

The Returns and Risk of Dynamic Investment Strategies: A Simulation Comparison

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1. Introduction

Historical stock market returns and risk can be quite different from historical stock investors' returns and risk because of different timing and magnitude of investment flows. As Dichev (2007) points out, investors' returns equal stock market returns only if the investors choose a buy-and-hold (BH) strategy; other strategies involve different timing and magnitude of investment flows, which create differences. With BH, investors do nothing no matter how stock prices change during the period of investment. With a contrarian or concave strategy, they buy more as prices fall and sell as they rise, whereas with a momentum or convex strategy, they sell as prices fall and buy as they rise. Contrarian and momentum strategies both make adjustments depending on price changes; therefore, both often make investment returns and risk unequal to the BH strategy.

How, specifically, are the returns and risk different? As Perold and Sharpe (1988) demonstrate, while BH gives straight-line payoff diagrams, momentum and contrarian strategies create convex and concave payoff curves, respectively. These two strategies generally yield very different return distributions. Momentum and contrarian strategies can be seen as mirror images of one another on either side of the BH strategy. Therefore, it is interesting to know how these two dynamic strategies change returns and risk.

Through Monte Carlo simulations, this paper compares the investment returns and risk of three strategies: (1) BH, (2) a contrarian strategy, constant-mix (CM), and (3) a momentum strategy, constant-proportion portfolio insurance (CPPI). The simulated results show that CM has the highest median returns followed by BH and then CPPI. In terms of value-at-risk, CPPI has the lowest risk, while CM has the

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highest. Therefore, there exists a risk-return tradeoff across the strategies, in terms of the median return and the value-at-risk, but not in terms of mean return and standard deviation. Because the return distributions generated are not all symmetric and normally distributed, the mean return is not the most likely return and the standard deviation is not an appropriate risk measure. As the risk measure focusing on downside risk measurement, here value-at-risk is more appropriate than the standard deviation. For the three strategies, the momentum strategy gives the most downside protection, while the contrarian strategy has the least downside protection.

The remainder of the note is organized as follows. Section 2 presents the stock price model, the investment strategies, and how the investment simulation is set up. In Section 3, the simulated results and discussions are given.

2. The Investment-Simulation Design

To generate the return distributions of the three strategies for further analysis, Monte Carlo simulation is set up following the framework of Cesari and Cremonini (2003). The three key elements are an initial portfolio, a stock-price and interest-rate model, and the rebalancing mechanisms for CM and CPPI.

All strategies have the same initial 50-50 stock-cash portfolio. The cash in the portfolios for the CM and CPPI strategies is used as a reserve for investment inflows and outflows during investment periods. No additional cash input or any withdrawal is allowed after the initial portfolio is set up. Thus, all strategies are self-financing. In this way, the three strategies are compared with the same investment scale of over a five-year period.

The stock price is assumed to follow geometric Brownian motion. The stock price model can be written as:

$$dS_t = \mu S_t dt + \sigma S_t dw_t, \quad (1)$$

where S_t is the stock price at time t , w_t is a Wiener process, and μ and σ are the drift and volatility parameters. Based on this equation and given the values of the initial stock price and the parameters, stock market scenarios can be generated by Monte Carlo simulations. Here, μ and σ are set to be 10% and 20% respectively. These numbers are close to the long-term average index return and the return volatility of the S&P 500. In addition, a lower average with $\mu = 5\%$ and a higher volatility with $\sigma = 30\%$ are considered. In total, there are four market situations: normal-return-normal-volatility ($\mu = 10\%$ and $\sigma = 20\%$), low-return-normal-volatility ($\mu = 5\%$ and $\sigma = 20\%$), normal-return-high-volatility ($\mu = 10\%$ and $\sigma = 30\%$), and low-return-high-volatility ($\mu = 5\%$ and $\sigma = 30\%$). For each situation, 10,000 scenarios are created. The cash is assumed to earn a fixed continuous compounding interest rate. Under this setup, the source of risk comes from the stock-market risk only. The interest rate is set to 3%, which is close to the long-term average return of US treasury bills.

For the two dynamic strategies, CM and CPPI, the rebalancing mechanisms,

which determine the timings of capital inflows and outflows, are set up based on the price principle, with rebalancing only when the new simulated price has increased or decreased no less than 2.5% percent since the last rebalancing. The stock prices are simulated weekly (i.e., $\Delta = 1/52$). The transaction costs of the stock are 0.3% of the value traded. When rebalancing, the portfolio will be back to the initial 50-50 stock-cash mix for CM. For CPPI, when rebalancing, the stock and cash positions are reset according to the following formulas:

$$SP = m(V - f) \quad (2)$$

$$CP = V - SP, \quad (3)$$

where SP and CP are stock and cash positions respectively, V is the value of the portfolio at the time of rebalancing, f is the floor, and m is the multiplier. However, when V is less than f , the portfolio will be held in cash only; when $m(V - f)$ is larger than V , to avoid the possibility of leverage, the new stock position will be set to V . To implement simulations, f is set to 75% of the portfolio value, and m is set to 2. Under this setup, the initial portfolio mix is the same for all strategies.

3. Simulated Results and Discussion

After Monte Carlo simulations, the return distributions under the three strategies are generated, and then statistics are presented for return and risk analysis. Corresponding to the four market situations, the statistics are tabulated in Tables 1–4, with the first four central moments of the simulated return distributions (mean, standard deviation, skewness, and excess kurtosis) being given. Also, the Jarque-Bera statistics, which are used to test the normality of a distribution, are calculated and listed in the tables. Finally, the fifth percentile, which is also the value-at-risk, is included.

The return analysis is done by comparing the mean returns and median returns across the three strategies. Overall, based on the mean returns, there is no best choice of the three for all market situations. However, by looking at the median returns for all cases, with or without the transaction costs, CM does best, followed by BH, and then CPPI. Though the median return is seldom used in the financial literature, it gives consistent ranking across the cases here.

In analyzing risk, it is important to check the distributions' normality because skewness and excess kurtosis can make the standard deviation inappropriate for measuring risk. By using the Jarque-Bera test for normality in the four cases, the normality hypothesis is only accepted for CM (with transaction costs or not), but rejected for BH and CPPI at the 5% significant level (the critical value is 5.99). In fact, the standard deviation of CPPI returns is the largest among the three strategies. If we use standard deviation as a risk measure, CPPI will be the riskiest strategy. This is not consistent with the downside protection just mentioned above. The large standard deviation of CPPI is simply due to the great upside potential; the standard

deviation penalizes this potential because it deviates a lot from the average. By using an appropriate risk indicator, such as the value-at-risk, we can clarify the riskiness of the three strategies. In terms of this indicator, CPPI has the lowest risk and CM has the highest risk.

Last but not the least is the issue of risk-return tradeoff for the three strategies. The results show that, while the mean return and the standard deviation do not indicate a risk-return tradeoff, the median return and the value-at-risk do. CM, which bears the highest risk in terms of the value-at-risk, has the highest median return; CPPI, which bears the lowest risk, has the lowest median return. However, by looking at the mean returns and standard deviations, there is no regularity in the risk-return tradeoff.

Table 1. Statistics of Simulated Return Distributions under Normal-Return-Normal-Volatility

	BH	CM	CPPI
Mean	6.18%	6.04% (5.98%)	6.74% (6.40%)
Median	5.68%	6.01% (5.95%)	5.24% (4.88%)
Standard Deviation	5.06%	4.47% (4.47%)	6.77% (6.70%)
Skewness	0.5691	0.0288 (0.0285)	0.8798 (0.9183)
Excess Kurtosis	0.3475	-0.0151 (-0.0151)	0.4170 (0.5006)
Jarque-Bera statistic	590.02	1.47 (1.44)	1362.52 (1509.85)
VaR	3.35%	4.32% (4.40%)	3.11% (3.20%)

Notes: The numbers in parentheses are calculated assuming 0.3% transaction costs.

Table 2. Statistics of Simulated Return Distributions under Low-Return-Normal-Volatility

	BH	CM	CPPI
Mean	3.53%	3.54% (3.48%)	3.45% (3.17%)
Median	3.02%	3.52% (3.45%)	1.99% (1.72%)
Standard Deviation	4.54%	4.47% (4.47%)	5.55% (5.47%)
Skewness	0.6448	0.0287 (0.0284)	1.2494 (1.2956)
Excess Kurtosis	0.4942	-0.0148 (-0.0149)	1.5682 (1.7376)
Jarque-Bera statistic	794.80	1.46 (1.43)	3626.47 (4055.42)
VaR	4.79%	6.82% (6.90%)	3.79% (3.86%)

Notes: The numbers in parentheses are calculated assuming 0.3% transaction costs.

Table 3. Statistics of Simulated Return Distributions under Normal-Return-High-Volatility

	BH	CM	CPPI
Mean	5.42%	5.44% (5.33%)	5.35% (4.87%)
Median	4.32%	5.41% (5.30%)	2.29% (1.81%)
Standard Deviation	7.27%	6.70% (6.70%)	8.96% (8.78%)
Skewness	0.8363	0.0298 (0.0295)	1.3044 (1.3693)
Excess Kurtosis	0.7855	-0.0147 (-0.0147)	1.4007 (1.6138)
Jarque-Bera statistic	1422.57	1.57 (1.54)	3653.21 (4210.36)
VaR	6.57%	10.12% (10.24%)	4.61% (4.66%)

Notes: The numbers in parentheses are calculated assuming 0.3% transaction costs.

Table 4. Statistics of Simulated Return Distributions under Low-Return-High-Volatility

	BH	CM	CPPI
Mean	2.91%	2.94% (2.83%)	2.58% (2.19%)
Median	1.81%	2.91% (2.80%)	-0.10% (-0.44%)
Standard Deviation	6.53%	6.70% (6.70%)	7.44% (7.24%)
Skewness	0.9416	0.0298 (0.0295)	1.6990 (1.7784)
Excess Kurtosis	1.0785	-0.0144 (-0.0145)	3.0059 (3.3733)
Jarque-Bera statistic	1962.34	1.57 (1.54)	8573.15 (10012.85)
VaR	7.44%	12.60% (12.73%)	4.85% (4.88%)

Notes: The numbers in parentheses are calculated assuming 0.3% transaction costs.

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