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Intra Industry Trade and Product Quality: China and Eight Developing Countries

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Abstract

This study aims to measure the export product quality improvement for trade between China and eight selected developing countries: Argentina, Brazil, Chile, Egypt, South Africa, Mexico, Turkey, and the Russian Federation based on disaggregated four-digit SITC data for the two periods 2000–2004 and 2004–2008. Vertical intra-industry trade high (VIITH) is considered as high quality products. The results revealed that the highest benefits of intra-industry trade with China are for South Africa in the first period and for Egypt in the second period. The highest positive growth of high quality products during the two periods is reported for Russia in the first period and for Brazil and Argentina in the second period. In conclusion, there is a massive potential for economic cooperation between China and these three partners.

Key words: intra industry trade; product quality value; China

JEL classification: F10; L15

1. Introduction

In the trade literature, product quality is mostly discussed in relation to the concept of intra-industry trade (IIT). IIT refers to the exporting and importing of products belonging to the same industry or the same types of goods and services, such as trade in automobiles, foodstuffs, computers, and minerals. IIT can be divided into two categories: horizontal intra-industry trade (HIIT) refers to different varieties of a specific product with similar quality and at the same stage of processing and vertical intra-industry trade (VIIT) is driven by differences in skill content or at different stages of processing. In VIIT, where goods are differentiated by quality, high quality goods require more high skilled labor as compared to low quality goods. Product quality is closely associated to vertical differentiation since

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the latter is defined as the behavior of firms producing goods of different quality (Shaked and Sutton, 1987).

The aim of this study is to measure product quality improvement in trade between China and selected developing countries for the periods 2000–2004 and 2004–2008. Using the methodology of Azhar and Elliott (2003, 2006), we calculate the S index to measure the dynamic IIT and the product quality value (PQV) index. We split VIIT and HIIT for every product that has an S index between \pm 0.4 to reveal the variability in vertical and horizontal IIT. Azhar and Elliot (2003, 2006) used the PQV index to differentiate whether the product is high or low in VIIT. High VIIT represents the home country exporting high quality products, while low VIIT indicates that the home country is importing high quality products.

This study focuses on China's trade with eight newly industrialized countries (NICs) that have a similar level of development: Argentina, Brazil, Chile, Egypt, South Africa, Mexico, Turkey, and Russia. The NICs have higher trade costs compared to other developing countries since they are countries with economies that have not yet reached First World status but that have, in a macroeconomic sense, outpaced their developing counterparts. China is the largest country in the world with 19.2% of the world population in 2011. It was also the world's second largest exporter of merchandise in 2011. With the increase in volume of high quality products in the world trade market, developing countries such as China have begun to increase their trade in this sector. During the past decade, trade between China and other developing countries has grown rapidly. Developing countries are significant trade partners and the annual growth rates of China's bilateral trade are increasing more quickly than that of China's total trade. IIT in manufacturing plays an important role in China's foreign trade. This study contributes to the trade quality literature since, while some research has examined the quality of exports from China, empirical tests of the recent graphical method of PQV indexes remains an untouched area. This study uses an empirical examination that represents the improvements of export product quality of China and developing countries during the period of study that is important to the foundation of its trade quality. There is massive potential for wider economic cooperation between China and its selected developing partners. The existing scope of high quality IIT between China and developing countries show the remarkable power of China's exports, and it is worthwhile for China's economy to focus more on exporting high quality products, particularly in manufacturing.

2. Literature Review

Initial works on IIT measurement focused on the degrees and patterns of IIT. The first measurement was proposed by Balassa (1966), who suggested using the amount to which exports of a given good are offset by imports of an equivalent good. He proposed an index with a value of one for IIT and zero otherwise. Subsequent studies determined the extent of IIT using an improved method based on Balassa's index, established by Grubel and Lioyd (1975). Their study used an index that

Mastooreh Eshraghi and Normaz Wana Ismail

clearly shows IIT as the fraction of trade that is not inter-industry. If there is no IIT the index is zero, and if all trade involves IIT the index is one. Newer models introduced by Dixit and Stiglitz (1977) and Lancaster (1979) offer formal analysis on IIT through alternative representations of horizontal differentiation. Falvey (1981) established a model of product quality in a multi-product industry with a restricted output range that is obtained from a specific kind of capital. This study found that IIT is a natural result of this structure, with no requirement of increasing returns to scale or an imperfect competitive market.

Helpman and Krugman (1987) proposed the Chamberlin-Heckscher-Ohlin (CHO) model, which included factor endowment, falling costs, and horizontal product differentiation to examine both intra- and inter-industry trade. Falvey and Kierzkowski (1987) expanded a model for north-south trade based on vertical product differentiation, which produced rich patterns of trade dynamics. A similar model by Flam and Helpman (1987) illustrated inter-temporal changes in intra-industry and inter-sectoral trade. IIT occurs since consumers with different incomes demand products of different qualities since domestic products may not offer choice variability in qualities. Based on the differences in technology and income distribution, these IIT models showed that the south exports low quality and low cost varieties, whereas the north exports high quality and high cost varieties.

In Abd-el-Rahman (1991), microeconomic and macroeconomic determinants are mixed to analyze foreign trade through the presence of a firm's individual performances. He concluded that there are three kinds of foreign trade: one-way trade, intra-range or vertical differentiation trade, and two-way trade in similar commodities or horizontal differentiation trade, either bilateral or triangular. Since then, there has been a large increase in empirical research on splitting IIT flows into HIIT and VIIT. Greenaway et al. (1994) developed a model based on the methodology of Abd-el-Rahman (1991) while another model was established by Fontagne and Freudenberg (1997) based on an earlier approach from Abd-el-Rahman (1986). Both models use the ratio of export to import unit values to show quality differences. The two models differ in defining IIT and the degree of trade overlap that is required for an organization involved in IIT.

Greenaway and Hine (1991) reported that the link between IIT and adjustment costs could not be supported with conclusive empirical evidence. They compared three proxy measures of adjustment costs and considered the significance of different IIT formations by measuring VIIT and marginal intra-industry trade (MIIT). Fontagne et al. (1997) explored the theory of monopolistic competition and increasing returns in IIT between similar countries. They proposed that comparative advantage is useful for countries that are divided by high economic distance, different factor endowments, or technology levels.

Later, Azhar and Elliott (2003) introduced a method of measuring the trade induced adjustment. They extended an instrument and an index to show the changes in trade patterns, which can be used to analyze time series or cross sectional data for bilateral trade flows. Azhar and Elliott (2006) summarized the previous method in differentiating trade flows between HIIT and VIIT, compared the model with

Fontagne et al. (1997) and Greenaway et al. (1994), and proposed a third method with a PQV index based on the Grubel and Lloyd index. Applying the PQV index, the extent of quality differences at the product level associated with various bilateral trade relationships are quantified for measuring product quality changes. Complementing Greenaway et al. (1994) and Fontagne et al. (1997), their geometric instrument represents the range VIIT and HIIT in a diagram and shows import and export unit values as Cartesian coordinates. Azhar and Elliott (2008) formulated the marginal quality (MQ) index to measure the quality changes in matched trade changes that support the dynamic measures of MIIT or volume based IIT. Azhar et al. (2008) reviewed the main distinguishing factor of the three existence approaches and made a systematic comparison of them using data for China trade with its East Asian neighbors. They determined that the main difference in using Azhar's method is that it classifies more intra-industry trade as horizontal intra-industry trade. Azhar and Elliott (2011) further expanded their S and MQ indexes by introducing a new VQ index which combined changes in the volume and quality in IIT.

3. Methodology

Our data is obtained from the UN COMTRADE database, which gives imports and exports in quantities and values for the 4-digit Standard International Trade Classification (SITC 5-8). To separate HIIT and VIIT there are three approaches: Greenaway et al. (1994), Fontagne et al. (1997), and Azhar and Elliott (2003, 2006). This study is based on the methodology of Azhar and Elliott (2003, 2006) to show the extent of dynamic changes of intra-industry trade in China's trade flows and its quality improvements that were evident over the time period of study. The justification for choosing this method relates back to the scaling problem or disproportionate scaling of the unit value ratio in both the Greenaway et al. (1994) and Fontagne et al. (1997) approaches (Appendix A). We use a two-stage approach to measure product quality changes and analyze quality differentiated trade patterns for various type of goods, during two calendar periods and in many types of industries, to measure the share of product quality in VIIT and HIIT. In the first stage, the S index is used to measure dynamic IIT or to measure products that may have experienced a large increase or decrease in matched trade during the time of analysis. The S index is calculated using the formula:

$$\mathbf{S} = \frac{1}{2\mathbf{L}} (\Delta \mathbf{X} - \Delta \mathbf{M}) = \frac{\Delta \mathbf{X} - \Delta \mathbf{M}}{2 \left(\max\{ |\Delta X|t, |\Delta M|t \} \right)}, \text{ for } t \in N = 1, ..., n,$$
(1)

where L is the largest change in exports (X) and imports (M) over the study period and $-1 \le S \le 1$. However, based on Devadason (2008), L is the largest change in exports and imports over the period studied, and for inferring about adjustments posed by matched trade, products with little IIT change and those that represent inter-industry trade are removed. Therefore the S index values are restricted to $-0.4 \le S \le 0.4$. An S index of 0 means X and M are exactly matched. At the extremes, X

Mastooreh Eshraghi and Normaz Wana Ismail

63

and M move in opposite directions in favor of the exporter or home country with S indices of +0.4 or -0.4, respectively. Thus, S=0 means there is a similar quality of exports and imports and there is no benefit or loss in trade. A positive S index means that the quality of exports is higher than the quality of imports and the benefit of trade is for the exporter, while the negative S index means that the quality of the import is greater than the quality of export and so the benefit of trade is for the single that the quality of export and so the benefit of trade is for the importer. Although the S index shows some information about the direct benefits of trade in bilateral trade, the results of the PQV index are more suitable for use by policymakers or economists.

In the second stage, we use the restricted S index to calculate the PQV index. The PQV index that is adopted to differentiate horizontal and vertical intra-industry trade high and vertical intra-industry trade low and represents a measure of the dispersion of product quality in IIT flows. The PQV is given as:

$$PQV=1+\frac{\left(UV^{X}-UV^{M}\right)}{\left(UV^{X}+UV^{M}\right)} \text{ with } 0 < PQV < 2,$$
(2)

where unit value (UV) is a proxy for quality and is calculated as the ratio of total value to total volume of traded commodities. Price plays a role in this measure through factor price (which indicates quality) and trade costs. This approach is rooted in the belief that higher quality goods sell at higher prices and thus higher unit price denotes higher quality. Hallak (2006) found a positive relationship between price and quality. In Greenaway et al. (1995), Schott (2004), and Hummels and Skiba (2004) also recognized that unit value is positively related to vertical differentiation, justifying the use of unit values as an indicator of quality.

The UVM is the unit value of imports and UVX is the unit value of exports. If exports and imports share at least 85% of their costs, products are defined as HIIT. Therefore, the PQV index is one if every two-way trade is equivalent in quality and VIIT is equal to zero (PQV =1 if VIIT=0). The interval 0.85–1.15 is used to determine whether the PQV index is suitably far from one to specify quality differences (see Fontagné and Freudenberg, 1997; Greenaway et al., 1994; Chiarlone, 2000). The range of HIIT is as follows:

$$0.85 \le PQV \le 1.15.$$
 (3)

Products are considered as VIIT if imports and exports share only 50% of their costs. The VIIT is categorized as high or low (VIITH and VIITL) based on whether the quality of exports is higher than imports. Specifically, VIIT is categorized as high if:

$$PQV > 1.15.$$
 (4)

Products are considered as VIITL if the home country is exporting quality and:

$$PQV < 0.85.$$
 (5)

VIITH means that the home country is exporting quality products and VIITL means that the home country is importing quality products.

4. Results and Discussion

This study measures product quality improvement in trade between China and other developing countries: Argentina, Brazil, Chile, Egypt, South Africa, Mexico, Turkey, and the Russian Federation during the two periods 2000–2004 and 2004–2008. Table 1 reports the S indexes, given the total number of products and the IIT numbers. Trade flows between China and Argentina yield 303 products that changed significantly in matched trade in the first period and 144 products in the second period. Based on the percentage of positive S indexes, the second period performs better with 56.9% as compared to 49.8% in the first period. This means that 56.9% of trade between China and Argentina in the second period benefits China. Meanwhile, the quality of exports from China to Argentina is greater than the quality of its imports. While the IIT decreased from 246 to 160 in bilateral trade between China and Brazil, the percentage of positive S indexes increased from 47.2% to 61.%. This result supports Brazil as a profitable partner for China.

Table 1. The S Indexes for Trade between China and Eight Developing Countries

Time Period	2000-2004				2004-2008			
China's	Total	IIT	Number	Percentage	Total	IIT	Number	Percentage
Trading	Numbers	Number	of	of	Numbers	Number	of	of
Partner	of		Products	Positive S	of		Products	Positive S
	Products		with	Index	Products		with	Index
			Positive				Positive	
			\mathbf{S} Index				S Index	
Argentina	3110	303	151	49.83	3320	144	82	56.94
Brazil	3350	246	116	47.15	3465	160	98	61.25
Chile	3160	216	119	55.09	3315	157	95	60.50
Egypt	3270	264	148	56.06	3350	190	161	84.73
South Africa	3435	211	124	58.76	3490	190	112	58.94
Mexico	2580	178	77	43.25	3400	218	134	61.46
Turkey	3300	215	117	54.41	3430	213	127	59.62
Russia	3510	291	118	40.54	3515	161	104	64.59

Notes: Calculated from the UN COMTRADE using the S index. The IIT represents the total number of products that have experienced significant changes in matched trade based on the S index ($-0.4 \le S \le 0.4$).

The total numbers of products traded between China and Chile increased from 3160 in the first period to 3315 in the second period. Although the number of products that had large changes in IIT decreased from 216 to 157, the percentage of positive S indexes increased from 55.1% in the first period to 60.5% in the second period. This shows that the benefits of trade for China improved between the two periods. As for China's trade with Egypt, the percentage of positive S indexes increased from 56.1% to 84.7%, implying that IIT with Egypt during these periods is also profitable for China. Trade with South Africa shows a decrease in both IIT

65

numbers and number of products with positive S indexes in the second period. In both periods, benefits of trade to China reach approximately 58%. The IIT numbers for products traded between China and Mexico increased from 178 in the first period to 218 in the second period. The number of products with positive S indexes rose from 77 to 134, and the percentage of positive S indexes increased from 43.3% to 61.5%, indicating benefits of trade in favor of China. Trade benefits to China are also evident in the bilateral trade with Turkey and Russia where the percentage of positive S indexes increased from 54.4% to 59.6% and 40.5% to 64.6%, respectively.

Table 2. The PQV Index for Trade between China and Eight Developing Countries

Time Period	2000-	-2004	2004-	Growth of	
China's	Low Quality	High Quality	Low Quality	High Quality	High Quality
Partner	Products (%)				
Argentina	57.14	11.42	40	13.4	17.34
Brazil	52.78	8.34	41.81	10.9	30.69
Chile	33.34	0	33.34	0	0
Egypt	44.44	11.12	55.55	11.12	0
South Africa	34.69	18.36	45.83	16.67	-9.20
Mexico	41.86	9.3	58.66	8	-13.97
Turkey	42	14	50	9.75	-30.35
Russia	41.50	5.66	25	25	341.69

Notes: Calculated from the UN COMTRADE using the PQV index.

Table 2 and Figure 1 compare the improvement of product quality using the PQV index for trade flows between China and the eight selected trade partners. Figure 1 is divided to four quadrants: VIITH products in quadrant I, products that shifted from VIITL to VIITH in quadrant II, VIITL products in quadrant III, and products that shifted from VIITH to VIITH to VIITL in quadrant IV.

The percentage of low quality products (quadrant III) in both periods is higher than the percentage of high quality products (quadrant I) for all countries. In the case of Egypt, South Africa, Mexico, and Turkey, the percentage of low quality products in the second period increased. Mexico reported the highest percentage of low quality products with 58.7% in the second period. This means that 58.7% of traded products between China and Mexico in this period are classified as low quality products. In terms of high quality products, Russia has the highest positive growth of high quality products of 341.7% per cent between the two periods. Brazil and Argentina are in second and third places, with 30.7% and 17.3% growth, respectively. South Africa, Mexico, and Turkey have a negative growth of high quality products. This indicates that China exported more quality products to these countries in the first period. Meanwhile, Chile and Egypt reported zero growth in high quality products.

It is not surprising that Russia has the highest positive growth for importing high quality products from China. As neighboring countries, their bilateral trade is

expected to increase over time. Furthermore, Russia recorded its ninth straight year of annual growth by the end of 2008, averaging 7% between 2000 and 2008, and trade between Russia and China continues to recover after the global economic crisis. In 2011, the Russian prime minister expected that trade with China will reach US\$100 billion by 2015 and US\$200 billion by 2020 (Novo-Ogaryovo, 2011).

According to the International Monetary Fund and the World Bank reports in 2011, Brazil has the seventh largest purchasing power parity in the world, and its economy has been predicted to become one of the five largest in the world in the decades to come. Its positive growth of high quality products is evident as the total trade between Brazil and China expanded strongly during the past 12 years. Furthermore, Brazil's exports and imports with China grew by a compound annual growth rate of 46.9% and 37.8% from 1999 to 2010, respectively.

Trade between China and Argentina grew rapidly, and China became Argentina's second largest trading partner in 2009. As shown in Table 2, the percentage of high quality products in Argentina increased from 11.4% in the first period to 13.4% in the second period, showing positive growth between these two periods. However, the product quality of bilateral trade between China and Chile was unchanged between periods. All traded products are classified as HIIT or low quality IIT, where China imports quality products from Chile and the percentage of high quality products in both periods are zero. Finally, trade between China and Egypt has an increasing percentage of low quality products, while the percentage of high quality products in both periods remains at 11.1%.

5. Conclusion

This study focuses on the measurement of product quality improvement in trade between China and selected trading partners during the two periods 2000-2004 and 2004–2008. Examining the PQV index, the highest positive growth of high quality products during the two periods is reported by Russia, while Brazil and Argentina place second and third. Trade between Russia and China is rapidly recovering after the global slowdown in 2008. China is Brazil's biggest export market, and its large market size is a prospective key destination for China's high quality exports. According to International Monetary Fund and World Bank Reports (2011), Brazil was the seventh largest country in the world in terms of GDP and PPP in 2009, and it is expected to become one of the five largest economies in the next decade. China was also noted as Argentina's second largest trading partner in 2009. The emerging economies of both Argentina and Brazil play important roles in world trade, leading to agreements to improve their partnership. In conclusion, there is a massive potential for wider economic cooperation between China and its three major partners. The recent increase in high quality IIT between China and other developing countries also shows the remarkable power of China's exports.



Figure 1. PQV Index, Trade between China and Eight Developing Countries

67







Note: Calculated by authors using UN COMTRADE. Products that fall within the T-Zone represent horizontally differentiated goods (HIIT). Quadrant I represents products that remains as VIITH, Quadrant II represents products that shifted from VIITL to VIITH, Quadrant III represents products that remains as VIITL, and Quadrant IV represents products that shifted from VIITL to VIITH, to VIITL.

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Appendix A: Scaling Problem

The unit value ratio for Greenaway et al. (1994) and Fontagne et al. (1997) is:

$$\mathbf{r}=\frac{UV^{X}}{UV^{M}}.$$

For all $UV^x > 0$, $UV^M > 0$, and $UV^x > UV^M$ we have $1 < r < \infty$ and when $UV^x < UV^M$ we have 0 < r < 1; the function r can be described as being heavy on its denominator. A description of the disproportionate scaling of r is given by noticing that:

$$\mathbf{r} = f(UV^{\scriptscriptstyle M}, UV^{\scriptscriptstyle X}) = \frac{UV^{\scriptscriptstyle X}}{UV^{\scriptscriptstyle M}} \,.$$

and so:

$$\left(\frac{\partial r}{\partial UV^{M}}\right)UV^{X} = -\frac{UV^{X}}{\left(UV^{M}\right)^{2}} \Rightarrow \text{ as } UV^{M} \to 0, \text{ then, } \left(\frac{\partial r}{\partial UV^{M}}\right)UV^{X} \to \infty,$$
$$\left(\frac{\partial r}{\partial UV^{X}}\right)UV^{M} = \frac{1}{UV^{M}} \Rightarrow \text{ as } UV^{X} \to 0, \text{ now, } \left(\frac{\partial r}{\partial UV^{X}}\right)UV^{M} = \frac{1}{UV^{M}}.$$

Therefore, the rates of change of r with respect to UV^x and UV^M are not same; this is the so-called disproportionate scaling or proportionality effect, while for the PQV index of Azhar and Elliott we have PQV = $1 + (UV^x - UV^M)/(UV^x - UV^M)$. Here, for all $UV^x > 0$, $UV^M > 0$, and $UV^x > UV^M$ we have 1 < PQV < 2 and when $UV^x < UV^M$ we have 0 < PQV < 1. A description of the proportionate scaling of PQV is given by noticing that $PQV = f(UV^M, UV^x) = 1 + (UV^x - UV^M)/(UV^x - UV^M)$. Then:

$$\left(\frac{\partial PQV}{\partial UV^{M}}\right)UV^{X} = -\frac{2UV^{X}}{\left(UV^{X} + UV^{M}\right)^{2}} \Rightarrow \text{ as } UV^{M} \to 0, \ \left(\frac{\partial PQV}{\partial UV^{M}}\right)UV^{X} = \frac{-2}{UV^{X}}.$$

$$\left(\frac{\partial PQV}{\partial UV^{x}}\right)UV^{M} = \frac{2UV^{M}}{\left(UV^{x} + UV^{M}\right)^{2}} \Rightarrow \text{as } UV^{x} \rightarrow 0, \left(\frac{\partial PQV}{\partial UV^{x}}\right)UV^{M} = \frac{2}{UV^{M}}.$$

Hence, the PQV exhibits proportionate scaling.