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Asset Pricing with Overlapping Generations and the Housing Market

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Abstract

This paper investigates a consumption-based asset pricing model in a three-period overlapping generations (OLG) setting. In each period, there is a representative agent from one of three age groups – young, middle-aged, and old. As the agent grows older, he becomes more risk averse. The consumption of the representative agents includes housing, and perishable goods and non-housing service. The higher risk aversion in the old cohort and the additional risk in the consumption composition may help to explain the risk-free rate puzzle and the risk premium puzzle with lower relative risk aversion.

1. Introduction

The U.S. housing market crash of 2007 came as an unwelcome reminder of how changes in housing prices can affect consumption. Looking forward, the aging of the U.S. population raises important questions about the impact of demographic changes on the economy in general and also on the financial markets that include a large portion of retirement assets. However, what about the combined impact of changes in housing prices and the aging population? In the context of these critical long-range trends, this paper seeks to offer a new perspective on the risk-free rate puzzle - a question that can have an important impact on the equity premium and financial markets in the coming decades.

Mehra and Prescott (1985) are the first to coin the equity risk premium puzzle, which has stimulated volumes of research in the past 25 years. They demonstrate that the observed difference between the returns from risky equity and risk-free bonds is too high to be justified by a reasonable degree of risk aversion in a time-separable, expected-utility framework. In their standard consumption-based model, the focus on the consumption risk stems from the co-movements of consumption growth and stock price. Since very high-risk aversion means very low intertemporal elasticity of substitution, the representative agent has a strong desire to smooth the

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inter-temporal consumption by borrowing. This leads to a much higher risk-free rate than observed in data (Weil (1989)).

Since a high equity risk premium and low risk-free rate are the result of either an investor's high-risk aversion or perceived high risk, researchers initially have focused on changing one or both of them. Kocherlakota (1996) provides an excellent review of the literature on the equity risk premium puzzle and the risk-free rate puzzle. This paper studies both factors by incorporating a more risk-averse old generation living on retirement income, as reflected in the U.S. demographic trend, and an additional risk in consumption composition from the consumption bundle, including perishable good and housing consumption.

Investors' risk aversion varies with income ((Bosche-Domenech and Silvestre (1999)) and age ((Palsso (1996), Baskshi and Chen (1994), Morin and Suarez (1983)). As the U.S. older population is growing faster than the young and the middle-age population, the older cohort's influence in the market is also increasing. DaSilva and Giannikos (2006) incorporate the higher risk aversion of the old population in their three-period overlapping generation model and are able to generate a higher equity risk premium and a lower risk-free rate.

Piazzesi et al. (2007) introduce a consumption composition risk into the consumption asset pricing model by incorporating housing consumption. Investors now not only face the uncertainty of aggregate consumption growth, but also the risk of the consumption composition between perishable goods and non-housing service on the one hand and the housing service on the other. The increased perceived risk helps to explain the equity premium puzzle.

This paper confirms the results of DaSilva and Giannikos (2006) and Piazzesi et al. (2007). The higher risk aversion also implies that the older generation is less willing to substitute consumptions over time. The lower intertemporal elasticity of substitution leads to more volatility in consumption and the consumption composition ratio when the agent retires. The middle-aged agent, who is the only participant in both the stock and bond markets, perceives higher risk ahead and demands a higher risk premium. A possible avenue for future research is to construct simulations on the joint process of the consumption basket and wage distribution between age cohorts. We would expect an improvement on the fit of the model to the historical data. The housing consumption composition is the additional risk to DaSilva and Giannikos (2006). The unfavorable consumption composition happens not only right before the recession as in Piazzesi et al. (2007), but also happens when the agent ages normally.

Park (2015) also studies the pricing kernel with consumption composition risk under overlapping generations. She uses housing service as the numeraire. Our model is distinct from this approach in its allowance for the heterogeneity of the risk aversion between age cohorts and the use of standard perishable goods (conceived to include both perishable goods and non-housing services) as the numeraire.

2. The Model

We consider a three-period OLG model, following the framework of Constantinides et al. (2002) and DaSilva and Giannikos (2004), where each generation lives through youth, middle-age, and seniority. There is one representative agent in each age group. The agent becomes more risk averse as he ages.

The consumption of the representative agents includes housing service, and perishable goods and non-housing service, which serve as the numeraire. (For the rest of the paper, when we talk about the perishable good, it includes non-housing service.) There is a risky stock with dividend payments d_t in the form of perishable goods. This risky stock is considered as the market portfolio. There are also discount bonds guaranteeing the numeraire perishable consumption when the agent moves to the next age cohort.

The representative agent born in period t has zero endowment in financial securities and earns a fixed low wage, $W^0 > 0$, in period t. He faces a borrowing constraint and is not allowed to borrow to invest in equity. When he is middle-aged, he earns a stochastic wage of $\widetilde{w}_{t+1} > 0$, in t+1. He buys houses and participates in both bond and equity markets. We assume that the total supply of bond, equity, and housing is one each in the market. The middle-aged agent holds fractions, $z^H_{t+1,1}$, $z^e_{t+1,1}$, and $z^b_{t+1,1}$, of the total market supply of housing, bonds, and equity. When he is old, without any labor income, he sells his houses and all his investments and pays rents. He will consume all his wealth in this period. The shares of the shelter occupied by three generations are $\eta_1 : (1 - \eta_1 - \eta_2) : \eta_2$.

The borrowing constraints imposed on the young generation make the model a lot simpler than the one without the constraint. If the young agent is allowed to borrow, then he would like to smooth his intertemporal consumption by borrowing against future labor income to participate in the equity market, especially when the correlation between the equity premium and the future wage is low (Constantinides et al. (2002)). However, he is young, and his human capital cannot serve as reliable collateral to borrow against the future. Furthermore, even if he is allowed to borrow, the marginal utility of his low consumption from his low wage is so high that he is not very willing to take the intertemporal substitution. The borrowing constraint assumption is supported by the calibration of Constantinides et al. (2002). With reasonable parameters, even when the young agent is allowed to borrow, "a young consumer is unwilling to sacrifice even one dollar of immediate consumption to put up as margin for the purchase of equity worth $56."

The representative agents in all age groups have the same utility function:

$$U = \frac{\left[\left(C_t^{\frac{\varepsilon-1}{\varepsilon}} + \omega S_t^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]^{\frac{\sigma-1}{\sigma}}}{\frac{\sigma-1}{\sigma}}, \tag{1}$$

where:

c_t is the consumption from the perishable good;
s_t is the service provided by the shelter;
ω is a constant, representing the relative importance of the housing service; and
σ is the intertemporal elasticity of substitution, and 1/ σ also represents the relative risk aversion.

The old generation is more risk averse. Thus, $\sigma_{old} < \sigma_{middle}$. It also means that the old generation is less willing to substitute its consumption basket over time. Its consumption basket includes the housing service and perishable goods and service excluding housing service.

The intratemporal elasticity of substitution between the perishable goods and housing service is captured by ε. A higher ε means two goods are stronger substitutes, and when ε approaches infinity, they become perfect substitutes. Agents are more willing to substitute the two goods in the same period with a higher ε. When ε= σ, the utility is time separable.

This is the utility function used in Piazzesi et al. (2007). When this utility function incorporates a consumption basket of perishable goods and housing service in the non-time separable CES utility function capsulated in the standard constant relative risk aversion environment, the utility function is now non-time-separable.

The life-time utility of the agent born in period t is:

$$E(\sum_{i=0}^{2} \beta^{i} U(c_{t,i};\ s_{t,i}|I_t)), \tag{2}$$

where β is the subjective discount rate, and I_t is the information set available at period t.

His budget constraints are:

$$W^0 \geq C_{t,0} + \rho \eta_1 z_{t,1}^{H} q_t^{H}, \tag{3a}$$

$$\widetilde{W}_{t+1}^{1} + \rho(\eta_1 + \eta_2) z_{t+1,1}^{H} q_{t+1}^{H} \geq C_{t+1,1} + z_{t+1,1}^{H} q_{t+1}^{H} + z_{t+1,1}^{e} q_{t+1}^{e} + z_{t+1,1}^{b} q_{t+1}^{b}, \tag{3b}$$

$$z_{t,1}^{e}(q_{t+2}^{e} + d_{t+2}) + z_{t,1}^{b} + (1-\Omega) z_{t+1,1}^{H} q_{t+2}^{H} \geq C_{t+2,2} + \rho \eta_2 z_{t+2,1}^{H} q_{t+2}^{H}. \tag{3c}$$

Here, $C_{i,j}$ and $s_{i,j}$ are the perishable consumption and housing service for an agent at age j in period i, respectively (when j=0, the agent is young; when j=1, the agent is middle- aged; and when j=2, the agent is old);
q_t^H is the price of housing in period t;
ρ is the rent, which is a proportion of the housing price;
q_t^e is the price of the risky equity;
q_t^b is the price of the risk-free bond;
d_{t+2} is the dividends issued by risky equity in period t+1;
Ω is the depreciation rate of the housing stock.

Let p_t^s be the price of each unit of service flow from the housing service. The consumption of the housing service for the middle-aged agent born in period t is then his investment in the house at time t+1 adjusting for his rental income:

$$s_{t+1} p_{t+1}^{s} = z_{t+1,1}^{H} p_{t+1}^{H} - (\eta_1 + \eta_2) \rho z_{t+1,1}^{H} p_{t+1}^{H}. \tag{4}$$

When the market clears, $z^H_{t+1,1} = z^e_{t+1,1} = z^b_{t+1,1} = 1$. At equilibrium, the market prices are $\{1, p^s_{t+1}, q^H_{t+1}, q^e_{t+1}, q^b_{t+1}\}$[1] such that the consumption bundles $\{c_t, s_t\}$ and the investment holdings $z^H_{t+1,1} = z^e_{t+1,1} = z^b_{t+1,1} = 1$ maximize agents' utility (1) subject to the budget constraints (3a-3c).

The middle-aged group is the only group investing in the financial market. Given that perishable consumption is the numeraire, the static first-order condition leads to

$$p^s_{t+1} = \frac{u_2(c_{t+1}, s_{t+1})}{u_1(c_{t+1}, s_{t+1})} = \omega\left(\frac{s_{t+1}}{c_{t+1}}\right)^{-\frac{1}{\varepsilon}}. \tag{5}$$

The perishable consumption ratio of the agent born in period t is:

$$\alpha_{t+1} = \frac{C_{t+1}}{C_{t+1} + p^s_{t+1} S_{t+1}} = \frac{C_{t+1}}{C_{t+1} + \omega\left(\frac{s_{t+1}}{c_{t+1}}\right)^{-\frac{1}{\varepsilon}} S_{t+1}} = \frac{1}{1 + \omega\left(\frac{s_{t+1}}{c_{t+1}}\right)^{1-\frac{1}{\varepsilon}}}. \tag{6}$$

The pricing kernel in the standard Mehra and Prescott (1985) is:

$$M_{t+1} = \beta\frac{u'(c_{t+2})}{u'(c_{t+1})} = \beta\left(\frac{C_{t+2}}{C_{t+1}}\right)^{-\frac{1}{\sigma}}. \tag{7}$$

The price of the risky asset is determined by consumption growth. When consumption growth is very smooth, as it is in the U.S. data, it requires very high-risk aversion to reproduce the equity premium observed.

Our pricing kernel (SDF) is:

$$M_{t+1} = \beta\left(\frac{C_{t+2}}{C_{t+1}}\right)^{-\frac{1}{\sigma_{middle}}} C_{t+2}^{\left(\frac{1}{\sigma_{middle}} - \frac{1}{\sigma_{old}}\right)} \left(\frac{1+\omega\left(\frac{S_{t+2}}{C_{t+2}}\right)^{\frac{\varepsilon-1}{\varepsilon}}}{1+\omega\left(\frac{S_{t+1}}{C_{t+1}}\right)^{\frac{\varepsilon-1}{\varepsilon}}}\right)^{\frac{\sigma_{middle}-\varepsilon}{\sigma_{middle}(\varepsilon-1)}} \left(\frac{1}{\alpha_{t+2}}\right)^{\left(\frac{\sigma_{old}-\varepsilon}{\sigma_{old}(\varepsilon-1)} - \frac{\sigma_{middle}-\varepsilon}{\sigma_{middle}(\varepsilon-1)}\right)}$$

$$= \beta \underbrace{\left(\frac{C_{t+2}}{C_{t+1}}\right)^{-\frac{1}{\sigma_{middle}}}}_{1} \underbrace{C_{t+2}^{\left(\frac{1}{\sigma_{middle}} - \frac{1}{\sigma_{old}}\right)}}_{2} \underbrace{\left(\frac{\alpha_{t+2}}{\alpha_{t+1}}\right)^{\frac{\varepsilon-\sigma_{middle}}{\sigma_{middle}(\varepsilon-1)}}}_{3} \underbrace{(\alpha_{t+2})^{\left(\frac{\varepsilon-\sigma_{old}}{\sigma_{old}(\varepsilon-1)} - \frac{\varepsilon-\sigma_{middle}}{\sigma_{middle}(\varepsilon-1)}\right)}}_{4} \tag{8}$$

The first term of the SDF is the same one in the standard one-good model with a power utility function. It captures the perishable good's consumption risk. In the state when the middle-aged agent expects a lower growth rate in the perishable consumption when he grows older, the perishable payoff will be highly valued.

The second to the fourth terms in our pricing kernel are additions to the standard pricing kernel. The second term is the same as the one in DaSiva and Giannikos (2006), and the third term looks similar to the one in Piazzesi et al. (2007), but it will generate a larger risk premium. The last term is new with our model.

Since the relative risk aversion for the retired is higher than that of the middle-aged, the second term actually decreases the mean SDF and leads to a higher risk-free rate. The intertemporal elasticity of substitution is the inverse of the risk aversion. The agent is less willing to substitute the perishable consumption across the time. The lack of incentive to postpone consumption drives up the equity return, but unfortunately at the same time increases the risk-free rate.

Simulations in DaSiva and Giannoks (2006) show that the consumption volatility of the old is much higher than that of the middle-aged, due to lower intertemporal elasticity of substitution in the second term. This will be a feedback to the first term. The middle-aged, knowing that they would face higher risk in consumption in the future in their life cycle when they dislike risk more, would like to purchase the risk-free bond to hedge their future perishable consumption risk. This will lower the risk-free rate and increase the equity premium. The impact on the risk-free rate from the feedback is greater than the undesirable direct impact from low intertemporal elasticity of substitution.

The third term involves the consumption composition risk of the agent as he ages. The relative housing consumption will decrease and the consumption composition (α) will increase when the agent moves into the old-age cohort. If intertemporal and intra-temporal elasticities of substitutions are the same, $\sigma = \varepsilon$, then consumption composition risk will not play any role in the price of the financial assets.

When the agent is more willing to substitute housing consumption with other consumption within the same period than he is to substitute the whole consumption basket across the time period, the intertemporal consumption smoothing is more important than the intertemporal substitution; $\sigma<\varepsilon$. Following Piazzesi et al. (2007), housing consumption and perishable consumption are substitutes; $\varepsilon > 1$. The third term increases the mean SDF. When expecting lower relative housing consumption in the old-age cohort, the middle-age agent would value perishable consumption more as a substitute for housing consumption. The risk-free bond with guaranteed payoff in the numeraire consumption good when the agent is old is very desirable and produces a lower interest rate. The risk-free bond is desirable not only when the perishable consumption is expected to be low in the future, but also when the relative shares of housing consumption decrease as the agent ages. The higher perceived future consumption composition risk will also increase the equity risk premium.

By incorporating overlapping generations, the improvement to Piazzesi et al. (2007) is that a lower risk-free rate and higher risk premium do not only happen in recessions when the agent expects a lower relative housing consumption in the future, but also in a normal state of the economy as long as the middle-aged agent wants to compensate for his lower relative housing consumption when he gets old.

The last term in the pricing kernel is a new one. It has similar effects as in the second term. On the surface, it decreases the SDF with the parameters in the model, $\varepsilon > 1$, $\sigma < \varepsilon$, and $\sigma_{old} < \sigma_{middle}$. It reflects the consumption composition risk faced by the agent when aging. Like the second term agent, the middle-aged agent is less willing to postpone his consumption to smooth his consumption basket over time. The price of the risk-free securities goes down and the risk-free rate increases. However, this low inter-temporal substitution elasticity also causes the old to face a more volatile relative housing consumption. It will become a feedback with greater impact to the third term. The middle-aged agent, now expecting higher consumption composition risk when he gets older, decides to buy more bonds to hedge the risk. The increase in precautionary savings will drive up bond prices and lead to a drop in their yield. This middle-aged agent's higher perceived consumption composition risk will thus increase the equity premium.

3. Conclusion

This paper investigates the risk-free rate puzzle and equity premium puzzle with three-period overlapping generations in an economy with two consumption goods: perishable goods and housing. The higher risk aversion and lower intertemporal elasticity of substitution of the older generation and the consumption composition risk help to increase the equity premium and reduce the risk-free rate. It may be promising to extend the current study with simulations of the joint process of consumption on two goods (perishable and housing) and the wage distributions across age cohorts. This simulation may show that the model is able to reproduce the basic statistics in the financial market with lower relative risk aversion ($1/\sigma$).

Notes

1. For the price of the perishable good, the numeraire is 1.

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