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Negative Royalty in Duopoly and Definition of License Fee: General Demand and Cost Functions

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Abstract

We extend the analysis about negative royalty in a duopoly with an outside innovator under linear demand and cost functions by Liao and Sen (2005) to a situation with general demand and cost functions. Moreover, we consider a case where an innovator has an option to enter the market and show that the optimal royalty rate for the innovator when it does not have an option to enter the market is smaller than that when it can enter the market. The sign of the optimal royalty rate depends on whether firms' goods are strategic substitutes or strategic complements. We provide a concise example of welfare analysis, which suggests that the prohibition of entry of the innovator into the market may be an appropriate policy.

1. Introduction

Liao and Sen (2005) analyze the problem of licensing by a combination of a royalty per output and a fixed fee under an oligopoly with an outside or an incumbent innovator, showing that when there are one licensee and one non-licensee, the innovator imposes a negative royalty (they call it a *subsidy*) with a positive fixed fee on the licensee, but they assume linear demand and cost functions. We extend the analysis to a situation with general demand and cost functions. This is similar to Kamien and Tauman (1986), who define the total license fee in the outside innovator case by the difference between the profit of the licensee when it buys the license and its profit when it does not buy the license and that the other incumbent firm (non-licensee) buys the license. However, if the outside innovator has an option to sell a license to the other incumbent firm and at the same time enter the market when the potential licensee refuses to buy the license, then we shall use a different definition of license fee.

Liao and Sen (2005) consider two license schemes.

(1) FR: Upfront fee plus royalty.

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(2) AR: Auction plus royalty.

In the FR policy, Liao and Sen (2005) state that the willingness to pay for the licensee is the difference between its profit as a licensee and the profit when no firm buys the license. This is smaller than the willingness to pay in the AR policy.[1] Thus, for the innovator the AR policy yields a higher payoff than the FR policy when one firm buys the license, and considering AR policy is sufficient.

In this paper we consider two scenarios about the license fee.

Scenario 1: If the potential licensee refuses the payment of a license fee, the other incumbent firm buys the license, and the innovator does not enter the market, then the willingness to pay for the licensee is the difference between its profit as a licensee and the profit of a non-licensee. This is the definition in Liao and Sen (2005).

Scenario 2: If the potential licensee refuses the payment of a license fee, the other incumbent firm buys the license, and at the same time the innovator enters the market, then it may involve more severe punishment than selling the license to the other firm without entering the market. The willingness to pay for the licensee is the difference between its profit as a licensee and its profit when the innovator enters the market with a license to the other incumbent firm.

We consider a model of an oligopoly in which firms produce substitutable goods under general demand and cost functions,[2] and that the innovator has a new cost-reducing technology that can be commonly used by all firms. We present the following results.

1. The optimal royalty rate in Scenario 1 is smaller than that in Scenario 2.
2. If the goods of the firms are strategic substitutes, then the optimal royalty rates in Scenario 1 and Scenario 2 are negative.
3. If the goods of the firms are strategic complements, then the optimal royalty rate in Scenario 1 may be positive or negative, but the one in Scenario 2 is positive.
4. When the non-licensee drops out of the market at the optimal royalty rate, Scenario 1 and Scenario 2 are equivalent.

In the related literature, no study has ever considered a negative royalty. Katz and Shapiro (1985) represent one of the earliest papers in the field and use general demand and cost functions, but only consider a fixed fee. Subsequently, Kamien et al. (1992) compare the pure auction, fixed fee, and royalty with general demand function and linear cost functions. Their results show that a royalty is inferior to the other two, and an auction is superior to the other two when the magnitude of innovation is not too small. Moreover, Wang (1998) note that a pure royalty may be superior to a fixed fee in a model of one incumbent innovator and one licensee. Similarly, Sen (2005) state that a pure royalty may be superior to the other two licensing schemes for an outside innovator. Wang (1998) and Sen (2005) use linear demand and cost functions, while Sen and Tauman (2007) discuss the optimal combinations of an auction and royalty. However, their paper employs linear demand and cost functions, but do not consider the threat of entry by the licensor. Fosfuri and Roca (2004) compare the royalty and the fixed fee. Using a model with linear demand and cost functions, they show that although a royalty is optimal for the incumbent innovator when it sells licenses to all other incumbent firms, a fixed fee is optimal when only some of the other incumbent firms buy licenses.

This present research considers only the outside innovator case. In the incumbent innovator case there is no problem of a definition of license fee, because in that case the innovator does not have an option whether its enters a market or not. We also analyze only a problem of a possibility of a negative royalty with one licensee and one non-license. For an outside innovator with two potential licensees, whether it sells a license to one firm or sells licenses to two firms is an important problem. However, analyses of the innovator's optimal choice as well as analyses of social welfare and public policy about licensing may be complicated under general demand and cost functions. We will study such problems in future research.

The rest of the paper runs as follows. Sectoin 2 describes the paper's model. Section 3 presents the main results summarized in three propositions. Section 4 considers a simple example with linear demand and cost functions and offers a brief analysis about social welfare. We will show that, in such an example, the social welfare in Scenario 1 is larger than the social welfare in Scenario 2. Thus, prohibition of entry of the innovator into the market may be an appropriate policy. Section 5 provides concluding remarks.

2. The Model

There are three firms: one outside innovator and two incumbent firms. The innovator has a superior cost reducing technology that can be commonly used by all firms and licenses its technology to the licensee. Another incumbent firm is the non-licensee. The licensee is called Firm A, the non-licensee is called Firm B, and the innovator is called Firm I. Firm I is an outside innovator at present, but it may enter the market if Firm A refuses to buy the license. Therefore, we consider the possibility of entry by Firm I so as to determine the license fee imposed on Firm A. Firm I does not really enter the market.

Denote the outputs of Firms A and B by x_A and x_B. The output of Firm I when it enters the market is x_I. The firms produce substitutable goods. The prices of the goods of Firms I, A, and B are p_I, p_A, p_B, and the inverse demand functions are written as $p_A(x_A, x_B)$ and $p_B(x_A, x_B)$, or $p_I(x_I, x_A, x_B)$, $p_A(x_I, x_A, x_B)$, and $p_B(x_I, x_A, x_B)$. We assume that all partial derivatives of the inverse demand functions are negative and are twice differentiable. The cost function of Firm A with a license of the new technology is $c_A(x_A)$. The cost function of Firm B is $c_B(x_B)$, and the cost function of Firm I when it enters the market is $c_A(x_I)$. They are increasing and twice differentiable. We assume $c_A(x_A) < c_B(x_B)$ and $c'_A(x_A) < c'_B(x_B)$ for $x_A = x_B$. Firm I imposes a royalty per output and a fixed fee on Firm A. We denote the royalty rate by r.

We calculate the profit of Firm A net of the royalty and the profit of Firm B as:

$$\pi_A = p_A x_A - c_A(x_A) - r x_A,$$

and

$$\pi_B = p_B x_B - c_B\left(x_B\right).$$

To determine the total license fee we consider the two following scenarios.

Scenario 1: The innovator does not enter the market. According to the auction policy by the innovator in Liao and Sen (2005), if Firm A refuses the payment of a license fee, then Firm B buys the license, and the willingness to pay for Firm A is the difference between its profit as a licensee and the profit of a non-licensee - that is, $\pi_A - \pi_B$. Let L be the fixed license fee, and thus we have:

$$L = \pi_A - \pi_B.$$

The payoff of the innovator is the sum of the royalty and the fixed license fee, denoted by:

$$\varphi_1 = L + rx_A = p_A x_A - c_A\left(x_A\right) - \left(p_B x_B - c_B\left(x_B\right)\right).$$

Scenario 2: The innovator has an option to enter the market when Firm A refuses to buy a license. Firm B then buys the license and at the same time Firm I enters the market. Firm A must pay the difference between its profit as a licensee when Firm I does not enter and its profit when Firm I enters the market with a license to Firm B. The fixed license fee, L, is:

$$L = \pi_A - \pi_A^e.$$

Here, π_A^e denotes the profit of Firm A when Firm I enters the market and Firm B buys the license. Firm A is then a non-licensee. The Appendix includes a concise description of the equilibrium in that case.

We denote the payoff of the innovator in this case by:

$$\varphi_2 = L + rx_A = p_A x_A - c_A\left(x_A\right) - \pi_A^e.$$

Note that π_A^e is a constant - that is, it does not depend on the royalty rate when Firm I does not enter, because the optimal royalty rate when Firm I enters the market is different from (and independent of) that when it does not enter. On the other hand, π_B in φ_1 is not constant. It depends on the optimal royalty rate without entry of Firm I.

Severity and Credibility of Punishment

If $\pi_A^e < \pi_B$, then the entry of Firm I with a license to Firm B is more severe punishment for Firm A than a license to Firm B without entry of Firm I when it does not buy the license. Here, π_B is the profit of a non-licensee in a duopoly. On the other hand, π_A^e is the profit of a non-licensee in an oligopoly with three firms in which two firms, Firm I and B, use the new technology. Thus, we can see that π_A^e is usually smaller than π_B

When $\pi_A^e < \pi_B$, we have $\varphi_2 > \varphi_1$, and Scenario 2 is more profitable than Scenario 1 for Firm I. However, its entry is not necessarily a credible threat. The payoff of Firm I when it enters the market with a license to Firm B is the sum of its profit as a firm in an oligopoly and the licensee fee imposed on Firm B (not Firm A). If it is larger than φ_1, then entry into the market is a credible threat.

We assume that the output of Firm B, x_B, is positive when the royalty rate is zero. However, if the innovator imposes a negative royalty on Firm A, then x_B may be zero. We consider two cases about demand functions. One case is when the goods are strategic substitutes, and the other case is when the goods are strategic complements. We further consider two cases about the innovation. The first is when the non-licensee continues to operate after a negative royalty is imposed on the licensee, and the second is when the non-licensee drops out of the market with the optimal royalty rate. In the former case the innovation is non-drastic, and in the latter case it is drastic. We shall show that in the case where the non-licensee drops out of the market, Scenario 1 and Scenario 2 are equivalent.

3. The Results

3.1 Firm Behavior

The first-order conditions for profit maximization of Firm A and Firm B are:

$$p_A + \frac{\partial p_A}{\partial x_A} x_A - c'_A(x_A) - r = 0, \tag{1}$$

and

$$p_B + \frac{\partial p_B}{\partial x_B} x_B - c'_B(x_B) = 0. \tag{2}$$

Their second-order conditions are:

$$2\frac{\partial p_A}{\partial x_A} + \frac{\partial^2 p_A}{\partial x_A^2} x_A - c''_A(x_A) < 0,$$

and

$$2\frac{\partial p_B}{\partial x_B} + \frac{\partial^2 p_B}{\partial x_B^2} x_B - c''_B(x_B) < 0.$$

Differentiating (1) and (2) with respect to r yields:

$$\left(2\frac{\partial p_A}{\partial x_A}+\frac{\partial^2 p_A}{\partial x_A^2}x_A-c''_A(x_A)\right)\frac{dx_A}{dr}+\left(\frac{\partial p_A}{\partial x_B}+\frac{\partial^2 p_A}{\partial x_A x_B}x_A\right)\frac{dx_B}{dr}=1,$$

and

$$\left(\frac{\partial p_B}{\partial x_A}+\frac{\partial^2 p_B}{\partial x_A x_B}x_B\right)\frac{dx_A}{dr}+\left(2\frac{\partial p_B}{\partial x_B}+\frac{\partial^2 p_B}{\partial x_B^2}x_B-c''_B(x_B)\right)\frac{dx_B}{dr}=0.$$

From them we obtain:

$$\frac{dx_A}{dr}=\frac{2\frac{\partial^2 p_B}{\partial x_B^2}+\frac{\partial^2 p_B}{\partial x_B^2}x_B-c''_B(x_B)}{\Delta},$$

and

$$\frac{dx_B}{dr}=-\frac{\frac{\partial p_B}{\partial x_A}+\frac{\partial^2 p_B}{\partial x_A x_B}\mathrm{x}_B}{\Delta},$$

where

$$\Delta=\left(2\frac{\partial^2 p_B}{\partial x_B^2}+\frac{\partial^2 p_B}{\partial x_B^2}x_B-c''_B(x_B)\right)\left(2\frac{\partial p_A}{\partial x_A}+\frac{\partial^2 p_A}{\partial x_A^2}x_A-c''_A(x_A)\right)$$
$$-\left(\frac{\partial p_B}{\partial x_A}+\frac{\partial^2 p_B}{\partial x_A x_B}x_B\right)\left(\frac{\partial p_A}{\partial x_B}+\frac{\partial^2 p_A}{\partial x_A x_B}x_A\right).$$

We assume that:

$$\left|2\frac{\partial p_A}{\partial x_A}+\frac{\partial^2 p_A}{\partial x_A^2}x_A-c''_A(x_A)\right|>\left|\frac{\partial p_A}{\partial x_B}+\frac{\partial^2 p_A}{\partial x_A x_B}x_A\right|, \tag{4}$$

and

$$\left|2\frac{\partial p_B}{\partial x_B}+\frac{\partial^2 p_B}{\partial x_B^2}x_B-c''_B(x_B)\right|>\left|\frac{\partial p_B}{\partial x_A}+\frac{\partial^2 p_B}{\partial x_A x_B}x_B\right|. \tag{5}$$

They imply that:

$\Delta > 0$.

We derive these assumptions from the stability conditions for a duopoly (see Seade (1980) and Dixit (1986)). Equations (4) and (5) mean that the absolute values of the slopes of the firms' reaction curves are smaller than one. Therefore, we get:

$$\frac{dx_A}{dr} < 0,$$

and

$$\left|\frac{dx_A}{dr}\right| > \left|\frac{dx_B}{dr}\right|,$$

The goods of the firms are strategic substitutes when $\partial p_B/\partial x_A + (\partial^2 p_B/\partial x_A x_B)\, x_B < 0$ and strategic complements when $\partial p_B/\partial x_A + (\partial^2 p_B/\partial x_A x_B)\, x_B > 0$. Thus, we state the following.

1. When the goods of the firms are strategic substitutes, $dx_B/dr > 0$.
2. When the goods of the firms are strategic complements, $dx_B/dr < 0$.

3.2 Comparison of Two Scenarios

Suppose that Firm B does not drop out of the market.

Scenario 1: The innovator does not enter the market.

The condition for maximization of φ_1 with respect to r is:

$$\begin{aligned}\frac{d\varphi_1}{dr} &= \left(p_A + \frac{\partial p_A}{\partial x_A}x_A - c'_A(x_A) - \frac{\partial p_B}{\partial x_A}x_B\right)\frac{dx_A}{dr} \\ &- \left(p_B + \frac{\partial p_B}{\partial x_B}x_B - c'_B(x_B) - \frac{\partial p_A}{\partial x_B}x_A\right)\frac{dx_B}{dr} \\ &= \left(r - \frac{\partial p_B}{\partial x_A}x_B\right)\frac{dx_A}{dr} + \frac{\partial p_A}{\partial x_B}x_A\frac{dx_B}{dr} = 0.\end{aligned} \tag{6}$$

We get the optimal royalty rate for the innovator as follows.

$$\tilde{r}_1 = \frac{1}{\frac{dx_A}{dr}}\left(\frac{\partial p_B}{\partial x_A}x_B\frac{dx_A}{dr} - \frac{\partial p_A}{\partial x_B}x_A\frac{dx_B}{dr}\right). \tag{7}$$

Scenario 2: The innovator enters the market when Firm A refuses to buy a license.

The condition for maximization of φ_2 with respect to r is:

$$\begin{aligned}\frac{d\varphi_2}{dr} &= \left(p_A + \frac{\partial p_A}{\partial x_A}x_A - c'_A\left(x_A\right)\right)\frac{dx_A}{dr} + \frac{\partial p_A}{\partial x_B}x_A\frac{dx_B}{dr} \\ &= r\frac{dx_A}{dr} + \frac{\partial p_A}{\partial x_B}x_A\frac{dx_B}{dr} = 0.\end{aligned} \tag{8}$$

We then get the optimal royalty rate for the innovator as follows.

$$\tilde{r}_2 = -\frac{1}{\frac{dx_A}{dr}}\frac{dx_B}{dr}\frac{\partial p_A}{\partial x_B}x_A. \tag{9}$$

Suppose that $\tilde{r}_1 = \tilde{r}_2$ and (8) is satisfied. Here dx_A/dr , and $(\partial p_A/\partial x_B)x_A(dx_B/dr)$ in (6) and those in (8) are equal. Thus, we have:

$$\frac{d\varphi_1}{dr} = -\frac{\partial p_B}{\partial x_A}x_B\frac{dx_A}{dr} < 0.$$

Therefore, $\tilde{r}_1 < \tilde{r}_2$. We have shown the following proposition.

Proposition 1 *The optimal royalty rate in Scenario 1 is smaller than that in Scenario 2.*

If both optimal royalty rates are negative, then the absolute value of the optimal royalty in Scenario 1 is larger than that in Scenario 2.

From (7) and (9) we can show the following result.

Proposition 2 *Suppose that Firm B does not drop out of the market.*

1. *If the goods of the firms are strategic substitutes, then the optimal royalty rates in Scenario 1 and Scenario 2 are negative.*

2. *If the goods of the firms are strategic complements, then the optimal royalty rate in Scenario 1 may be positive or negative, but that in Scenario 2 is positive.*

Proof.

1. If the goods of the firms are strategic substitutes we have $\frac{dx_B}{dr} > 0$, and then

$\tilde{r}_1 < 0$, because $\frac{dx_A}{dr} < 0$. We also have $\tilde{r}_2 < 0$.

2. If the goods of the firms are strategic complements, then $\frac{dx_B}{dr} < 0$, and we have $\tilde{r}_1 > 0$ or $\tilde{r}_1 < 0$ depending on $\frac{\partial p_B}{\partial x_A} x_B \frac{dx_A}{dr} - \frac{\partial p_A}{\partial x_B} x_A \frac{dx_B}{dr} < 0$ or $\frac{\partial p_B}{\partial x_A} x_B \frac{dx_A}{dr} - \frac{\partial p_A}{\partial x_B} x_A \frac{dx_B}{dr} > 0$. On the other hand, $\frac{dx_B}{dr} < 0$ means $\tilde{r}_2 > 0$. If x_B is sufficiently smaller than x_A, then even when Firm B does not drop out, it is likely that $\frac{\partial p_B}{\partial x_A} x_B \frac{dx_A}{dr} - \frac{\partial p_A}{\partial x_B} x_A \frac{dx_B}{dr} < 0$ and $\tilde{r}_1 > 0$. (Q.E.D.)

A Case where Firm B Drops Out

Suppose that at some royalty rate Firm B drops out of the market. In Scenario 1 we then have:

$$\left.\frac{d\varphi_1}{dr}\right|_{x_B=0} = r\frac{dx_A}{dr} + \frac{\partial p_A}{\partial x_B} x_A \frac{dx_B}{dr}.$$

From (8) also in Scenario 2 we get the same relation. Thus, when Firm B drops out of the market, Scenario 1 and Scenario 2 are equivalent, and we obtain the following results.

Proposition 3 *In the case where Firm B drops out of the market we obtain the following results.*

1. *If the goods of the firms are strategic substitutes, then the optimal royalty rate is negative.*
2. *If the goods of the firms are strategic complements, then the optimal royalty rate is positive.*

Proof.

1. If

$$\left.\frac{d\varphi}{dr}\right|_{x_B=0} = r\frac{dx_A}{dr} + \frac{\partial p_A}{\partial x_B} x_A \frac{dx_B}{dr} > 0,$$

then $x_B > 0$ at the optimal state for the innovator, and we have the case in the previous proposition where φ is φ_1 or φ_2. On the other hand, if $\frac{d\varphi}{dr} \le 0$ when $x_B = 0$, then the licensee is a monopolist and the optimal royalty rate for the innovator is one such that $x_B = 0$. This is negative, because $x_B > 0$ with zero royalty and $\frac{dx_B}{dr} > 0$.

2. If $\frac{d\varphi}{dr} < 0$ at $x_B = 0$, then $x_B > 0$ at the optimal state for the innovator, and we have the case in the previous proposition. On the other hand, if $\frac{d\varphi}{dr} \geq 0$ at $x_B = 0$, then the licensee is a monopolist and the optimal royalty rate for the innovator is one such that $x_B = 0$. This is positive, because $x_B > 0$ with zero royalty and $\frac{dx_B}{dr} < 0$. (Q.E.D.)

4. An Example and Policy Implications

Now let us consider an example. Assume that the goods of the firms are homogeneous. The inverse demand function is:

$$p = a - x_A - x_B.$$

The cost functions of Firm A and Firm B after adoption of the new technology by Firm A are respectively $(c - \varepsilon)x_A$ and cx_B. Here, c and ε are positive constants such that $c > \varepsilon$, and we assume $a - c > 2\varepsilon$. Next, we have:

$$x_A = \frac{a - c - 2r + 2\varepsilon}{3}, x_B = \frac{a - c + r - \varepsilon}{3}.$$

From this, we get:

$$\frac{dx_A}{dr} = -\frac{2}{3}, \frac{dx_B}{dr} = \frac{1}{3}.$$

The optimal royalty rate for the innovator in Scenario 1 and that in Scenario 2 are:

$$\tilde{r}_1 = -\frac{a - c}{2} < 0,$$

and

$$\tilde{r}_2 = -\frac{a - c + 2\varepsilon}{4} < 0.$$

Comparing them, we have:

$$\tilde{r}_1 - \tilde{r}_2 = -\frac{a-c-2\varepsilon}{4} < 0. \tag{10}$$

With $r = \tilde{r}_1$, the equilibrium values of the outputs are:

$$x_A = \frac{2(a-c+\varepsilon)}{3}, x_B = \frac{a-c-2\varepsilon}{6}.$$

With $r = \tilde{r}_2$, the equilibrium values of the outputs are:

$$x_A = \frac{a-c+2\varepsilon}{2}, x_B = \frac{a-c-2\varepsilon}{4}.$$

When $r = \tilde{r}_1$, the total output is:

$$X_1 = x_A + x_B = \frac{5a-5c+2\varepsilon}{6}.$$

When $r = \tilde{r}_2$, the total output is:

$$X_2 = x_A + x_B = \frac{3a-3c+2\varepsilon}{4}.$$

Comparing them, we find:

$$X_1 - X_2 = \frac{a-c-2\varepsilon}{12} > 0.$$

Thus, the total output in Scenario 1 is larger than that in Scenario 2, because the optimal royalty rate in Scenario 1 is smaller than that in Scenario 2 as shown in (10).

The welfare, which is the sum of the consumers' surplus and the total profits of the firms including the innovator, in Scenario 1 is:

$$\tilde{W}_1 = \frac{65a^2 - 130ac + 65c^2 + 124a\varepsilon - 124c\varepsilon + 68\varepsilon^2}{72}.$$

The welfare in Scenario 2 is:

$$\tilde{W}_2 = \frac{21a^2 - 42ac + 21c^2 + 60a\varepsilon - 60c\varepsilon + 52\varepsilon^2}{32}.$$

Comparing them, we find:

$$\tilde{W}_1 - \tilde{W}_2 = \frac{(a-c-2\varepsilon)(71a-71c+98\varepsilon)}{288} > 0 .$$

This means that, in this example, the welfare in Scenario 2 is smaller than the welfare in Scenario 1. Thus, for example, the prohibition of entry of the innovator into market may be an appropriate policy.

5. Concluding Remarks

This paper has extended an analysis of a negative royalty in a duopoly under linear demand and cost functions by Liao and Sen (2005) to a situation with general demand and cost functions. We further have considered two cases for the determination of license fees: Scenario 1) the innovator does not have an option to enter the market; and Scenario 2) the innovator has an option to enter the market when the licensee refuses to buy a license. We have shown that the optimal royalty rate in Scenario 1 is smaller than that in Scenario 2, and that the sign of the optimal royalty rate depends on whether the goods are stratgic substitutes or complements. The strategy of entry threat by the licensor used in Scenario 2) reduces the licensor's profit and is socially undesirable.

In future research we want to analyze the optimal strategy of the innovator - for example, whether it sells licenses to one firm or two firms in an oligopoly with two incumbent firms. We also want to examine social welfare and public policy about licensing under general demand and cost functions.

Notes

1. When two firms buy the licenses, the FR policy and the AR policy coincide.
2. Sen and Stamatopoulos (2016) present an analysis of a royalty and fixed fee in a duopoly under general demand and cost functions.

Appendix: Main Points of the Case where the Innovator Enters the Market with a License to Firm B

Suppose that Firm A refuses to buy the license, and Firm I enters the market with a license to Firm B. Firm B then uses the new technology, and Firm A uses the old technology. Denote the output and the profit of the innovator by x_I and π_I. The firms' profits are:

$$\pi_I = p_I x_I - c_A(x_I),$$
$$\pi_B = p_B x_B - c_A(x_B) - r x_B,$$

and

$$\pi_A = p_A x_A - c_B\left(x_A\right).$$

In this case Firm B pays the license fee. The first-order conditions for profit maximization of Firms I, A, and B are:

$$p_I + \frac{\partial p_I}{\partial x_I} x_I - c'_A\left(x_I\right) = 0, \tag{10}$$

$$p_B + \frac{\partial p_B}{\partial x_B} x_B - c'_A\left(x_B\right) - r = 0, \tag{11}$$

$$p_A + \frac{\partial p_A}{\partial x_A} x_A - c'_B\left(x_A\right) = 0. \tag{12}$$

The second-order conditions are:

$$2\frac{\partial p_I}{\partial x_I} + \frac{\partial^2 p_I}{\partial x_I^2} x_I - c''_A\left(x_I\right) < 0,$$

$$2\frac{\partial p_B}{\partial x_B} + \frac{\partial^2 p_B}{\partial x_B^2} x_B - c''_A\left(x_B\right) < 0,$$

and

$$2\frac{\partial p_A}{\partial x_A} + \frac{\partial^2 p_A}{\partial x_A^2} x_A - c''_B\left(x_A\right) < 0.$$

Differentiating (10), (11), and (12) with respect to r yields:

$$\lambda_I \frac{dx_I}{dr} + \sigma_{IB}\frac{dx_B}{dr} + \sigma_{IA}\frac{dx_A}{dr} = 0,$$

$$\sigma_{BI}\frac{dx_I}{dr} + \lambda_B\frac{dx_B}{dr} + \sigma_{BA}\frac{dx_A}{dr} = 1,$$

and

$$\sigma_{AI}\frac{dx_I}{dr} + \sigma_{AB}\frac{dx_B}{dr} + \lambda_A\frac{dx_A}{dr} = 0.$$

From them we obtain:

$$\frac{dx_I}{dr} = -\frac{\lambda_A\sigma_{IB} - \sigma_{AB}\sigma_{IA}}{\Gamma},$$

$$\frac{dx_B}{dr} = \frac{\lambda_A\lambda_I - \sigma_{AI}\sigma_{IA}}{\Gamma},$$

$$\frac{dx_A}{dr} = -\frac{\sigma_{AB}\lambda_I - \sigma_{AI}\sigma_{IB}}{\Gamma},$$

where

$$\Gamma = \lambda_I \lambda_B \lambda_A - \lambda_I \sigma_{BA}\sigma_{AB} - \lambda_B \sigma_{IA}\sigma_{AI} - \lambda_A \sigma_{IB}\sigma_{BI} + \sigma_{IB}\sigma_{BA}\sigma_{AI} + \sigma_{IA}\sigma_{BI}\sigma_{AB},$$

$$\lambda_I = 2\frac{\partial p_I}{\partial x_I} + \frac{\partial^2 p_I}{\partial x_I^2} x_I - c"_A\left(x_I\right),$$

$$\lambda_B = 2\frac{\partial p_B}{\partial x_B} + \frac{\partial^2 p_B}{\partial x_B^2} x_B - c"_A\left(x_B\right),$$

$$\lambda_A = 2\frac{\partial p_A}{\partial x_A} + \frac{\partial^2 p_A}{\partial x_A^2} x_A - c"_B\left(x_A\right),$$

$$\sigma_{IB} = \frac{\partial p_I}{\partial x_B} + \frac{\partial^2 p_I}{\partial x_I x_B} x_I,$$

$$\sigma_{IA} = \frac{\partial p_I}{\partial x_A} + \frac{\partial^2 p_I}{\partial x_I x_A} x_I,$$

$$\sigma_{BI} = \frac{\partial p_B}{\partial x_I} + \frac{\partial^2 p_B}{\partial x_I x_B} x_B,$$

$$\sigma_{BA} = \frac{\partial p_B}{\partial x_A} + \frac{\partial^2 p_B}{\partial x_B x_A} x_B,$$

$$\sigma_{AI} = \frac{\partial p_A}{\partial x_I} + \frac{\partial^2 p_A}{\partial x_I x_A} x_A,$$

and

$$\sigma_{AB} = \frac{\partial p_A}{\partial x_B} + \frac{\partial^2 p_A}{\partial x_B x_A} x_A.$$

Firm B must pay the following total license fee.

$$rx_B + \pi_B - \pi_A.$$

If Firm B refuses to buy the license, then Firm A in turn buys the license. The fixed fee is $\pi_B - \pi_A$. The payoff of Firm I is:

$$\varphi_e = \pi_I + rx_B + \pi_B - \pi_A = p_I x_I - c_A\left(x_I\right) + p_B x_B - c_A\left(x_B\right) - \left(p_A x_A - c_B\left(x_A\right)\right).$$

The condition for maximization of φ_e with respect to r is:

$$\frac{d\varphi_e}{dr}=\left(p_I+\frac{\partial p_I}{\partial x_I}x_I-c'_A\left(x_I\right)+\frac{\partial p_B}{\partial x_I}x_B-\frac{\partial p_A}{\partial x_I}x_A\right)\frac{dx_I}{dr}?$$
$$+\left(p_B+\frac{\partial p_B}{\partial x_B}x_B-c'_A\left(x_B\right)+\frac{\partial p_I}{\partial x_B}x_I-\frac{\partial p_A}{\partial x_B}x_A\right)\frac{dx_B}{dr}?$$
$$-\left(p_A+\frac{\partial p_A}{\partial x_A}x_A-c'_B\left(x_A\right)-\frac{\partial p_I}{\partial x_A}x_I-\frac{\partial p_B}{\partial x_A}x_B\right)\frac{dx_A}{dr}$$
$$=\left(\frac{\partial p_B}{\partial x_I}x_B-\frac{\partial p_A}{\partial x_I}x_A\right)\frac{dx_I}{dr}+\left(r+\frac{\partial p_I}{\partial x_B}x_I-\frac{\partial p_A}{\partial x_B}x_A\right)\frac{dx_B}{dr}$$
$$+\left(\frac{\partial p_I}{\partial x_A}x_I+\frac{\partial p_B}{\partial x_A}x_B\right)\frac{dx_A}{dr}=0.$$

We express the optimal royalty rate for the innovator as:

$$\tilde{r}=-\frac{1}{\frac{dx_B}{dr}}\left[\left(\frac{\partial p_B}{\partial x_I}x_B-\frac{\partial p_A}{\partial x_I}x_A\right)\frac{dx_I}{dr}+\left(\frac{\partial p_I}{\partial x_B}x_I-\frac{\partial p_A}{\partial x_B}x_A\right)\frac{dx_B}{dr}\right.$$
$$\left.+\left(\frac{\partial p_I}{\partial x_A}x_I+\frac{\partial p_B}{\partial x_A}x_B\right)\frac{dx_A}{dr}\right].$$

We denote the profit of Firm A in this case by π_A^e.

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