

Free Entry Oligopoly, Cournot, Bertrand and Relative Profit Maximization

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Abstract

We study a symmetric free entry oligopoly in which firms produce differentiated goods so as to maximize their relative profits. The relative profit of each firm is the difference between its profit and the average of the profits of other firms. We show that, whether firms determine their outputs or prices, the equilibrium price when firms maximize their relative profits is lower than the equilibrium price when firms maximize their absolute profits, but the equilibrium number of firms under relative profit maximization is smaller than the equilibrium number of firms under absolute profit maximization. This is because each firm is more aggressive and produces larger output under relative profit maximization than under absolute profit maximization.

Key words: free entry; oligopoly; relative profit maximization

JEL classification: D43; L13; L21

1. Introduction

In recent years, maximizing relative profit instead of absolute profit has aroused the interest of economists. For analyses of relative profit maximization, see Gibbons and Murphy (1990), Lu (2011), Matsumura et al. (2013), Satoh and Tanaka (2013, 2014a, 2014b), Schaffer (1989), Tanaka (2013a, 2013b), and Vega-Redondo (1997).

In the analysis of the delegation problem, such as Miller and Pazgal (2001), the weight on the relative profit is treated as a means of the owner of a firm to control its firm, and the owner's objective itself is still the absolute profit of its firm. But in this paper we have an interest in the case where the owners of firms themselves seek to maximize the relative profits, so that we do not consider the delegation problem.

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In Vega-Redondo (1997), it was shown that the equilibrium in an oligopoly with a homogeneous good under relative profit maximization is equivalent to the competitive equilibrium. Referring to Alchian (1950) and Friedman (1953), he argued that it is relative rather than absolute performance which should in the end prove decisive in the long run.

With differentiated goods, however, the equilibrium in an oligopoly under relative profit maximization is not equivalent to the competitive equilibrium. In Tanaka (2013a), it was shown that, under the assumption of linear demand and cost functions, when firms in a duopoly with differentiated goods maximize their relative profits, the Cournot equilibrium and the Bertrand equilibrium are equivalent. Satoh and Tanaka (2014a) extended this result to an asymmetric duopoly in which firms have different cost functions. Satoh and Tanaka (2013) showed that, in a Bertrand duopoly with a homogeneous good under relative profit maximization and quadratic cost functions, there exists a range of the equilibrium price, and this range is narrower and lower than the range of the equilibrium price in duopolistic equilibria under absolute profit maximization shown by Dastidar (1995). Tanaka (2013b) showed that, under relative profit maximization, the choice of strategic variables, price or quantity, is irrelevant to the equilibrium of duopoly with differentiated goods. Usually, the relative profit of a firm in a duopoly or an oligopoly is defined as the difference between the absolute profit of this firm and the absolute profit of the rival firm (or the average of the absolute profits of the rival firms). Alternatively, we can define the relative profit as the ratio of the profit of a firm to the total profit in the industry. In Satoh and Tanaka (2014b), we compare these two definitions in a duopoly.

We think that seeking relative profit or utility is based on the nature of man. Even if a person earns big money, if his brother/sister or close friend earns more money, he is not sufficiently happy and may be disappointed. On the other hand, even if he is very poor, if his neighbor is poorer, he may be consoled by that fact. Similarly, firms in an industry not only seek to improve their own performances but also want to outperform the rival firms. The TV audience-rating race and market-share competitions by breweries, automobile manufacturers, convenience store chains, and mobile-phone carriers, especially in Japan, are examples of such behavior of firms.

In this paper, we study a symmetric free entry oligopoly in which firms produce differentiated goods so as to maximize their relative profits. The relative profit of each firm is the difference between its profit and the average of the profits of other firms. We show that, whether firms determine their outputs or prices, the equilibrium price when firms maximize their relative profits is lower than the equilibrium price when firms maximize their absolute profits, but the equilibrium number of firms under relative profit maximization is smaller than the equilibrium number of firms under absolute profit maximization. This is because each firm is more aggressive and produces larger output under relative profit maximization than under absolute profit maximization. Also we show that Cournot and Bertrand equilibria under relative profit maximization are equivalent.

An equilibrium of a free entry oligopoly is defined as a sub-game perfect equilibrium of the following two-stage game:

1. There are many potential firms. In the first stage of the game, each firm determines whether to enter or not to enter into the industry. If a firm does not enter, its absolute profit is zero.
2. In the second stage, each firm that entered in the first stage determines the output or the price of its good.

In the next section we present the model, in Section 3 we analyze Cournot and Bertrand equilibria under relative profit maximization, and in Section 4 we compare relative and absolute profit maximization.

2. The Model

There are n firms ($n \geq 2$). The firms produce differentiated substitutable goods. The output and the price of the good of firm i , $i = 1, \dots, n$, are denoted x_i and p_i . The marginal cost $c > 0$ is common. There is a fixed cost $f > 0$, which is also common to all firms.

The inverse demand functions of the goods produced by the firms are:

$$p_i = a - x_i - b \sum_{j=1, j \neq i}^n x_j, \quad i = 1, \dots, n, \quad (1)$$

where $a > c$ and $0 < b < 1$. Here b is a substitution parameter. The larger the value of b , the more substitutable the goods are. These inverse demand functions are symmetric. By symmetry, we can assume that all x_j for all $j \neq i$ are equal at any equilibrium. Differentiating (1) with respect to p_i yields:

$$1 = -\frac{\partial x_i}{\partial p_i} - (n-1)b \frac{\partial x_j}{\partial p_i},$$

and

$$0 = -b \frac{\partial x_i}{\partial p_i} - [1 + (n-2)b] \frac{\partial x_j}{\partial p_i}.$$

Then, we obtain:

$$\frac{\partial x_i}{\partial p_i} = -\frac{1 + (n-2)b}{1 + (n-2)b - (n-1)b^2},$$

and

$$\frac{\partial x_j}{\partial p_i} = \frac{b}{1 + (n-2)b - (n-1)b^2}, j \neq i.$$

Thus,

$$\frac{\partial x_i}{\partial p_i} - \frac{\partial x_j}{\partial p_i} = -\frac{1 + (n-1)b}{(1-b)[1 + (n-1)b]} = -\frac{1}{1-b}. \quad (2)$$

3. Relative Profit Maximization

3.1 Cournot Equilibrium

The absolute profit of firm i is written as:

$$\pi_i = (a - x_i - b \sum_{j=1, j \neq i}^n x_j)x_i - cx_i - f, \quad i = 1, \dots, n.$$

We denote the relative profit of firm i by Π_i . It is written as follows:

$$\begin{aligned} \Pi_i &= \pi_i - \frac{1}{n-1} \sum_{j=1, j \neq i}^n \pi_j, \\ &= [a - x_i - b \sum_{j=1, j \neq i}^n x_j - c]x_i - f \\ &\quad - \frac{1}{n-1} \sum_{j=1, j \neq i}^n \{[a - x_j - b \sum_{k=1, k \neq j}^n x_k - c]x_j - f\}. \end{aligned}$$

The condition for maximization of Π_i with respect to x_i is:

$$a - 2x_i - b \sum_{j=1, j \neq i}^n x_j - c + \frac{b}{n-1} \sum_{j=1, j \neq i}^n x_j = 0.$$

By symmetry, we can assume that all x_i 's are equal. Then, this equation can be rewritten as:

$$a - [2 + (n-1)b]x_i - c + \frac{b}{n-1}(n-1)x_i = a - [2 + (n-2)b]x_i - c = 0.$$

And the equilibrium outputs and prices are:

$$\tilde{x}_i^c = \frac{a - c}{2 + (n-2)b},$$

and

$$\tilde{p}_i^c = \frac{(1-b)a + [1 + (n-1)b]c}{2 + (n-2)b}.$$

where the superscript C indicates *Cournot*.

3.2 Bertrand Equilibrium

The absolute profit of firm i is written as:

$$\pi_i = (p_i - c)x_i - f, \quad i = 1, \dots, n.$$

The relative profit of firm i , Π_i , is written as follows:

$$\begin{aligned} \Pi_i &= \pi_i - \frac{1}{n-1} \sum_{j=1, j \neq i}^n \pi_j \\ &= (p_i - c)x_i - f - \frac{1}{n-1} \sum_{j=1, j \neq i} [(p_j - c)x_j - f]. \end{aligned}$$

The condition for maximization of Π_i with respect to p_i is:

$$x_i + (p_i - c) \frac{\partial x_i}{\partial p_i} - \frac{1}{n-1} \sum_{j=1, j \neq i} (p_j - c) \frac{\partial x_j}{\partial p_i} = 0.$$

By symmetry, we can assume that all $\partial x_j / \partial p_i$'s for $j \neq i$ are equal, and all p_i 's are equal. Then, this equation can be rewritten as:

$$x_i + (p_i - c) \left(\frac{\partial x_i}{\partial p_i} - \frac{\partial x_j}{\partial p_i} \right) = 0. \quad (3)$$

Substituting (2) into (3), we get:

$$x_i - \frac{a - [1 + (n-1)b]x_i - c}{1-b} = 0.$$

This equation can be rewritten as:

$$a - [2 + (n-2)b]x_i - c = 0.$$

This is equal to the first-order condition at the Cournot equilibrium. We provide the reason why the equivalence of Cournot and Bertrand equilibria holds in the next subsection.

The equilibrium outputs and prices are obtained as follows,

$$\tilde{x}_i^B = \frac{a-c}{2+(n-2)b},$$

and

$$\tilde{p}_i^B = \frac{(1-b)a + [1+(n-1)b]c}{2+(n-2)b}.$$

The superscript *B* indicates *Bertrand*. We have $\tilde{x}_i^B = \tilde{x}_i^C$ and $\tilde{p}_i^B = \tilde{p}_i^C$. Thus, when firms maximize their relative profits, Cournot and Bertrand equilibria are equivalent.

We denote the equilibrium output and price of the good of each firm under relative profit maximization by \tilde{x}_i and \tilde{p}_i . The equilibrium absolute profit of each firm is expressed by:

$$\pi_i = \frac{(\tilde{p}_i - c)^2}{1-b} - f = (1-b)\tilde{x}_i^2 - f = \frac{(1-b)(a-c)^2}{[2+(n-2)b]^2} - f.$$

The relative profit of an entered firm is the difference between its absolute profit and the average of the absolute profits of other firms, including non-entering firms. So, if the absolute profit of an entered firm in the second stage of the game is positive (negative), its relative profit in the first stage of the game is also positive (negative), and the free entry condition in the first stage is that the absolute profit of an entered firm in the second stage is zero. Thus, the condition for free entry of firms, ignoring the fact that the number of firms is an integer, is:

$$\frac{(1-b)(a-c)^2}{[2+(n-2)b]^2} = f, \quad (4)$$

or

$$\frac{(\tilde{p}_i - c)^2}{1-b} = f,$$

or

$$(1-b)\tilde{x}_i^2 = f.$$

Therefore, the equilibrium output and price of the good of each firm are:

$$\tilde{x}_i = \sqrt{\frac{f}{1-b}},$$

and

$$\tilde{p}_i = \sqrt{(1-b)f} + c.$$

Solving (4) for n , we get:

$$\tilde{n} = \frac{(a-c)\sqrt{(1-b)f} - 2(1-b)f}{bf}.$$

Here \tilde{n} denotes the equilibrium number of firms in the case of relative profit maximization.

3.3 Discussion

Comparing the first-order conditions for relative profit maximization in the Cournot oligopoly and those in the Bertrand oligopoly, we can provide the reason why the equivalence of Cournot and Bertrand equilibria holds. The first-order conditions at the Cournot equilibrium are:

$$\frac{\partial \Pi_i}{\partial x_i} = 0, \quad i = 1, \dots, n. \quad (5)$$

The first-order conditions at the Bertrand equilibrium are:

$$\frac{\partial \Pi_i}{\partial p_i} = \frac{\partial \Pi_i}{\partial x_i} \frac{\partial x_i}{\partial p_i} + (n-1) \frac{\partial \Pi_i}{\partial x_j} \frac{\partial x_j}{\partial p_i} = 0, \quad i = 1, \dots, n, \quad j \neq i. \quad (6)$$

From the property of the relative profits, the following relation holds:

$$\sum_{i=1}^n \Pi_i = 0.$$

At equilibrium, this can be rewritten as:

$$\Pi_i = -(n-1)\Pi_j, \quad (7)$$

and, by symmetry, the following relation holds:

$$\frac{\partial \Pi_i}{\partial x_j} = \frac{\partial \Pi_j}{\partial x_i}, \quad j \neq i. \quad (8)$$

Then, using (2), (7), and (8), (6) can be rewritten as:

$$\frac{\partial \Pi_i}{\partial x_i} \left(\frac{\partial x_i}{\partial p_i} - \frac{\partial x_j}{\partial p_i} \right) = \frac{\partial \Pi_i}{\partial x_i} \left(-\frac{1}{1-b} \right) = 0, \quad i = 1, \dots, n, \quad j \neq i.$$

Since $0 < b < 1$, this implies (5).

4. Comparison between Relative and Absolute Profit Maximization

4.1 Cournot Equilibrium under Absolute Profit Maximization

The condition of absolute profit maximization for firm i with respect to x_i is:

$$a - c - 2x_i - b \sum_{j=1, j \neq i}^n x_j = 0, \quad i = 1, \dots, n.$$

The equilibrium outputs, prices, and profits of the firms are:

$$x_i^c = \frac{a - c}{2 + (n - 1)b},$$

$$p_i^c = \frac{a - c}{2 + (n - 1)b} + c,$$

and

$$\pi_i^c = \left[\frac{a - c}{2 + (n - 1)b} \right]^2 - f.$$

Again, the superscript C indicates *Cournot*. The condition for free entry of firms, ignoring the fact that the number of firms is an integer, is:

$$\left[\frac{a - c}{2 + (n - 1)b} \right]^2 = f, \quad (9)$$

or

$$(x_i^c)^2 = f,$$

or

$$(p_i^c - c)^2 = f.$$

Solving (9) for n , we get:

$$n^c = \frac{(a - c)\sqrt{f} - (2 - b)f}{bf}.$$

Here n^c denotes the equilibrium number of firms at the Cournot equilibrium under

absolute profit maximization. Also we have:

$$p_i^c = \sqrt{f} + c,$$

and

$$x_i^c = \sqrt{f}.$$

4.2 Bertrand Equilibrium under Absolute Profit Maximization

The condition of absolute profit maximization for firm i with respect to p_i is:

$$x_i + (p_i - c) \frac{\partial x_i}{\partial p_i} = x_i - (p_i - c) \frac{1 + (n-2)b}{1 + (n-2)b - (n-1)b^2} = 0.$$

The equilibrium outputs, prices, and profits of the goods of the firms are:

$$x_i^B = \frac{[1 + (n-2)b](a-c)}{[1 + (n-1)b][2 + (n-3)b]},$$

$$p_i^B = \frac{(1-b)(a-c)}{2 + (n-3)b} + c,$$

and

$$\pi_i^B = \left[\frac{1 + (n-2)b}{1 + (n-1)b} \right] \frac{(p_i^B - c)^2}{1-b} - f.$$

Again, here B indicates *Bertrand*. The condition for free entry of firms, ignoring the fact that the number of firms is an integer, is:

$$\frac{(1-b)[1 + (n-2)b](a-c)^2}{[1 + (n-1)b][2 + (n-3)b]^2} = f, \quad (10)$$

or

$$(x_i^B)^2 = \frac{[1 + (n-2)b]}{(1-b)[1 + (n-1)b]} f,$$

or

$$(p_i^B - c)^2 = \frac{(1-b)[1 + (n-1)b]}{1 + (n-2)b} f.$$

Denote the number of firms which satisfies (10) by n^B . It is the equilibrium number of firms at the Bertrand equilibrium under absolute profit maximization. Also we have:

$$p_i^B = \sqrt{\left[\frac{1+(n^B-1)b}{1+(n^B-2)b} \right] (1-b)f + c},$$

and

$$x_i^B = \sqrt{\left[\frac{1+(n^B-2)b}{1+(n^B-1)b} \right] \left(\frac{f}{1-b} \right)}.$$

4.3 Comparison of the Equilibrium Prices

Comparing \tilde{p}_i with p_i^B implies:

$$\tilde{p}_i - p_i^B = \sqrt{(1-b)f} - \sqrt{\left[\frac{1+(n^B-1)b}{1+(n^B-2)b} \right] (1-b)f}.$$

Since $[1+(n^B-1)b/1+(n^B-2)b] > 1$, we have $\tilde{p}_i - p_i^B < 0$. Comparing p_i^B with p_i^C implies:

$$p_i^B - p_i^C = \sqrt{\left[\frac{1+(n^B-1)b}{1+(n^B-2)b} \right] (1-b)f} - \sqrt{f}.$$

Since:

$$\left[\frac{1+(n^B-1)b}{1+(n^B-2)b} \right] (1-b) - 1 = \frac{-(n^B-1)b^2}{1+(n^B-2)b} < 0,$$

we have $p_i^B - p_i^C < 0$. Therefore, we have shown that:

$$\tilde{p}_i < p_i^B < p_i^C.$$

4.4 Comparison of the Equilibrium Outputs per Firm

Comparing \tilde{x}_i with x_i^B implies:

$$\tilde{x}_i - x_i^B = \sqrt{\frac{f}{1-b}} - \sqrt{\left[\frac{1+(n^B-2)b}{1+(n^B-1)b} \right] \left(\frac{f}{1-b} \right)}.$$

Since $[1+(n^B-2)b/1+(n^B-1)b] < 1$, we have $\tilde{x}_i - x_i^B > 0$. Comparing x_i^B with x_i^C implies:

$$x_i^B - x_i^C = \sqrt{\left[\frac{1+(n^B-2)b}{1+(n^B-1)b} \right] \left(\frac{f}{1-b} \right)} - \sqrt{f}.$$

Since:

$$\left[\frac{1+(n^B-2)b}{1+(n^B-1)b} \right] \left(\frac{1}{1-b} \right) - 1 = \frac{(n^B-2)b^2}{1+(n^B-1)b} > 0,$$

we have $x_i^B - x_i^C > 0$. Therefore, we have shown:

$$\tilde{x}_i > x_i^B > x_i^C.$$

4.5 Comparison of the Equilibrium Numbers of Firms

Compare \tilde{n} with n^B . The equation for the equilibrium number of firms in the Bertrand oligopoly under absolute profit maximization is a cubic equation, and its closed-form solution is very complicated. So, implicit comparison is appropriate. Assume that the number of firms at the Bertrand equilibrium under relative profit maximization and that at the Bertrand equilibrium under absolute profit maximization are equal. Then, from (4) and (10), we have:

$$\frac{(1-b)[1+(n-2)b]}{[1+(n-1)b][2+(n-3)b]^2} - \frac{1-b}{[2+(n-2)b]^2} > 0. \quad (11)$$

Since $(1-b)/[2+(n-2)b]^2$ is a decreasing function of n , (11) means that the equilibrium number of firms under relative profit maximization is smaller than that at the Bertrand equilibrium under absolute profit maximization.

Compare n^C and n^B . Assume that the number of firms at the Cournot equilibrium and that at the Bertrand equilibrium under absolute profit maximization are equal. Then, from (9) and (10) we have:

$$\left[\frac{1}{2+(n-1)b} \right]^2 - \frac{(1-b)[1+(n-2)b]}{[1+(n-1)b][2+(n-3)b]^2} > 0. \quad (12)$$

Since $1/[2+(n-1)b]^2$ is a decreasing function of n , (12) means that the equilibrium number of firms at the Cournot equilibrium under absolute profit maximization is larger than that at the Bertrand equilibrium under absolute profit maximization. Therefore, we have shown:

$$\tilde{n} < n^B < n^C.$$

The following proposition summarizes the results.

Proposition 1. Whether firms determine their outputs or prices,

1. The equilibrium price when firms maximize their relative profits is lower than the equilibrium price when firms maximize their absolute profits.
2. The equilibrium output per firm under relative profit maximization is larger than the equilibrium output per firm under absolute profit maximization.
3. The equilibrium number of firms under relative profit maximization is smaller than the equilibrium number of firms under absolute profit maximization.

The reason why the equilibrium number of firms under relative profit maximization is smaller than that under absolute profit maximization is that each firm is more aggressive and produces larger output under relative profit maximization than under absolute profit maximization.

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