Increasing Returns in Matching and Labour Market Dynamics: Comments on Indeterminacy and Search Theory

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1. Introduction

In the context of the matching framework à la Pissarides (2000), a number of scholars have addressed the issue of indeterminacy. For instance, Burda and Weder (2002) derive indeterminacy in a matching economy relying on labour market institutions. Hashimzade and Ortigueira (2005) find out the possibility of sunspot equilibria by augmenting the matching model with capital accumulation. Zanetti (2006) builds a New Keynesian model with search by showing that the shape of monetary policy functions may drive indeterminacy. Krause and Lubik (2010) develop a DSGE model with matching by arguing that extrinsic uncertainty can actually cause business cycles, but the required parametrization lies at the boundaries of empirical plausibility.

A common feature of those contributions is the assumption of constant-returns-to-scale in the matching function. However, it is well known that indeterminacy has been often framed in the context of increasing returns (cf. Benhabib and Farmer, 1994). Confirmation of this is a work by Giammarioli (2003) that sets forth a search model with matching externalities in which a representative agent chooses the optimal vacancy rate while (un)employment evolves according to the rules of a standard matching framework.

Relying on that theoretical setting, Giammarioli (2003) argues that an aggregate matching function with increasing returns with respect to vacancies is a sufficient requirement for indeterminacy. This dynamic characterization arises when the stationary solution of the model is a sink, i.e., a situation in which the unstable manifold has dimension zero. In this way, sunspot equilibria become possible and the economy may justify the "animal spirits" hypothesis of business cycles (cf. Farmer and Guo, 1994).

In this note, building on the framework set forth by Giammarioli (2003), I argue that increasing returns with respect to vacancies is only a necessary condition for indeterminacy. Specifically, I show that the required bifurcation is actually obtained by imposing an additional condition between the "perceived" (or private) and the "aggregate" elasticity of the matching function with respect to unemployment.

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This note is arranged as follows. Section 2 analyses the dynamics of the benchmark model. Section 3 concludes.

2. Dynamics of the Benchmark Model

The system of differential equations derived from the solution of the optimization problem developed by Giammarioli (2003) can be easily rearranged as:

$$\dot{l} = \exp\left(\frac{\alpha}{1-\alpha}\left(\ln(a)+\lambda\right)-\frac{(1-\alpha)(1-L)+\beta L}{(1-\alpha)(1-L)}l\right)-\delta \tag{1}$$

$$\dot{\lambda} = \delta+\rho-\exp(-l-\lambda)+b\frac{\exp\left(\dfrac{\alpha}{1-\alpha}\left(\ln(a)+\lambda\right)-\dfrac{\beta L}{(1-\alpha)(1-L)}l\right)}{1-\exp(l)}, \tag{2}$$

where l is the logarithm of the state variable (L), λ is the logarithm of the co-state variable, δ is the separation rate, ρ is the discounting rate, while a and b (α and β) are, respectively the elasticities of the private (aggregate) matching function with respect to vacancies and unemployment (Giammarioli, 2003, p. 21).

The Jacobian matrix (J) of the linearized system is:

$$J \equiv \begin{bmatrix} -\dfrac{\left((1-\alpha)(1-L)+\beta L\right)\delta}{(1-\alpha)(1-L)} & \dfrac{\alpha\delta}{1-\alpha} \\ \rho+\delta+b\delta u-\dfrac{\beta L}{(1-\alpha)(1-L)}bu\delta+u^2 b\delta & \dfrac{\alpha u b\delta}{1-\alpha}+\rho+\delta+b\delta u \end{bmatrix}, \tag{3}$$

where $u \equiv L(1-L)^{-1}$.

The trace of J is given by:

$$\text{TR}(J) = \frac{\alpha u b\delta-u\beta\delta}{1-\alpha}+\rho+b\delta u. \tag{4}$$

Resorting to numerical simulations, Gimmarioli (2003) sets $\beta=b$. Consequently, (4) reduces to:

$$\text{TR}(J) = \rho. \tag{5}$$

As far as the result in (5) is concerned, it is possible to conclude that whenever $\beta=b$, the trace of J is exactly equal to the discount rate, which is usually positive. Therefore, if $\beta=b$, then the trace of J is positive regardless of value of α. Since the trace of J provides the sum of the two eigenvalues associated to the dynamic system in (1)–(2), it seems hard to configure the possibility of indeterminate

equilibrium paths, unless admitting an implausible negative discounting rate; indeed, such a dynamic characterization requires a negative value of $\text{TR}(J)$.

In order to find the requirements for indeterminacy, I consider the more general case in which β is different from b. Hence,

$$\text{TR}(J) = \frac{u\delta(\alpha b - \beta)}{1-\alpha} + \rho + b\delta u . \tag{6}$$

Whenever $\alpha > 1$, the condition under which $\text{TR}(J)$ is negative is:

$$\alpha b - \beta > 0 \text{ or } \alpha > \frac{\beta}{b} . \tag{7}$$

This is exactly the condition provided by Giammarioli (2003, p. 23). However, the inequality in (7) is necessary but not sufficient; indeed, it must also happen that:

$$\left| \frac{u\delta(\alpha b - \beta)}{1-\alpha} \right| > \rho + b\delta u . \tag{8}$$

Straightforward algebra suggests that whenever $\alpha > 1$, the inequality in (8) is verified if and only if $b > \beta$. This means that a sink, i.e., indeterminacy of equilibrium paths, is actually obtained when the aggregate matching function displays increasing returns with respect to vacancies while, at the private level, the reactivity of the matching function to unemployment is higher than its aggregate counterpart. This statement is confirmed by the figures in Table 1, in which the two eigenvalues of the system in (1)–(2) are retrieved for different values of a, b, α, and β (MATLAB code is available from the author).

Table 1. Eigenvalues of the System ($\rho = 0.03$ and $\delta = 0.1$)

β	b	a	α	EIGENVALUE #1	EIGENVALUE #2	DYNAMICS
0.40	0.50	0.60	0.80	−1.191	1.552	*Saddle*
0.40	0.50	0.60	0.90	−1.441	2.022	*Saddle*
0.40	0.50	0.60	0.99	−3.0765	7.8553	*Saddle*
0.40	0.50	0.60	1.01	−2.285−4.244i	−2.285+4.244i	*Sink*
0.40	0.50	0.60	1.05	−0.4179−2.034i	−0.417+2.034i	*Sink*
0.40	0.50	0.60	1.10	−0.1861−1.393i	−0.186+1.393i	*Sink*

As far as the unemployment elasticity is concerned, the analytical and numerical findings derived above are somehow counterintuitive. In fact, in their seminal article on indeterminacy and increasing returns, Benhabib and Farmer (1994) show that in the optimal-growth model indeterminacy requires social increasing returns with respect to the control variable (labour) and an aggregate output elasticity with respect to the stock variable (capital) higher than its private counterpart (cf. Guerrazzi, 2012).

3. Concluding Remarks

In this note, building on Giammarioli (2003), I show that, within the textbook matching model augmented with externalities, indeterminacy of the equilibrium paths requires increasing returns at the aggregate level with respect to vacancies and social elasticity of the matching function with respect to unemployment lower than its private counterpart. The result for the unemployment elasticity is at odds with the requirements for indeterminacy that hold in the standard model of optimal growth with productivity externalities (cf. Benhabib and Farmer, 1994; Guerrazzi, 2012).

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