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Ambiguity and the Excess Consumption Growth Puzzle

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Abstract

Using a two-period non-stochastic life-cycle model, Hauenschild and Stahlecker (2001) show that when information about future labor income is ambiguous, individuals may engage in precautionary savings even if their marginal utility is not convex. We extend the methodology of Houenschild and Stahlecker to a model with standard preferences and demonstrate the precautionary savings that consumers accumulate due to ambiguity and fuzzy decision-making possibly explain the "excess consumption growth puzzle."

Key words: ambiguity; precautionary savings; consumption growth puzzle; fuzzy sets; intertemporal life-cycle models

JEL classification: E21; D11; D91

1. Introduction

A large family of life-cycle representative rational agent models predicts that personal consumption must decline whenever an individual's coefficient of time preference exceeds the real rate of interest. Since consumers prefer present consumption to future consumption and thus have a positive coefficient of time preference, this condition automatically holds whenever the real interest rate turns negative. Deaton (1986, 1992), however, reports some puzzling evidence that contradicts this prediction: there was a more than 30-year period following World War II when U.S. real aggregate consumption continued to grow, even though the real interest rate remained on average negative. Responding to this "excess consumption growth" puzzle, Caballero (1990) proposes that precautionary motives can go a long way toward explaining excessive savings and therefore excessively growing consumption. However, as Deaton (1992) notes, the puzzle remains even with precautionary motives accounted for; precautionary savings that risk-averse individuals accumulate from the risk associated with future earnings are not sufficient to explain the excess consumption growth puzzle. It can also be shown

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that the puzzle remains even when the magnitude of precautionary savings is guided by extreme *subjective* pessimism and doubt in the subjective distribution of consumption growth.

The current study is based on the premise that while precautionary savings engendered by pure risk do not explain the excess consumption growth puzzle, precautionary savings due to other sorts of uncertainty may. Following an emerging body of literature, we recognize that there is an important distinction between two types of uncertainty: risk and ambiguity.[1,2] Risk is often used to denote uncertainty when the probability distribution of all possible outcomes is known. Ambiguity, on the other hand, refers to the uncertainty when the decision-maker does not have sufficient information about external events, so that the probability distribution cannot even be estimated. The literature presents a number of approaches to modeling choice in the presence of ambiguity.[3] In order to quantify the effect of ambiguity on precautionary motives, we rely on the methodology advanced by Hauenschild and Stahlecker (2001). These authors model intertemporal consumer choice under an ambiguous income constraint using fuzzy sets and membership functions. Using a simple two-period model, they demonstrate that when information about future labor income is ambiguous, individuals may engage in precautionary savings even when their preferences are quadratic, and thus the marginal utility is not convex.

The question that we address in this exploratory note is whether the fuzzy decision-making mechanism of Hauenschild and Stahlecker (2001) and precautionary savings caused by ambiguity of future wealth can explain the excess consumption growth puzzle. To calibrate the theoretical model of Hauenschild and Stahlecker (2001) to the post-World War II data, we modify their framework in two key respects. First, we represent consumer preferences by a more general, constant relative risk aversion (CRRA) utility function. Second, we specify a particular triangular form for the membership function used in conjunction with fuzzy sets. These modifications lead us to an Euler equation that is trackable and lends itself to numerical investigation. The results of our investigation suggest that when future wealth is ambiguous, the fuzzy decision-making mechanism of Hauenschild and Stahlecker (2001) may successfully explain the post-World War II U.S. data.

There are at least two important reasons why, even after thirty years since Deaton's (1986) publication, the excess consumption growth puzzle is still very important and relevant. First, any progress made toward understanding this fundamental inconsistency between theory and empirical evidence may shed light on other more widely researched puzzles, such as that of excess sensitivity of consumption (Flavin, 1981), the equity premium puzzle (Mehra and Prescott, 1985), and the risk-free rate puzzle (Weil, 1989). Second, even though Deaton's (1986) original findings are based on the U.S. data from the 1950s to the 1980s, negative short-term rates (real and nominal) are once again a part of the modern-day reality in the U.S. and around the world.[4] Will real consumption continue to grow, as it did during the three decades following 1950, or will it start declining, as the standard economic theory predicts? Our findings suggest that consumption will likely continue to grow so long as consumer predictions about future wealth remain sufficiently ambiguous and consumers respond to this ambiguity by making fuzzy, imprecise forecasts.

The rest of this paper is organized as follows. Section 2 develops a simple two-period model of consumer intertemporal choice under an ambiguous income constraint. Section 3 demonstrates that this model can be successfully calibrated to match the post-war average consumption growth and real interest rate.

2. Two-Period Model with Ambiguity and Fuzzy Expectations

Consider a representative agent who has to decide her current level of consumption, c_1, to maximize a two-period time-separable utility function:

$$U(c_1, c_2) = u(c_1) + \frac{1}{1+\delta} u(c_2). \tag{1}$$

In the utility function above, c_2 is the individual's second-period consumption, while δ is her discount factor with which she discounts future utility. The individual starts off the first period with no wealth, but receives some labor income y_1 and y_2 at the beginning of each period. She allocates some part of y_1 to the first-period consumption, c_1, and saves the rest for the future at some risk-free interest rate, r. The principal and earned interest plus her second-period income, y_2, are what the consumer will have available for her second-period consumption, c_2. Thus, the objective function (1) must be maximized subject to the following constraint:

$$A_2 = y_2 + (1+r)(y_1 - c_1), \tag{2}$$

where A_2 is the second-period wealth. The consumer must leave no savings beyond period two. Thus, the following transversality condition must hold:

$$c_2 = A_2. \tag{3}$$

Even though in our settings y_2 and r are not risky, the decision maker does not have complete and precise information about their magnitudes. This may, for example, be due to a complicated income tax system that the individual does not attempt to understand. Thus, the disposable levels of income and interest are ambiguous. As a result, the consumer must decide on her first-period optimal consumption by knowing that her second-period wealth is going to be *around* some perfectly known value $\tilde{A}_2$, which we refer to as the *benchmark* level of second-period wealth. The word "around" encompasses the total uncertainty due to all sources of ambiguity such as the complex income tax system. We assume that it is very costly or impossible to eliminate ambiguity completely, and so subjective projections of future wealth remain fuzzy.[5] In what follows, we refer to the approximate statements that a decision maker expresses linguistically in terms of fuzzy descriptors such as "around x"or "in the vicinity of x" as fuzzy projections or forecasts.

In order to choose optimal levels of c_1 and c_2, the multivalued nature of fuzzy projection "around $\tilde{A}_2$" needs to be resolved. For this purpose, we use the defuzzification strategy proposed by Hauenschild and Stahlecker (2001) that relies on the notions of fuzzy sets and membership functions.[6] Generally speaking, a membership function of a fuzzy set is a way of functionally assigning a grade of membership to any given element of the system in the fuzzy set. Membership functions are a generalization of the indicator function in the classical set theory and has a continuous range between zero and one: $\mu: \mathbb{R} \rightarrow [0,1]$. This study chooses a triangular membership function to describe the association between future wealth values A_2 and the fuzzy set described by the sentence "wealth around $\tilde{A}_2$."[7]

$$\mu(A_2) = \begin{cases} \dfrac{A_2 - (1-z)\tilde{A}_2}{\tilde{A}_2 z} & \text{if } (1-z)\tilde{A}_2 \leq A_2 \leq \tilde{A}_2 \\ \dfrac{(1+z)\tilde{A}_2 - A_2}{z\tilde{A}_2} & \text{if } \tilde{A}_2 \leq A_2 \leq (1+z)\tilde{A}_2 \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

The membership function (4) assigns an exact degree of membership between 0 and 1 to every possible second-period wealth A_2 to the fuzzy projection "around $\tilde{A}_2$." The parameter $z \in [0,1]$ governs the relative degree of ambiguity: the greater the value of z is, the wider the membership function and the more ambiguity there is in the model. Conversely, ambiguity vanishes as z approaches zero. Henceforth, we refer to this parameter as the *coefficient of relative ambiguity*. It represents mathematically what consumers have in mind when they make a fuzzy projection of the form "around $\tilde{A}_2$." Figure 1 plots four examples of membership functions (4) around $\tilde{A}_2 = \$1{,}000$, letting z take four different values: $\{0.05, 0.3, 0.5, 0.75\}$. The interpretation of z is straight-forward: for a given level of $\tilde{A}_2$, the consumer considers any value of A_2 in the interval $\tilde{A}_2 \pm \tilde{A}_2 z$ as "around $\tilde{A}_2$." The shapes of the membership functions in Figure 1 are unimodal, implying that values of A_2 far from $\tilde{A}_2$ are less obviously described as "around $\tilde{A}_2$," and thus the less is the membership of A_2 in the fuzzy set described by fuzzy "around $\tilde{A}_2$." The coefficient of relative ambiguity is one of the four parameters we use to calibrate the model, wherein we restrict its values to the range $[0,0.75]$.

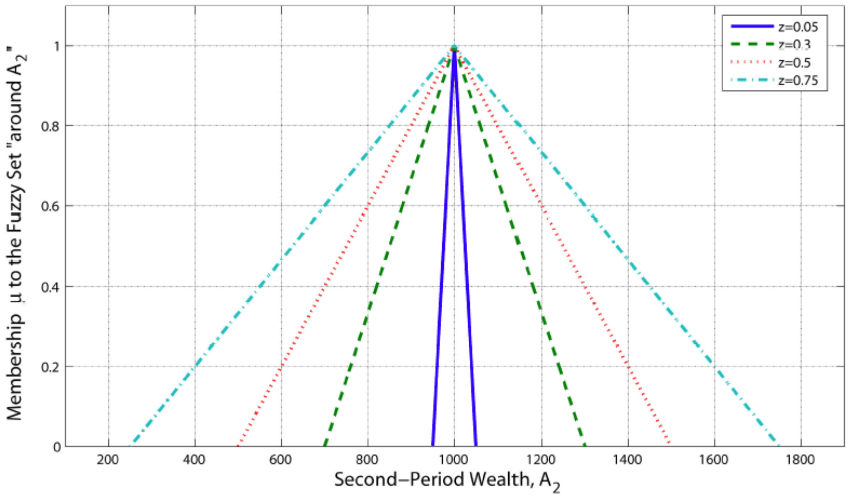
Definition: An α-cut of a fuzzy set Ω and its corresponding membership function μ is given by the following family of crisp sets:

$$P_\alpha = \{A_2 \in \mathbb{R}_+ | \mu(A_2) \geq \alpha\}, \tag{5}$$

where $\alpha \in (0,1]$.
It can be shown that the α-cut of the fuzzy set described by "around $\tilde{A}_2$" and the membership function (4) is a crisp set of all real numbers within the range:

$$[\tilde{A}_2 - \tilde{A}_2 z(1-\alpha), \tilde{A}_2 + \tilde{A}_2 z(1-\alpha)]. \tag{6}$$

Figure 1. The Graph of the Membership Function in (4) Parametrized by a Benchmark Level of Wealth $\tilde{A}_2 = \$1,000$ and Varous Levels of Ambiguity, z



Given the α-cut at $\alpha = \alpha^*$, an extremely optimistic consumer believes that her second-period wealth is going to be:

$$A_2(\alpha^*) = \tilde{A}_2 + \tilde{A}_2 z(1-\alpha^*). \tag{7}$$

An extremely pessimistic consumer projects that her second-period wealth is going to be

$$A_2(\alpha^*) = \tilde{A}_2 - \tilde{A}_2 z(1-\alpha^*). \tag{8}$$

We rewrite the subjective projected utility function of an optimistic consumer as follows. First, substitute (7) into (3). Second, substituting the resulting expression into (1) yields:

$$U(c_1) = u(c_1) + \frac{1}{1+\delta} u(\tilde{A}_2 + \tilde{A}_2 z(1-\alpha^*)). \tag{9}$$

Similarly, the utility of a pessimistic individual is given by:

$$U(c_1) = u(c_1) + \frac{1}{1+\delta} u\left(\tilde{A}_2 - \tilde{A}_2 z(1-\alpha^*)\right). \tag{10}$$

The utility of an individual whose optimism falls between these two extremes is:

$$U(c_1) = u(c_1) + \frac{1}{1+\delta}\left\{qu\left((1 - z(1-\alpha^*))\tilde{A}_2\right) + (1-q)u\left((1 + \right.\right. \tag{11}$$

$z(1-\alpha^*))\tilde{A}_2\big)\big\}$.

Here, q is the Arrow-Hurwicz optimism-pessimism index (Arrow and Hurwicz, 1972) that takes a value between 0 and 1 and represents extreme optimism and extreme pessimism, respectively, at these extremes. In these partial equilibrium settings, we assume that q is an exogenous free parameter that depends on socioeconomic events outside of the model. Aggregating (11) over all possible α-cuts, we get a version of the utility function in (1) that incorporates the effect of ambiguity:

$$U(c_1) = u(c_1) + \frac{1}{1+\delta}\{q\int_0^1 u((1-z(1-\alpha))\tilde{A}_2)\ d\alpha + (1-q)\int_0^1 u((1+z(1-\alpha))\tilde{A}_2)\ d\alpha\}. \tag{12}$$

We obtain the first-order necessary condition by recognizing that the wealth constraint (2) should hold for any level of wealth, including $\tilde{A}_2$. We then substitute the constraint $\tilde{A}_2 = y_2 + (1+r)(y_1 - c_1)$ into (12), differentiate the resulting expression with respect to c_1, set it equal to zero, and rearrange terms. After introducing $\tilde{A}_2$ back into the expression, the result is:

$$u'(c_1) = \frac{1+r}{1+\delta}\{q\int_0^1 (1-z(1-\alpha))u'((1-z(1-\alpha))\tilde{A}_2)\ d\alpha + (1-\ q)\int_0^1 (1+z(1-\alpha))u'((1+z(1-\alpha))\tilde{A}_2)\ d\alpha\}. \tag{13}$$

In order to get to a more usable version of (13), we assume that the consumer's preferences are described by a conventional constant relative risk aversion (CRRA) utility function of the form $u(c) = c\hat{}(1-\lambda)/(1-\lambda)$. Here, c is real consumption, and λ is the consumer's coefficient of relative risk aversion.[8] We substitute this utility function into (13) and rearrange terms:

$$c_1^{-\lambda} = \frac{1+r}{1+\delta}(\tilde{c}_2)^{-\lambda}\{q\int_0^1 (1-z(1-\alpha))^{1-\lambda}\ d\alpha + (1-q)\int_0^1 (1+\ z(1-\alpha))^{1-\lambda}\ d\alpha\}, \tag{14}$$

where $\tilde{c}_2$ is what the second-period consumption would have been if the consumer had a perfect (i.e. non-ambiguous) foresight about her future wealth being equal to the benchmark level of wealth, $\tilde{A}_2$. After evaluating the integrals explicitly, (14) simplifies to:

$$c_1^{-\lambda} = \frac{1+r}{1+\delta}(\tilde{c}_2)^{-\lambda}\left[\frac{(2q-1)-q(1-z)^{2-\lambda}+(1-q)(1+z)^{2-\lambda}}{z(2-\lambda)}\right]. \tag{15}$$

By taking the logarithm on both sides of the above expression and assuming that the interest rate and the rate of time preference are reasonably small, we approximate (15) as:

$$\Delta \ln c_2 = \frac{1}{\lambda}(r-\delta) + \frac{1}{\lambda}\ln\left(\frac{(2q-1)-q(1-z)^{2-\lambda}+(1-q)(1+z)^{2-\lambda}}{z(2-\lambda)}\right), \quad (16)$$

where $\Delta \ln c_2 = \ln \tilde{c}_2 - \ln c_1$. It can be shown that the Euler equation (14) and its simplified version (16) carry a few interesting features. First, it may shed light why the consumer confidence index is a better predictor of consumption growth than labor income or any other major macroeconomic variable (see Acemoglu and Scott, 1994). Second, in (14) we see that as z approaches zero, the fuzzy Euler equation (14) approaches its non-fuzzy, perfect foresight counterpart:

$$(c_1)^{-\lambda} = \frac{1+r}{1+\delta}(\tilde{c}_2)^{-\lambda} \Rightarrow \Delta \ln c_2 = \frac{1}{\lambda}(r-\delta). \quad (17)$$

Thus, the fuzzy model presented in this study nests the traditional perfect foresight model. Finally, we can show (see Balagyozyan and Giannikos, 2006) that as extreme optimism or pessimism becomes widespread, the heterogeneity in consumption patterns between more risk-averse and less risk-averse individuals disappears. This heterogeneity is most pronounced when individuals are optimism-pessimism neutral (i.e. when $q = 0.5$).

The second term on the right-hand side of (16) represents precautionary savings engendered by ambiguity. Equation (16) predicts that regardless of the sign of the first term on the right-hand side, consumption will continue to grow whenever the second term is positive and exceeds in absolute value the first term. As Balagyozyan and Giannikos (2016) demonstrate, for even negligible amounts of ambiguity (i.e. small values of z) the second term is positive whenever the individual is moderately risk averse and pessimistic. With more ambiguity, this term turns positive for even moderately optimistic individuals.

3. Calibration and Results

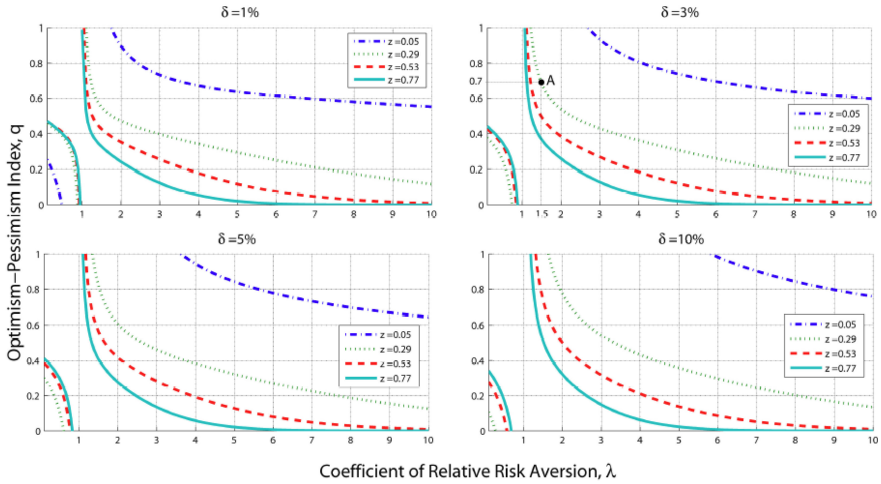
It is fairly straightforward to check whether the historical estimates of the average ex-post real interest rate and average consumption growth can be reconciled with (16). The second column of Table 1 reports the point estimates of average quarterly log consumption growth and the real interest rate, as reported by Deaton (1986, 1992) for the period between 3Q1953 and 4Q1984 (126 observations).

Table 1. Historical Moments of Quarterly Consumption Growth and the Real Rate of Interest between 3Q1953 and 4Q1984. These Numbers are Reported in Deaton (1992)

Statistics (3Q1953-4Q1984 N=126)	Point Estimates (as reported in Deaton, 1992)
$\overline{\Delta \ln c}$	0.005
$\overline{r}$	-0.0006

To calibrate the model to these post-World War II average values, we first substitute them into (16) and then numerically search for the optimism-pessimism index q, the coefficient of risk-aversion λ, the coefficient of time preference δ, and the coefficient of relative ambiguity z that solve the Euler equation (16). Figure 2 shows a graphical representation of these solutions. We consider scenarios with four different values of δ: $\delta = 1\%$, $\delta = 3\%$, $\delta = 5\%$, and $\delta = 10\%$ (per quarter). Each panel demonstrates the calibration results for one particular value of δ. The solution curves characterize all combinations of optimism-pessimism and risk-aversion that conform with (16), given the historical consumption growth pattern, ex-post real rate of interest, relative ambiguity, and rate of time preference. It can be readily observed in all the graphs that reasonable values exist for the optimism-pessimism index and risk aversion coefficient that match the model consumption growth with its empirical counterpart. For instance, if consumers discount future consumption at a quarterly rate of $\delta = 3\%$, (the top right panel), and the relative ambiguity is $z = 29\%$, then setting $\lambda = 1.5$ and $q = 0.7$ (point A) leads to a prediction of 0.5% quarterly consumption growth under the real quarterly rate of interest $r = -0.06\%$. It is interesting to see that when there is little ambiguity (e.g. $z = 0.05$), the model predicts excessive consumption growth for only pessimistic individuals. However, under greater amounts of ambiguity (e.g. $z > 0.29$), we observe excessive consumption growth for both pessimistic and optimistic individuals.

Figure 2. Values of the Optimism-pessimism Index, q, and Coefficients of Risk Aversion, λ, that Predict 0.5% Quarterly Consumption Growth when the Real Interest Rate is -0.06%



4. Conclusion

Deaton (1986) reports that there was a prolonged period in the U.S. post-World

War II history when real aggregate consumption continued to grow even though the real interest rate remained on average negative. This puzzling evidence contradicts the prediction of the standard economic theory. Our study is an inquiry into whether fuzzy decision-making and precautionary savings accumulated from ambiguity of future wealth can possibly explain this excess consumption growth puzzle. We modify the fuzzy decision-making methodology of Hauenschild and Stahlecker (2001) by representing preferences with a standard CRRA utility function and by proposing a particular triangular form for the membership function. Our results indicate that the model parametrized by even a negligible amount of ambiguity can successfully match the average rate of consumption growth of the post-World War II period. When there is little ambiguity, matching requires a considerable degree of consumer pessimism. However, with more ambiguity, pessimism is no longer required for addressing the puzzle. Thus, the model suggests that fuzzy decision-making, ambiguity, and/or general pessimism about future wealth might have been responsible for the excess consumption growth of the period.

Before concluding, we would like to offer a word of caution. Even though the model presented herein can match the historical moments of the real interest rate and consumption growth and is capable of doing so within fairly reasonable ranges of the model parameters, it must be clear that the model also imposes certain restrictions on the relationships between these parameters. In this study we do not attempt to advance any claims about the validity or plausibility of these relationships and leave that to future research. With this, however, our results imply that the puzzle of excess consumption growth disappears once ambiguity is properly accounted for by the proposed model.

Notes

1. See Nguyen, (2002) for an excellent discussion of the distinctions and connections between these two notions of uncertainty.
2. In this paper we treat the terms "vagueness" and "ambiguity" as synonyms and use them interchangeably.
3. Camerer and Weber (1992), Etner et al. (2012), and Guidolin and Rinaldi, (2013) offer extensive surveys of this literature.
4. According to the data from the FRED database of the U.S. Federal Reserve Bank of St. Louis, the 1-year ex-post real Treasury yield has been negative from the beginning of the 2007 recession until the time of this writing. (https://fred.stlouisfed.org/graph/?g=6aH7). In addition, between January 1, 2010 and July 1, 2016, the ex-ante real interest rate measured by the daily yield on a 5-year Treasury inflation-indexed security was on average -0.32% (https://fred.stlouisfed.org/graph/?g=62SO).
5. Individuals may choose to live with ambiguity, because the utility lost investigating the exact details can be tremendous. On the other hand, the utility loss from setting consumption according to a reasonable rule of thumb rather than the optimal permanent-income decision rule can be microscopic. See Cochrane (1989) for further support of this argument.
6. Fuzzy sets are a generalization of classical, crisp sets. In the case of crisp sets, a given element is

either a member of the set or not. In the case of fuzzy sets, on the other hand, the degree of membership of any element in a fuzzy set could be partial. A defuzzification strategy refers to the mathematical procedure of reducing a fuzzy set into a signleton scalar value. For further information on fuzzy sets, refer for instance to Zimmermann (1992) and Klir and Wierman (1999).

7. We choose this membership function, because of its mathematical simplicity. However, our subsequent analysis and results do not critically depend on this choice; any unimodal membership function will lead to the same results.
8. In order to ensure concavity of the utility function, λ is required to be greater than zero. Estimates of λ vary in the literature. Altug (1983) finds it to be near zero. Mankiw et al. (1985) establish that it is near 0.5. Hansen and Singleton (1983) estimate λ to be around 1. The estimate of Tobin and Dolde (1971) is 1.5. Friend and Blume (1975) find it to be around 2. Zeldes's (1989) estimates of λ are in the region of 2.3. Mankiw (1981, 1985) estimates λ to fall between 3 and 4. Barski et al. (1997) explore the results of a survey and find that the median coefficient of relative risk aversion is approximately 7.

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References

Acemoglu, D. and A. Scott, (1994), "Consumer Confidence and Rational Expectations: Are Agents' Beliefs Consistent with the Theory?" *The Economic Journal*, 104(422), 1-19.

Altug, S., (1983), "Gestation Lags and the Business Cycle: An Empirical Analysis," Carnegie-Mellon *Working Paper*, Presented at the Econometric Society Meeting, Stanford University (Carnegie-Mellon University, Pittsburgh, PA).

Arrow, K. and L. Hurwicz, (1972), "An Optimality Criterion for Decision-Making Under Ignorance," in *Uncertainty and Expectations in Economics: Essays in Honour of GLS Shackle, CF Carter and JL Ford*, Oxford: Basil Blackwell.

Balagyozyan, A. and C. Giannikos, (2006), "Modeling and Implications of Risk Aversion and Optimism-Pessimism in a Fuzzy Information Framework," *Working Paper*.

Balagyozyan, A. and C. Giannikos, (2016), "Ambiguity, Fuzzy Expectations, and the Equity Premium Puzzle," *Working Paper*.

Barski, R. B., F. T. Juster, M. S. Kimball, and M. D. Shapiro, (1997), "Preference Parameters and Behavioral Heterogeneity: An Expiremental in the Health and Retirement Study," *Quarterly Journal of Econoics*, 112(2), 537-579.

Caballero, R. J., (1990), "Consumption Puzzles and Precautionary Savings," *Journal of Monetary Economics*, 25, 113-136.

Camerer, C. and M. Weber, (1992), "Recent Developments in Modeling Preferences:

Uncertainty and Ambiguity," *Journal of Risk and Uncertainty*, 5, 325-370.

Cochrane, J. H., (1989), "The Sensitivity of Tests of the Intertemporal Allocation of Consumption to Near-Rational Alternatives," *The American Economic Review*, 79(3), 319-337.

Deaton, A., (1986), "Life-Cycle Models of Consumption: Is the Evidence Consistent with the Theory?" National Bureau of Economic Research, *Working Paper* No. 1910.

Deaton, A., (1992), *Understanding Consumption*, Oxford University Press Inc., New York.

Etner, J., M. Jeleva, and J.-M. Tallon, (2012), "Decision Theory under Ambiguity," *Journal of Economic Surveys*, 26(2), 234-270.

Flavin, M. A., (1981), "The Adjustment of Consumption to Changing Expectations about Future Income," *The Journal of Political Economy*, 89(5), 974-1009.

Friend, I. and M. Blume, (1975), "The Demand for Risky Assets," *American Economic Review*, 65(5), 900-922.

Goodman, I. R. and H. T. Nguyen, (2002), "Fuzziness and Randomness," in *Statistical Modeling, Analysis and Management of Fuzzy Data*, Springer, 3-21.

Guidolin, M. and F. Rinaldi, (2013), "Ambiguity in Asset Pricing and Portfolio Choice: A Review of the Literature," *Theory and Decision*, 74(2), 183-217.

Hansen, L. P. and K. Singleton, (1983), "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns," *Journal of Political Economy*, XCI, 249-265.

Hauenschild, N. and P. Stahlecker, (2001), "Precautionary Saving and Fuzzy Information," *Economics Letters*, 70(1), 107-114.

Klir, G. J. and M. J. Wierman, (1999), *Uncertainty-based Information: Elements of Generalized Information Theory*, Vol. 15, Springer.

Mankiw, N., (1981), "The Permanent Income Hypothesis and the Real Interest Rate," *Economic Letters*, 7(4), 307-311.

Mankiw, N., (1985), "Consumer Durables and the Real Interest Rate," *Review of Economics and Statistics*, 67(3), 353-362.

Mankiw, N., J. Rotemberg, and L. Summers, (1985), "Intertemporal Substitution in Macroeconomics," *The Quarterly Journal of Economics*, 104(2), 275-298.

Mehra, R. and E. Prescott, (1985), "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15(2), 145-161.

Tobin, J. and W. Dolde, (1971), "Wealth, Liquidity and Consumption," in *Consumer Spending and Monetary Policy: The Linkage*, Federal Reserve Bank of Boston, Boston, MA, 99-146.

Weil, P., (1989), "The Equity Premium Puzzle and the Risk-free Rate Puzzle," *Journal of Monetary Economics*, 24(3), 401-421.

Zeldes, S., (1989), "Consumption and Liquidity Constraints: An Empirical Investigation," *Journal of Political Economy*, 97(2), 305-346.

Zimmermann, H., (1992), *Fuzzy Set Theory and Its Applications*, Springer.