, the equilibrium values of environmental R&D efforts under two R&D regimes (non-cooperative R&D and cooperative R&D) are derived as follows:

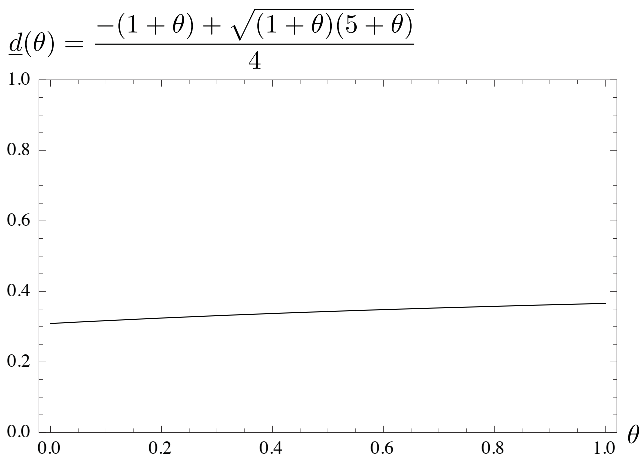
$$\hat{z}_{nc} = \frac{[(2d+1+\theta)(2d-1)+2d(1+\beta)]A}{\gamma(2d+1+\theta)^2 + d(1+\beta)[(2d+1+\theta)(2+\theta)(3+\beta)-4d(1+\beta)]},$$
$$\hat{z}_{erc} = \frac{(1+\beta)[(2d+1+\theta)(2d-1)+4d]A}{\gamma(2d+1+\theta)^2 + d(1+\beta)^2[(2+\theta)(2d+1+\theta)-2d]}.$$

The next step is to derive the condition to guarantee that $\hat{z}_{nc} > 0$ and $\hat{z}_{erc} > 0$. After some manipulation, we understand that, in the case of a differentiated Cournot duopoly, the interior solution of R&D stage is guaranteed by the following assumption.

Assumption (differentiated duopoly): $d > \underline{d}(\theta) \equiv [-(1+\theta)+\sqrt{(1+\theta)(5+\theta)}]/4$.

The critical value $\underline{d}(\theta)$ is the increasing function in $\theta \in [0,1]$. Figure 1 shows the function $\underline{d}(\theta)$.

Figure 1. The Critical Value of Environmental Damage Coefficient



Therefore, it is apparent that when $\theta \to 0$, the parameter set we must analyze expands. This result states that the regulatory environment we must examine widens as the degree of product differentiation becomes larger.

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