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An Oligopoly Game with Network Effects for Compatible and Incompatible Standards: As Applied to Short and Multimedia Message Services

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Abstract

We develop a game theoretical model to characterize how two oligopoly firms choose the levels of standards compatibility or interoperability in the context of short and multimedia message services (MMS). When the network effect is strong, both firms choose the ratified standards that are completely interoperable, as in the case of short message service (SMS) worldwide. In contrast, when the network effect is weak, as in the case of MMS in the U.S., it is more likely that firms pursue partially incompatible protocols that differ from the ratified standard. Such an equilibrium is similar to the prisoner's dilemma game as the firms reach the inefficient equilibrium. At the same time, when the government intervenes in choosing messaging protocols, it is also possible for the firms to reach the efficient equilibrium, as in the case of MMS in China.

Key words: Compatible and incompatible standards; interoperability; multimedia message service (MMS); Nash equilibrium; network effect; oligopoly, prisoner's dilemma game; short message services (SMS)

JEL classification: D21; D43; D62; L15; L96

1. Introduction

The short message service (SMS, a.k.a. text messaging) is a data protocol that conveys a message of up to 160 characters from one cell phone to another. The specification of SMS was ratified into the Global System for Mobile Communications in 1987, and the first SMS message was sent in 1992 (Le Bodic,

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2005). SMS provides wireless carriers and equipment manufacturers with significant revenue from value-added services (Bandera, 2016b): in 2011, 7.8 trillion SMS text messages were sent, which generated about $128 billion in revenue (MobiThinking, 2012), or roughly 1.6 cents per text message, with a profit margin of over 70% (Ahonen, 2012).

In 2002, the Open Mobile Alliance (OMA) ratified the multimedia messaging service (MMS, a.k.a. picture messaging) protocol which supports mobile multimedia services that are more engaging than what could be possible with text alone (Le Bodic, 2003). The OMA consists of 200 telecommunications companies including service providers and the manufacturers of handsets and infrastructure hardware. The OMA has also been built upon the contributions from the Internet Engineering Task Force and the World Wide Web Consortium. MMS is thus a more complex messaging protocol than SMS; the former conveys pictures, video, audio, and text (hundreds of thousands of characters) and supports multiple recipients, whereas the latter does not.

At that time and bolstered by the deployment of cell phones with cameras and graphics screens, the telecommunications industry projected that MMS usage would surpass that of SMS by 2005 (Lillie, 2012). However, industry projections did not materialize in the U.S. and Europe. In 2011, 207 billion MMS messages were sent, generating $31 billion in revenue (MobiThinking, 2012), or roughly 15 cents per text message, which is far below the adoption expectations initially set for MMS relative to SMS. The number of messages sent per active customer (per year) is also lower in MMS than SMS. Although MMS traffic globally has increased exponentially since 2010 at a rate of 33.5% per year, reaching 218 billion messages in the U.S. by 2015 (CTIA, 2016), MMS has not replaced SMS over time.

The main reason that MMS has not replaced SMS in popularity in the U.S. is the lack of interoperability among different wireless carriers (Bandera, 2016a). When carriers differentiate their multimedia value-added services (VAS) in ways that do not faithfully comply with the OMA MMS protocol, each carrier creates for its subscribers a walled garden that does not guarantee MMS interoperability with subscribers in the walled gardens of competing carriers (Cheng and Sun, 2012; Pohjola and Kilkki, 2005; and Wu, 2007). Carriers also use these walled gardens to compete with application and over-the-top providers such as Skype, Google and iTunes (Holzer and Ondrus, 2011). In addition, cell phone manufacturers similarly design multimedia features into their devices to help differentiate their products from those of the competitors, at the expense of interoperability. Consequently, a multimedia message sent from a mobile phone may be only *partially* viewable by a recipient on a different carrier, or even on the same carrier but using different devices. This lack of MMS interoperability has become an impediment to the population-wide business-to-person (B2P) messaging associated with the VAS to subscribers and the revenues to carriers (Gandhi, 2012; Samanta et al., 2009).

It is noteworthy that the aforementioned lack of MMS interoperability did not occur in China, in great part due to the government's control of the cell phone industry. This contrast between MMS interoperability in China and lack thereof outside China, especially in the U.S., can serve as the prime examples for us to compare different scenarios from a game theory perspective. As discussed by Economides and Flyer (1997), "firms have strong incentives to adhere to common technical compatibility standards, so that they reap the network externalities of the whole group. However, a firm also benefits from producing an incompatible product thereby increasing its horizontal product differentiation." It seems that the wireless carriers made the right decisions not to deviate from the ratified standard in the case of SMS at a time when cell phone adoption was just beginning. However, over a decade later when facing a more saturated market, these firms seemed to have made the wrong decisions to deviate from the ratified standard in the case of MMS in the U.S. In addition, as there are strategic interactions among these carriers, they fit perfectly into the market structure of oligopoly in which big firms sell differentiated products to increase the sizes of their networks of customers. Therefore, the main goal of our research is to apply game theory to characterize how two oligopoly firms choose the levels of compatibility or interoperability of their products in the cases of SMS and MMS, with the U.S. and China as the prime examples.

This research proceeds as follows: Section 2 is the literature review. Section 3 discusses some simple games to illustrate the core concepts of game theory as well as their applications to the context of the SMS and MMS. The result of the game-theoretical model is presented in Section 4. Section 5 discusses the business and policy implications of the model. Section 6 provides the conclusion and several directions for future research. The mathematical details of the model are presented in the Appendix.

2. Literature Review

In this Section, we review some research most relevant to our model as well as the developments of SMS and MMS. Bandera (2016a) analyzes the deviations from the ratified MMS protocol by wireless carriers and cell phone manufacturers. These deviations allow a cell phone user to create multimedia messages that are more appealing (e.g., higher resolution images and longer duration videos) than those which the ratified protocol supports. The motivation of wireless service providers and device manufacturers to offer such deviations is rooted in the competition for new customers.

Bandera (2016b) has further researched the impact of MMS interoperability on the two main MMS-based business models: (1) multimedia messages between cell phone users, and (2) multimedia messages between cell phone users and value-added service providers (VASPs) such as news and entertainment outlets. The first business model relies on person-to-person (P2P) MMS traffic and generates revenue from consumers, whereas the VASP business model relies on B2P MMS traffic for which carriers can charge more per message. The latter is particularly sensitive to interoperability because a VASP business model requires compatibility with all these carriers and their subscribers to deliver its content to a broad audience distributed across multiple carriers.

In the U.S., revenue from B2P MMS traffic is negligible compared to revenue from P2P MMS, whereas in China, where carriers and device manufacturers did not (or could not) deviate from the ratified MMS protocol, B2P revenue is roughly 2.3 times that of P2P revenue (Portio Research, 2012). By using China as a reference, Bandera (2016b) also estimates that the MMS market in the U.S. and Europe including P2P and B2P traffic would thus be 3.3 times greater but only if all carriers and device manufacturers outside China complied with the ratified MMS protocols. The author also estimates that this goal would not be achieved if just one of the top three carriers in any market failed to comply because the incompatibility that carrier would introduce to the VASP's target audience would adversely impact the VASP business model.

In game theory/oligopoly literature, Chen and Chen (2011) investigate a firm's equilibrium behavior under product compatibility and differentiation, as well as the network effect. The authors find that "a firm with a high degree of compatibility has a greater competitive disadvantage due to its higher spillover effect with other firms". When "firms can freely determine their own compatibility, each firm will choose the lowest degree of compatibility." "In contrast to the social optimum in which both firms choose the highest degree of compatibility, a social dilemma occurs." The outcome is similar to the development of the MMS in the U.S.

In contrast, Toshimitsu (2014) modifies the assumption of network size made by Chen and Chen (2011) and shows that the social dilemma will not arise. In addition, based on a simple model of compatibility choice under differentiated Cournot duopoly with network externalities, Toshimitsu (2018) shows that there are multiple equilibria involving imperfect and perfect compatibility. The author also demonstrates "the conditions for constructing such a network alliance so that firms provide perfectly compatible products," which is also socially optimal. Most importantly, Toshimitsu (2014) states that one possible extension for future research is to integrate his approach and that of Chen and Chen (2011). Our paper is one such attempt, with the focus on the context of the SMS and MMS.

Katz and Shapiro (1985) study the compatibility choice of competing firms in industries with network externalities when the firms in the industry have complete compatibility or complete incompatibility when there are J groups of firms. The brands of firms within a given group are mutually compatible with each other, but incompatible with any non-member brands. The authors also investigate the social vs. private incentives of compatibility choice as well as the welfare implications.

The pioneering work of Katz and Shapiro (1985) has been adopted and extended in many directions. For example, Barrett and Yang (2001) explore the issue of rational incompatibility with international product standards. Instead of assuming the complete compatibility or complete incompatibility among different standards, the authors allow partial compatibility. They find that "not only do incumbent firms using a different technology have an incentive to deviate from an international standard, but a host country government concerned for its consumers' welfare has no incentive to enforce the international standard, and may even value deviation from the international standard through technical barriers to trade."

In another extension, Belleflamme and Peitz (2010) analyze how two firms can compete with each other in a standardization game. Each firm has two strategies, *A* and *B*, representing two kinds of goods produced based on different standards. The larger firm prefers standard *A*, while the smaller firm prefers standard *B*. By assuming that two firms are asymmetric in terms of the installed base of customers (β) and the cost of adopting the competitor's technology (c), the authors use β and c to characterize the game into four possible outcomes – (1) (*A*, *A*) and (*B*, *B*), i.e., two firms compete *in* the market as in the Battle of the Sexes game, (2) no pure strategy Nash equilibrium (the larger firm prefers an incompatible standard while the small firms prefer compatible standards), (3) (*A*, *A*), straightforward standardization, and (4) (*A*, *B*), a standards war. The authors also discuss the policy implications of the model.

Although we adopt the model discussed by Belleflamme and Peitz (2010), there are important differences. First, we assume that both firms are symmetric in size, as in the context of SMS and MMS. Second, Belleflamme and Peitz (2010) assume that when a firm adopts the competitor's product, there will be 100% compatibility with an adopting (or adaptor) cost. In our paper, we assume that, as in the case of MMS, the products of different firms do not have complete compatibility although it is possible in the case of SMS. Third, the authors assume that the network effect is fixed, while we allow the network effect to vary in different situations.

Therefore, the contributions of our research are threefold:

(1) We develop a game-theoretical model to characterize how oligopoly firms can choose their levels of (partial) compatibility or interoperability in a standardization game, which is a contribution to the game theory/industrial economics literature. Particularly, our model can incorporate the possibilities of both efficient equilibrium and inefficient equilibrium, as discussed by Chen and Chen (2011), Toshimitsu (2014), and Toshimitsu (2018).

(2) As we apply the game theory model to the context of MMS vs. SMS, this research answers the important question regarding why large companies can make the wrong decisions in a paradoxical situation, as in the case of the U.S. This research, therefore, narrows the knowledge gaps between mathematical game theory models and the empirical studies of SMS and MMS, with the U.S. and China as the prime examples.

(3) Our research can serve as a starting point for many important extensions. On the theoretical side, our 2-player game can be extended to an *n*-player game, which can also incorporate the literature in evolutionary game theory and computer simulations. On the empirical side, our model, with some necessary modifications, can also be applied to other industries and countries, which can address important policy issues or questions for both the governments and firms.

3. The Games of Standardization

In this section, we provide some simple but important games to illustrate the core concepts of game theory as well as possible different equilibria in the standardization game. Our focus is the two-player game in which each player has two strategies, *A* and *B*, that represent two standards. The two players are firms (carriers) that choose their own standards to maximize payoffs. In the payoff matrix, the first number of the payoff is the payoff of firm 1, and the second number of the payoff is the payoff of firm 2. As there are two players with two strategies, there are four strategy combinations in each game.

The four strategy combinations imply that there are four possible scenarios. From a game theory perspective, as each firm has binary choices, only the relative payoffs matter to each firm when it needs to choose between the two strategies instead of the absolute payoffs unless the players engage in monetary transfers or side payments. Therefore, arbitrary numbers can be assigned to each player in every strategy combination if they can satisfy the assumptions of the relative sizes of payoffs. Nevertheless, instead of the numerical examples with arbitrary payoffs, in this Section, we will use parameters, w, x, y, z, to represent the payoffs of players when they choose the strategies in these simple games.

3.1 Dominant Strategies with an Efficient Equilibrium

For simplicity, we assume that both firms are symmetric in the sense that they will get the same payoffs when both firms choose the *A* strategy or the *B* strategy. When both firms choose the *A* strategy, the payoff of each firm is w. When both firms choose the *B* strategy, the payoff of each firm is z. When firms choose different strategies, the firm that chooses the *B* strategy will get x, while the other firm that chooses the *A* strategy will get y. Therefore, we have the following payoff matrix:

Table 1. The Combination of Dominant Strategies Leads to an *Efficient* Equilibrium

		Firm 2 *B*	*Firm 2* *B*
Firm 1	*A*	w, w	y, x
Firm 1	*A*	x, y	z, z

In the game depicted in Table 1, we assume that $w < x$, $y < z$, and $w < z$. When firm 2 chooses the *A* strategy, firm 1 has to choose between strategy *A* and *B*. As $w < x$ by assumption, firm 1 should choose the *B* strategy. When firm 2 chooses the *B* strategy, firm 1 should choose the *B* strategy as $y < z$. By symmetry, both players should choose the *B* strategy regardless of the strategy chosen by the other player. In this case, both players have the same *dominant strategy*, the *B* strategy. The equilibrium is the strategy combination (*B*, *B*), which is also the efficient equilibrium as $w < z$. The game is therefore a straightforward standardization when both firms choose the same standard *B*. In the market of multiple players, it is possible that some firms can choose standard *A* in the short run, but eventually, all firms will choose standard *B* in the long run. Such was the case when the option to advance from SMS to Enhanced Messaging Service (EMS) became available to carriers. In the context of SMS vs. EMS, the standard *B* is the SMS protocol, while standard *A* is the EMS protocol that became obsolete over time. There is no need for government intervention for firms to reach the efficient equilibrium.

More specifically, for the two players are AT&T and Verizon, the two largest wireless carriers in the U.S., when they needed to decide whether to adopt the EMS or continue to use the SMS in the early 2000s. As messaging is a source of revenue to carriers, each carrier tried to maximize the number of messages conveyed on its network. The 3rd Generation Partnership Project, a standards organization that develops protocols for mobile telephony, developed EMS in 2000 to compete with SMS by offering the ability to send rich content including formatted text and ringtones. EMS therefore is a messaging service with features more advanced than those of SMS, but slightly less advanced than those of MMS.[1] However, the lack of interoperability, the lack of support from wireless operators, and the rise of MMS as a superior alternative impeded the adoption of EMS (Le Bodic, 2005), without any government intervention. In the resulting equilibrium, both AT&T and Verizon chose SMS over EMS to maximize their numbers of subscribers. In 2015, AT&T and Verizon conveyed 578 and 631 billion SMS messages, respectively (CTIA, 2016; FierceWireless, 2018).

3.2 A Prisoner's Dilemma Game

Table 2. The Combination of Dominant Strategies Leads to an *Inefficient* Equilibrium

		Firm 2 *A*	*Firm 2* *B*
Firm 1	*A*	*w*, *w*	*y*, *x*
Firm 1	*B*	*x*, *y*	*z*, *z*

We now consider the game where the assumption of the payoffs is $w < x$, $y < z$, and $w > z$. When both players have the dominant strategy, the combination of dominant strategies does not always lead to an efficient equilibrium. In Table 2, both players have the dominant strategy, *B*, as $w < x$, and $y < z$. However, when both players choose the *B* strategy, each player's payoff is *z*. As $w > z$, (*B*, *B*) is not as efficient as (*A*, *A*). Therefore, the combination of dominant strategies leads to an inefficient equilibrium, which is the characteristic of a paradoxical situation in a prisoner's dilemma game.[2]

In the context of MMS, strategy *A* can represent the protocol ratified by international standards bodies, while strategy *B* means that firms deviate from the ratified protocol. When the firm deviates from the standard, the non-standard features can attract new consumer customers. However, when both firms deviate from the compatible standards, both firms are worse off as the lack of interoperability will prevent the firms from attracting commercial customers such as the VASPs. More importantly, this game also shows that when the government

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[1] See http://www.techopedia.com/definition/2772/enhanced-messaging-service-ems for details.

[2] In the original prisoner's dilemma game, both prisoners have two strategies – to confess or not to confess although both prisoners can be better off if they choose not to confess. Unfortunately, as the dominant strategy for both prisoners is to confess, both players will be worse off.

intervenes in the market, it is possible that the government can force both or all the firms to use strategy *A*, thus reaching the efficient equilibrium (*A*, *A*), which is similar to the case of China.

Furthermore, in China, the interoperability of MMS standards has led to a greater adoption of MMS, which enjoys roughly 82% of the world's MMS revenue. If the global per-capita adoption of MMS were the same as that in China, the global MMS market would increase by roughly $100 billion (in U.S. dollars) annually (Bandera, 2016b). Thus, while differentiating its products and services from those of competitors may appeal to companies as a dominant strategy in the short run, all the firms that choose different standards with low interoperability can be significantly worse off in the long run.

In the U.S., AT&T and Verizon chose to deviate from the ratified MMS protocol, and their payoffs in 2015 were 66.7 and 72.8 billion messages conveyed that year over their respective networks. This message traffic is predominantly P2P, i.e., consumers sending pictures and group chats to each other. If both carriers chose to follow the ratified MMS protocol, VASPs would be able to introduce new services in the U.S. in the form of premium priced B2P MMS messages similar to the services provided currently in China. Bandera (2016b) also estimates that this B2P MMS traffic in the U.S. would be 2.3 times that of the P2P traffic. Consequently, had both AT&T and Verizon chosen to conform to the ratified MMS protocol, their payoffs in 2015 would have been 200 and 218 billion messages conveyed that year over their respective networks.

3.3 Nash Equilibrium

Sometimes, the dominant strategy does not exist for both players. Then a different concept, such as the Nash equilibrium, is needed to determine the equilibrium. The Nash equilibrium is the strategy combination such that no player has the incentive to deviate from the strategy combination given the strategy chosen by the other player(s). One can easily verify that for the games in Table 1 and Table 2, (*B*, *B*) is the Nash equilibrium because $z > y$, which implies that no player has the incentive to choose the *A* strategy when the other player chooses the *B* strategy.

3.4 The Coordination Game

Table 3. The Coordination Game with No Winner or Loser in Equilibrium

		Firm 2 *A*	*Firm 2* *B*
Firm 1	*A*	*w*, *w*	*y*, *x*
Firm 1	*B*	*x*, *y*	*z*, *z*

In the game in Table 3, the underlying assumption is that $w > x$, $z > y$, and $w > z$; there are two Nash equilibria, (*A*, *A*) and (*B*, *B*). Neither player will deviate from the strategy combinations when the other player chooses the *A* or *B* strategy since $w > x$, and $z > y$. This also implies that both firms must choose the *same* standard to have higher payoffs or profits. In addition, in this game, (*A*, *A*) is more efficient than (*B*, *B*) as $w > z$. Such a game is called a coordination game in which both players need to coordinate with each player to choose the same strategy or standard, i.e., standard *A*, to reach the higher payoff, and there is no winner and loser in this game.[3] At the same time, government can also intervene in the market for the players or firms to reach the efficient equilibrium (*A*, *A*).

The players can also have different payoffs when they choose the same strategy. In that case, there will be a winner and a loser in the equilibrium of the game.[4]

Table 4. The Coordination Game with a Winner and a Loser in Equilibrium

		Firm 2 *A*	*Firm 2* *B*
Firm 1	*A*	*w*, *z*	*y*, *x*
Firm 1	*B*	*x*, *y*	*z*, *w*

For the game in Table 4, the assumption of the payoffs is $w > z > x$ and $w > z > y$. There are two Nash equilibria, (*A*, *A*) and (*B*, *B*) as $w > x$ and $z > x$, as well as $z > y$ and $w > y$. In (*A*, *A*), firm 1 is the winner, and in (*B*, *B*), firm 2 is the winner. This means that firm 1 benefits more from standard *A* than firm 2, but firm 2 benefits more from standard *B* than firm 1, if both firms can agree to choose the same standard. That is, both firms agree that the same standard is the best option for both firms, but they prefer different standards. As discussed by Belleflamme and Peitz (2010), this means that the two firms are competing *in* the same market instead of competing *for* the market.

3.5 Competing for the market

Table 5. Both Firms Compete for the Market

		Firm 2 *A*	*Firm 2* *B*
Firm 1	*A*	*w*, *z*	*y*, *x*
Firm 1	*B*	*x*, *y*	*z*, *w*

Players have more intensive competition in a game when both firms are competing *for* the market instead of competing *in* the market. For the game in Table 5, the assumption is that $w > x > z$ and $w > y > z$, and both players have the dominant strategy. The dominant strategy for firm 1 is strategy *A*, while the dominant strategy for firm 2 is strategy *B*. The only Nash equilibrium is (*A*, *B*). This implies that firm 1 prefers standard *A* while firm 2 prefers standard *B*. The two firms are therefore

[3] See Chou (2011) for another example of a coordination game and prisoner's dilemma game in the context of information systems development.

[4] This is similar to the famous game of the Battle of the Sexes in which the husband and wife must choose the same strategy to have positive payoffs. As the husband and wife have different payoffs, there are different winners and losers in different Nash equilibria.

competing in a *standards war* to be the *de facto* standard of the products. Famous examples include the competition between VHS and Betamax or Blu-ray and HD DVD.

Another possible interpretation of strategies is that strategy *A* is a proprietary standard incompatible with the ratified standard, while strategy *B* is the ratified standard. Firm 1 can be a larger firm that prefers incompatible standards, while firm 2 is a smaller firm that will benefit from producing the products that are compatible with firm 1 such that firm 2 can benefit from firm 1's installed base of customers through the network effect.

4. The Oligopoly/Game Theoretical Model

Instead of using parameters to represent the payoffs of players with arbitrary assumptions of relative payoffs, in this Section we develop a mathematical model that can endogenously determine the payoffs of players when both firms can choose to stick with the ratified standard or to deviate from the ratified standard. The two large firms, such as Verizon and AT&T, are competing in a standardization game. In addition, as *partially* compatible standards are also possible, each firm can choose its own level of compatibility or interoperability of its own products to maximize the profits in the standardization game. The mathematical details of the game are provided in the appendix.

4.1 The Basics of the Model

In the context of SMS or MMS, as firms (the carriers) are usually equally large, we assume that there are two firms that are symmetric in sizes. Both firms have two strategies – the *B* strategy to stay with the ratified standard, and the *A* strategy to deviate from the ratified standard. The payoff of each firm is its profit with revenues from the VASs.

The demand curve for firm 1 is $p_1 = \bar{p} + (q_1 + r_2q_2) - q_1 - q_2$, where α represents the level of network effect *per customer*, with $0 < \alpha < 1$. The q_1 is the number of customers or the size of the network for firm 1, and q_2 is the size of the network for firm 2. By following the partial compatibility assumption, as discussed by Jain (1989), Shy (1996), and Barrett and Yang (2001), we use the coefficient r_2 to show how compatible firm 2's products are with firm 1's products, with $0 < r_2 \leq 1$[5]. When $r_2 = 1$, firm 2's products have 100% compatibility with firm 1's products, and when $0 < r_2 < 1$, firm 2's products are less than 100% compatible with firm 1's products. Therefore, the extended network for firm 1 is $(q_1+r_2q_2)$, and the network effect to firm 1 is $\alpha(q_1+ r_2q_2)$.

The profit of firm 1 is $\pi_1 = p_1q_1 - cq_1 = [\bar{p} + \alpha(q_1+r_2q_2) - q_1 - q_2 - c]\, q_1$, where c

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[5] In MMS, a carrier can delete an image or video from incoming media messages. Videos can be re-sampled by the carrier, and can play poorly on the phone. It is also possible that media messages can be received, but they cannot be rendered on the phone. Therefore, there are different levels of partial compatibility. See Bandera (2016a) for more details.

is the marginal or average cost of producing products. For simplicity, we can normalize $\bar{p}$ to 1 and assume that $c = 0$ for both firms. Then the profit function for firm 1 is $\pi_1 = [1 + \alpha(q_1 + r_2 q_2) - q_1 - q_2]\, q_1$. By the assumption of symmetric firms, the profit function for firm 2 is $\pi_2 = [1 + \alpha(q_2 + r_1 q_1) - q_2 - q_1]\, q_2$.

4.2. The Profits of Firms

As each firm chooses the network size to maximize its profit, we can derive the solution via $\partial\pi_1/\partial q_1 = 0$ and $\partial\pi_2/\partial q_2 = 0$. With the mathematical details provided in the appendix, the profits of both firms are $\pi_1 = (1-\alpha)(1-2\alpha + \alpha r_2)^2/[4(1-\alpha)^2 - (1-\alpha r_1)(1-\alpha r_2)]^2$, and $\pi_2 = (1-\alpha)(1-2\alpha + \alpha r_1)^2/[4(1-\alpha)^2 - (1-\alpha r_2)(1-\alpha r_1)]^2$. These profit functions imply that when a firm i lowers the compatibility (r_i) of its products, it can increase its profit (π_i), if all other things remain unchanged.

When firm i chooses the B strategy with 100% compatibility with the ratified standard, $r_i = 1$, and when firm i chooses the A strategy with less than 100% compatibility, $r_i < 1$, for $i = 1, 2$. For simplicity, assume that when both firms choose the A strategy, $r_1 = r_2 = r$. When only one firm chooses the A strategy, $r_i = r < 1$, for $i = 1, 2$. This implies that the different standard has one-way backward compatibility to the ratified standard. But when both firms deviate from the ratified standard, their products are equally incompatible. Therefore, we have the payoff matrix as follows:

Table 6. Each Firm Chooses the Level of Compatibility to the Ratified Standard

		Firm 2 A ($r_2 < 1$)	*Firm* 2 B ($r_2 = 1$)
Firm 1	A ($r_1 < 1$)	$\pi_{1,(A,A)}$, $\pi_{2,(A,A)}$	$\pi_{1,(A,B)}$, $\pi_{2,(A,B)}$
Firm 1	B ($r_1 = 1$)	$\pi_{1,(B,A)}$, $\pi_{2,(B,A)}$	$\pi_{1,(B,B)}$, $\pi_{2,(B,B)}$

$\pi_{1,(A,A)} = \pi_{2,(A,A)} = (1-\alpha)(1-2\alpha + \alpha r)^2/[4(1-\alpha)^2 - (1-\alpha r)^2]^2$.

$\pi_{1,(B,A)} = \pi_{2,(A,B)} = (1-2\alpha + \alpha r)^2/[(1-\alpha)(3 - 4\alpha + \alpha r)^2]$

$\pi_{1,(A,B)} = \pi_{2,(B,A)} = (1-\alpha)/(3 - 4\alpha + \alpha r)^2$

$\pi_{1,(B,B)} = \pi_{2,(B,B)} = 1/[9(1-\alpha)]$

We can determine the Nash equilibrium if the following conditions are satisfied:

(1) For (A, A) to be a Nash equilibrium, $\pi_{1,(A,A)} \geq \pi_{1,(B,A)}$, and $\pi_{2,(A,A)} \geq \pi_{2,(A,B)}$ must hold.

(2) For (B, B) to be a Nash equilibrium, $\pi_{1,(B,B)} \geq \pi_{1,(A,B)}$, and $\pi_{2,(B,B)} \geq \pi_{2,(B,A)}$

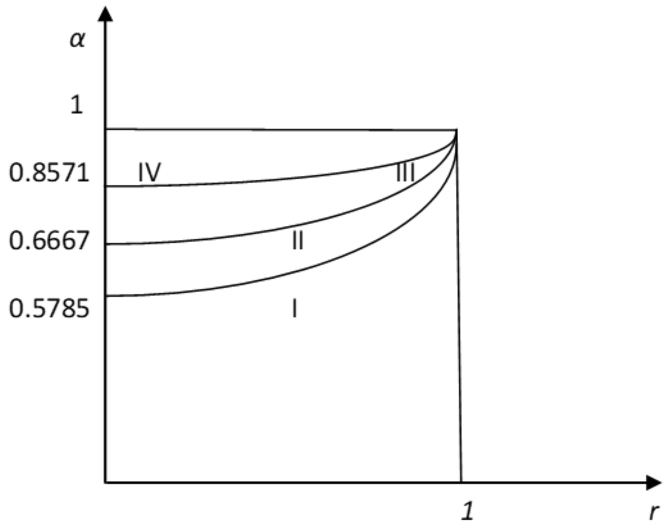
must hold.

(3) For (*A*, *B*) to be a Nash equilibrium, $\pi_{1,(A,B)} \geq \pi_{1,(B,B)}$, and $\pi_{2,(A,B)} \geq \pi_{2,(A,A)}$ must hold.

(4) For (*B*, *A*) to be a Nash equilibrium, $\pi_{1,(B,A)} \geq \pi_{1,(A,A)}$, and $\pi_{2,(B,A)} \geq \pi_{2,(B,B)}$ must hold.

Therefore, we can summarize the findings in terms of a graph on an $r\alpha$ plane in the following figure:

Figure 1. Different Nash Equilibria in Different Ranges of *r* and *α*.



(1) In Area I, (*A*, *A*) is the only Nash equilibrium. Both firms choose the *A* strategy as the dominant strategy, i.e., to deviate from the ratified standard. However, it is a prisoner's dilemma game with an inefficient equilibrium similar to the situation of MMS in the U.S.

(2) In Area II, there are two Nash equilibria, (*A*, *B*) and (*B*, *A*). This is the situation of a standards war with strategy *A* as the winning strategy. In an *n*-player game, this implies that if more and more firms will choose the *A* strategy, it may lower the network effect α over time.

(3) In Area III, there are two Nash equilibria, (*A*, *B*) and (*B*, *A*). This is the situation of a standards war with strategy *B* as the winning strategy. In an *n*-player game, this implies that if more and more firms choose the *B* strategy, it may raise the network effect α over time.

(4) In Area IV, (*B*, *B*) is the Nash equilibrium. Both firms choose the *B* strategy as the dominant strategy to adhere to the ratified standard. In this case, both players are better off as the Nash equilibrium is the efficient equilibrium similar to the situation of the SMS worldwide.

In other words, when the network effect is sufficiently strong, the Nash equilibrium is that both firms choose the ratified standards with 100% compatibility or interoperability, as in the case of SMS globally. This is when firms are competing *in* the market.

In contrast, when the network effect is sufficiently weak, as in the case of MMS in the U.S., it is more likely that firms choose different standards, which, however, is similar to the prisoner's dilemma game as both firms are worse off than the situation in which they both choose the ratified compatible standards. This is when firms compete *for* the market.

When the network effect is intermediate, both (*A*, *B*) and (*B*, *A*) are Nash equilibrium. This can be the beginning of the standards war when firms compete *for* the market. However, as the network effect may increase or decrease over time when more firms choose the *A* strategy or *B* strategy, eventually firms will reach either (*A*, *A*) or (*B*, *B*), with (*B*, *B*) as the efficient equilibrium. As in the real world, there may be a tipping point when one firm will make a critical difference about which standard will eventually become the *de facto* one.

5. Policy and Business Implications

If the deviation of firms from the ratified standards can result in the inefficient equilibrium of a prisoner's dilemma game, it may justify the *ex-ante* intervention of the government. A prime example is China where carriers are still run by the government which mandated interoperability and openness policies to VASPs (Kong and Luo, 2006), and where cell phone adoption is still far from saturated (Pasqua and Elkin, 2012). The Chinese wireless market is larger than that of any other country, and its state-run carriers are the largest in the world: China Mobile (62% Chinese market share and world's largest carrier with 19.3% of the world's cell phone subscribers), China Unicom (23% of the Chinese market), and China Telecom (15% of the Chinese market). In China, 70% of MMS messages are B2P (Ahonen, 2012); mobile subscribers reading newspapers via SMS and MMS equal nearly 40% of the country's daily newspaper circulation (Morgan Stanley Research, 2009). Another implication is that when the government wants to intervene in the market, the government should do so in the early stage of technology development. Once firms deviated from the ratified standards and developed technologies based on these standards, it will be difficult to force firms to switch back to the same ratified standards, as in the case of the U.S.

Patrick (2007) discusses the Korean and Japanese cases of achieving a uniform standard: "one option is the Korean approach – the Deus ex Machina – the government determines standards and pricing. A regulator can potentially take this role in-country. Another option could be Deus in Machina – a collaborative grouping coordinates behavior, or a dominant player drives behavior. Japanese DoCoMo is a blend of this". The "Korean example is by government fiat, the Japanese by "assertive collaboration"."

If there is a tipping point for the better technology standard to become the *de facto* standard, government can also intervene and enforce the standard in the industry. As discussed by Belleflamme and Peitz (2010), although "public authorities" can "refrain from intervening directly in the (*de facto* or *de jure*) standardization process, they can "still keep the possibility of controlling the process indirectly, *ex post*, mainly through competition (antitrust) policy." At the same time, governments may also intervene in the markets in favor of their domestic industries, as discussed by Barrett and Yang (2001). The full exploration of this topic is beyond the scope of our current paper, but it can be a direction for future research.

The most efficient equilibrium is when firms choose the same compatible standards, which may rely on government's intervention in the market. If that is not possible, as in the case of MMS in the U.S., carriers' MMS implementations and technologies should become more compatible with this ratified standard over time. Today, P2P SMS and MMS have become commodities, and carriers and VASPs agree that future growth and revenue will have to come from B2P messages, including automated notifications from companies and smart devices (Portio Research, 2014). One example is the growing number of online services that require their users to employ two-factor authentication for increased cyber-security; this authentication uses SMS and MMS. SMS and MMS are thus no longer just P2P messaging services, but enablers of a broad range of other more critical services.

For business/managerial implications, CEOs of the big firms should not be too over-confident in winning the standards war by choosing a different standard when there is already a ratified international standard. Maybe in the short run it is profitable for firms to do so. However, in the long run, as shown in the paper, profits from the VASPs are much lower in the U.S. than those in China. The decisionmakers of the firms should also understand game theory, as well as how to reach a more efficient equilibrium in case of a paradoxical prisoner's dilemma game.

6. Conclusion and Extensions

By adopting and modifying the model discussed by Belleflamme and Peitz (2010), we characterize the equilibrium of the standardization game in the context of the SMS and MMS. When the network effect per customer is strong, both firms choose 100% compatibility with the ratified standard and thus interoperability, as in the case of SMS, which is similar to the outcome of Toshimitus (2014). In contrast, if the network effect per customer is sufficiently weak, as in the case of MMS, it is more likely for firms to choose different standards, which, however, resembles the prisoner's dilemma game as both firms are worse off than in the situation when they both choose the ratified compatible standards, which is also consistent with the outcome of Chen and Chen (2011).

When the network effect per customer is intermediate, the game becomes a standards war in which firms are competing *for* the market. If to stay with the ratified standard is the winning strategy, i.e., the *B* strategy, in an *n*-player game, more and more firms will choose the compatible standard, which will increase the network effect over time. In this case, the Nash equilibrium will eventually become (*B*, *B, B,* ...) when all the firms choose the ratified standard. In contrast, if the incompatible standard is the winning strategy, i.e., *A* strategy, in an *n*-player game, the network effect will become even weaker. Therefore, the Nash equilibrium eventually will become (*A*, *A, A, …*) when all the firms choose a partially compatible standard.

For the past several decades, SMS has become the uniform standard for every firm, while EMS disappeared from history without any government intervention. The firms in the U.S. chose partially compatible or incompatible MMS standards, while the firms in China adopted the uniform compatible standard as a result of government intervention. The commonness of VASs as well as the profits generated from VAS in China are much greater than those in the U.S. These developments in SMS, EMS, and MMS over time serve as the prime examples for us to compare different equilibria from a game theory perspective. Although government intervention may be needed, as in the case of China, for all the firms to use the same MMS standards, the firms in the U.S. can still make the MMS implementations and technologies more compatible with the ratified standard. The future of SMS and MMS revenue is more dependent on B2P traffic, such as business notifications, 2-factor authentication, and messages from autonomous devices, which makes our research even more relevant and important to the carriers and VASP industry today.

This paper can be extended in several ways. First, we can explicitly make the network effect coefficient, α, endogenous, which can depend on the interoperability of standards, technology, policy, and market structure. Second, we can expand the two-player game into an *n*-player game in which networks or coalitions of firms compete in a standardization game, as discussed by Barrett and Yang (2001), among many others. For an *n*-player game, we can also apply evolutionary game theory to characterize how equilibrium changes over time, as discussed, for example, by Samuelson (2002). Third, we can include another coefficient to represent the installed base of locked-in customers and the cost of adopting alternative technology, and to determine their impacts to the (Nash) equilibrium. Fourth, another direction to extend the current model is to make the two firms asymmetric in terms of the installed base or the cost of adopting a different technology, as discussed by Belleflamme and Peitz (2010). When firms are asymmetric in size, we can use the model to address different issues when the larger or dominant firm prefers incompatible products while the smaller firm prefers compatible products, as discussed, for example, by Blind and Thurmm (2004), among others. We can even incorporate some perspectives of behavioral game theory, such as fairness, as discussed, for example, by Rabin (1993), or the issue of complacence, as discussed, for instance, by Chou et al. (2017). Finally, another direction to extend the paper is to include the government as a regulator to address the policy issues and whether government intervention or regulation can lead to a more efficient equilibrium, not only for the producers, but also for the consumers, in the framework of welfare analysis. However, most of these extensions will also make the paper much more mathematical and complicated.

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Appendix

When each firm chooses q_i to maximize its profits, $i = 1, 2$, we have $\partial\pi_1/\partial q_1 = 0$ and $\partial\pi_2/\partial q_2 = 0$. The second-order conditions are also satisfied such that the maximum exists. Hence, we have $q_1 = (1–2\alpha + \alpha r_2)/[4(1–\alpha)^2 – (1– \alpha r_2)(1–\alpha r_1)]$, and $q_2 = (1–2\alpha + \alpha r_1)/[4(1–\alpha)^2 – (1 – \alpha r_1)(1–\alpha r_2)]$. We also have $p_1 = (1–\alpha)(1–2\alpha + \alpha r_2) / [4(1–\alpha)^2 – (1– \alpha r_2)(1–\alpha r_1)]$, and $p_2 = (1–\alpha)(1–2\alpha + \alpha r_1)/[4(1–\alpha)^2 – (1– \alpha r_1)(1–\alpha r_2)]$. This implies that each firm i can have more customers q_i and higher price p_i when r_i decreases, all other things being equal. Then we can derive the profit functions for both firms, $\pi_1 = p_1q_1 = (1–\alpha)(1–2\alpha+\alpha r_2)^2/[4(1–\alpha)^2 – (1– \alpha r_1)(1–\alpha r_2)]^2$, and $\pi_2 = p_2q_2 = (1–\alpha)(1–2\alpha + \alpha r_1)^2/[4(1–\alpha)^2 – (1– \alpha r_2)(1–\alpha r_1)]^2$. Likewise, all other things equal, a firm i can increase its profit π_i by lowering its r_i.

When $r_1 = r_2= r$, $\pi_{1, (A, A)} = \pi_{2, (A, A)} = (1–\alpha)(1–2\alpha + \alpha r)^2/[4(1–\alpha)^2 – (1– \alpha r)^2]^2$. When $r_1 = 1$ and $r_2= r$, $\pi_{1, (B, A)} = (1–2\alpha + \alpha r)^2/[(1–\alpha)(3 – 4\alpha+\alpha r)^2]$, which is equal to $\pi_{2, (A, B)}$ when $r_1 = r$ and $r_2=1$. When $r_1 = r$ and $r_2 =1$, $\pi_{1, (A, B)} = (1–\alpha)/(3 – 4\alpha+\alpha r)^2$, which is equal to $\pi_{2, (B, A)}$ when $r_1 = 1$ and $r_2 = r$. When $r_1 = r_2 = 1$, $\pi_{1, (B, B)} = \pi_{2, (B, B)} = 1/[9(1– \alpha)]$.

To determine the Nash equilibrium of the game, we have to determine the signs of the following differences:

(1) $(1–\alpha)/(3 – 4\alpha+\alpha r)^2 – 1/[9(1– \alpha)] = (\alpha)(6 – 7\alpha+\alpha r)(1– r)/[9(1– \alpha)(3 – 4\alpha+\alpha r)^2]$. The term $(6 – 7\alpha+\alpha r)$ is an increasing function of r, and when $r = 0$, we need $\alpha = 6/7 = 0.8571$ to make $(6 – 7\alpha+\alpha r) = 0$. Hence, when $(6 – 7\alpha+\alpha r) < 0$, it defines the Area IV in which both firms should stay with the ratified standard with (B, B) as the Nash equilibrium.

(2) $(1–2\alpha + \alpha r)^2/[(1–\alpha)(3 – 4\alpha+\alpha r)^2] – (1–\alpha)(1–2\alpha + \alpha r)^2/[4(1–\alpha)^2 – (1– \alpha r)^2]^2 = [(3–2\alpha–\alpha r)(1–2\alpha+\alpha r) + (1–\alpha)(3–4\alpha+\alpha r)][(3–2\alpha–\alpha r)(1–2\alpha+\alpha r)–(1–\alpha)(3–4\alpha+\alpha r)] / [(1–\alpha)(3 – 4\alpha+\alpha r)^2][4(1–\alpha)^2 – (1– \alpha r)^2]^2$.

For the two terms in the product in the numerator, the second term $(3–2\alpha–\alpha r)(1–2\alpha+\alpha r) – (1–\alpha)(3–4\alpha+\alpha r) = \alpha(1–\alpha)(–1+\alpha r) < 0$. The first term $(3–2\alpha–\alpha r)(1–2\alpha+\alpha r) + (1–\alpha)(3–4\alpha+\alpha r) = 6 – 15\alpha + 8\alpha^2 +3\alpha r–\alpha^2 r–\alpha^2 r^2 = (8 – r – r^2)\alpha^2 – (15 – 3r)\alpha + 6 =\{\alpha – \{(15 – 3r) + [(15 – 3r)^2 – 24(8– r– r^2)]^{½}\}/[2(8 – r – r^2)]\}\{\alpha – \{(15 – 3r) – [(15 – 3r)^2 – 24(8– r– r^2)]^{½}\}/[2(8– r– r^2)]\}$.

One can show that for $0 \le r \le 1$, the term $\{(15 – 3r) + [(15 – 3r)^2 – 24(8 – r– r^2)]^{½}\}/[2(8 – r – r^2)]$ is a decreasing function of r, which implies that this term is greater or equal to 1. The other term $\{(15 – 3r) – [(15 – 3r)^2 – 24(8– r– r^2)]^{½}\}/[2(8– r– r^2)] \equiv f(r)$ is an increasing function of r. At the same time, we can show that when $r = 0$, $f(0) = 0.5785$. Therefore, for the area below the curve, i.e., Area I, we have $(1–2\alpha + \alpha r)^2/[(1–\alpha)(3–4\alpha+\alpha r)^2] < (1–\alpha)(1–2\alpha + \alpha r)^2/[4(1–\alpha)^2 – (1– \alpha r)^2]^2$. This implies that (A, A) is the Nash equilibrium. For the area above the curve, we have

$(1–2\alpha+\alpha r)^2$ / $[(1–\alpha)(3–4\alpha+\alpha r)^2]$ > $(1–\alpha)(1–2\alpha+\alpha r)^2$ / $[4(1–\alpha)^2 – (1– \alpha r)^2]^2$, which means that (*A*, *A*) is not the Nash equilibrium.

(3) $1/[9(1– \alpha)] – (1–\alpha)(1–2\alpha +\alpha r)^2/[4(1–\alpha)^2 – (1– \alpha r)^2]^2 = \{[4(1–\alpha)^2 – (1–\alpha r)^2]^2 – 9(1–\alpha)^2(1–2\alpha +\alpha r)^2\}/\{9(1– \alpha)[4(1–\alpha)^2 – (1– \alpha r)^2]^2\} = [6(1–\alpha)+\alpha(1–r)](\alpha)(1–\alpha)(1–2\alpha +\alpha r)^2/\{9(1– \alpha)[4(1–\alpha)^2 – (1– \alpha r)^2]^2 \} > 0$. Therefore, in Area I, (*A*, *A*) is the inefficient Nash equilibrium in a prisoner’s dilemma game because both players are worse off than in (*B*, *B*).

(4) $(1–\alpha)/(3–4\alpha+\alpha r)^2 – (1–2\alpha+\alpha r)^2/[(1–\alpha)(3–4\alpha+\alpha r)^2] = (2 – 3\alpha + \alpha r)(\alpha)(1–r)$ / $[(1–\alpha)(3 – 4\alpha+\alpha r)^2]$. Similar to (1), the term $(2–3\alpha +\alpha r)$ is an increasing function of r, and when $r = 0$, we need $\alpha = 2/3 = 0.6667$ to make $(2 – 3\alpha + \alpha r) = 0$. Hence, when $(2 – 3\alpha + \alpha r) < 0$, *B* strategy is the winning strategy, as in Area III, and when $(2 – 3\alpha + \alpha r) > 0$, *A* strategy is the winning strategy, as in Area II.