International Journal of Business and Economics, 2020, Vol. 19, No.2, 187-200

Modelling the Blind Principal Bid Mechanism: A Large Deviation Approach

Christos I. Giannikos *
Baruch College and Graduate Center, City University of New York, USA

Andreas Kakolyris
The O'Malley School of Business, Manhattan College, USA

Abstract

This paper focuses on the mechanism of blind principal bid (BPB) which is the result of an auction. An agent auctions a large basket of transactions in several stocks to brokers who do not know the individual stock names, but only some generic characteristics of the basket. The client delivers this large amount of sales and purchases to the winning broker who has to simultaneously trade the basket for a fixed commission fee. The commission is paid on the basis of the winning bid at the auction, usually submitted as cents per share. This study provides a new model, based on large deviations (LD) approximation results, that may facilitate brokers to formulate their bids. A numerical example is used to illustrate the methodology and the model is being compared with a Monte Carlo approximation.

Key words: Blind Principal Bid; Large Deviations; Portfolio Loss; Bidding-Auctions

JEL classification: G10; G20; D47

1. Introduction

One of the market mechanisms, which is frequently used for the trade of a large basket of stocks in a short period of time, is called program trading. In a complicated financial world, there is a plethora of reasons why investors need to transact immediately a large basket of stocks in the market. Thus, it is not surprising that program trading constitutes a significant percentage of the trading volume in major stock exchanges. A program trade takes two basic forms. In the first form, which is

*Correspondence to: Economics, The Graduate Center, City University of New York, USA, 365 5th Avenue, New York, NY 10016, USA. Email: cgiannikos@gc.cuny.edu.

We thank the editor, and two anonymous referees for their comments, and suggestions that helped us significantly improve the paper.

known as agency trade, a broker places trades on behalf of the client for a fixed commission. The client's large amount of sales and purchases generates a risk that is held always exclusively by the client and depends on the actual transaction price. The second form, known as principal bid (or principal trade), is a trade which is the result of blind auctions. In this case, the client transfers the ownership of a portfolio to the broker at the current market value for a fixed commission. The total fixed commission for the basket is paid on the basis of the submitted bid, usually expressed as cents per share. In a blind principal bid (BPB) auction, the client will provide the brokers with only specific characteristics of the portfolio. The individual names of the stocks are unknown to the participating brokers before the auction. If a bid is accepted, the client pays the net proceeds (purchases minus sales) based on the current market values of the assets plus the commission to the winning bidder who has a short period of time, usually three days, to transact all the sales and purchases ordered by the client. It is obvious that all the execution risk is held now by the broker. However, from the broker's point of view, the success of the BPB depends heavily on what happens after the auction. The broker's optimization problem involves the choice of the bid. The submitted bid should be low enough to win the auction, but high enough to generate profits from the subsequent trades. In this paper, we particularly concentrate on BPBs. The special features of principal bid's process, such as the uncertainty of the broker about the individual stock names and the transfer of all the execution risk from the client to the broker, make it attractive as a trade option. The blind procedure of the auction protects the client from the exposure to additional risks. Brokers who know the individual stock names of the basket could front-run the ordered trades. Consequently, the market impact of the front-run transactions could affect the transaction price of the assets and therefore the net proceeds paid by the client.

One of the first papers regarding principal trades is written by Almgren and Chriss (2003). This paper defines an optimal trajectory for the trading of one stock portfolio based on a simple market impact model. An efficient frontier and information ratio are introduced by means of the expected profit and variance of the portfolio's transaction, as functions of the execution time. The information ratio, analogous to Sharpe ratio, could be used as a pricing tool for program trades. Since the authors focus on establishing the information ratio, the important pieces of information for the participating broker are the expected profit and its variance. Hence, the blind procedure of the auction, which is a special feature of the principal trade, is not included in this analysis. The first empirical paper on this topic of BPB was written by Kavajecz and Keim (2005). The authors introduce an empirical regression which models the trading cost of BPB as a function of the basic characteristics of the basket communicated to the brokers. In another empirical study, Giannikos *et al.* (2012) try to better formulate the broker's bids. While Kavajecz and Keim (2005) use as independent variables only characteristics of the basket, Giannikos *et al.* (2012) enrich their model with variables related to broker's execution risk. An interesting part of this paper is the way in which the authors use the structural models developed by Stoll (1978a, b) and Bollen *et al.* (2004). These structural models were initially developed to explain the market maker's spread. By making the analogy of a market maker and a bidding broker, the authors apply a modified version of Stoll's original model of one stock to a basket consisting of several positions. In the last paper on this topic, Padilla and Van Roy (2012) observe that the blind process of the auction increases the cost of principal trade for the clients and the goal of their study is to improve the mechanism of the principal trade in order to lower the cost of the client and the uncertainty of the broker. The authors suggest the incorporation of a trusted intermediary between the client and the brokers at the auction process. The potential benefits are well demonstrated by the comparison of the Bayesian-Nash equilibrium for a standard and an intermediated principal bid auction. Their strong theoretical results were additionally supported by Monte Carlo simulations.

Continuing the line of the study of Giannikos *et al.* (2012) for the modeling of the bids in a blind principal auction, we show how a broker's bid could be formed on the basis of important characteristics of the basket such as the value and the volatility of the basket, the number of names and the weights in the basket. Large Deviations (LD) techniques are used in order to find the optimal bid level that should be submitted. In a principal trade, the lack of information about the individual stock names increases the uncertainty of the participating brokers. The brokers try to protect themselves from harmful rare events and as a result the level of the submitted bids is really high. Kavajecz and Keim (2005) report that 35 out of the 83 lowest bids in their study were ultimately rejected. This means that the cost of the principal bid trade for these cases was considered severely high by the clients. LD techniques have a natural application to the broker's problem as they provide information on probabilities of rare events and on the estimation of large losses.

LD theory is a very active area in applied probability, and its applications have been used in many papers in finance, risk management, insurance [see for instance Stutzer (2003) and Glasserman (2005)]. One of the papers that have concentrated on LD approaches in risk management is Dembo *et al.* (2004). The main goal of that paper is to calculate the total loss of a portfolio with a significant large number of positions. The authors examine a double-stochastic model of default timing. They introduce a common systematic risk factor as a single random variable. The defaults of different firms are considered as independent variables conditional on the value of the risk factor. Also, the amounts that will be lost in different default events are considered as independent. Our study is moving in the same direction as Dembo *et al.* (2004), in which a LD result regarding independent, identically distributed (i.i.d.) random variables is used to approximate a large portfolio loss. This LD argument involves ordinary sums of i.i.d. random variables. Dembo *et al.* (2004) do not concentrate on the establishment of an exponential rate of decay but on a precise closed form approximation solution for the probability of an event. For this paper, we apply a generalization of the above LD argument for weighted sums of i.i.d. random variables [see Book (1972)].

The paper is organized as follows. The general assumptions of our framework are given in section 2, where we also define the problem of the participating broker for the optimal bid. In section 3, we develop a LD argument and provide a closed form solution for the optimal bid. In section 4, a numerical example is used based on the summary statistics of the baskets published by Giannikos *et al.* (2012) and Kavajecz and Keim (2005) that forms an important basis for the comparison of the LD result with an approximation by Monte Carlo simulation. Section 5 provides concluding remarks.

2. Broker's Optimization Problem

Our analysis is developed from the perspective of the participating broker. For the broker's optimization problem, the optimal bid should be low enough to win the auction, but high enough to ensure positive profits following the execution of all the subsequent obligatory trades. We assume that the portfolio of an asset manager includes v stocks. The asset manager needs to sell this portfolio and buy a new one in a short period of time which includes u stocks. Thus, his order includes n transactions ($n = v + u$). Therefore, there are n stocks total to be transacted and for each stock $i, i = 1, \dots n$, the manager's order has N_i shares with price p_i per share. The basket is being auctioned to a number of K participating brokers. The individual stock names are unknown to the brokers who have to submit a bid, expressed as cents per share. b_k is the bid of the k^{th} broker with $k = 1, \dots K$. The asset manager pays the net proceeds (purchases minus sales) plus the commission to the winning bidder who has a short period of time, usually three days, to fulfill all the transactions ordered by the client. Let s_i be the amount of the current market value of the order for each stock i and R_i is the total amount that was actually received when selling or paid when buying for the fulfillment of the transaction related to the order of each stock i. The brokers are interested in the difference $R_i - s_i$. It is obvious that $s_i = N_i p_i$, where p_i, is most recent closing price for stock i. Therefore, in a BPB all the prices are predetermined. Net Proceeds are given as $\sum_{i=1}^{n} s_i$ while the commission is calculated by $b_k N$, where b_k is the winning bid and $N = \sum_{i=1}^{n} N_i$.

Let $\bar{x} = \frac{x_1 + x_1 + \dots + x_n}{n}$ where $x_i = R_i - s_i$, is the transaction gains or losses for the broker. Furthermore let l_i be the expected execution cost from slippage for asset i and so, $T = \sum_{i=1}^{n} l_i$ expresses the expected execution cost from slippage for the portfolio. The optimization problem for broker k is

Maximize E $b_k N$

Subject to (1)

$P(b_k N + n\bar{x} > 0) \geq \delta$

The parameter δ represents the maximum acceptable probability to experience a loss. The broker k believes that all $b_i, i \neq k$, are uniformly distributed between 0 and θ where θ could be considered as the average cost per share of the agency trade for this portfolio. This is the sum of the typical commission plus $\frac{T}{N}$. The above optimization problem has only one constraint where δ determines the confidence level of the broker that his final profit will be positive. We assume that $x_i, i = 1, \ldots n$, are independent normally distributed variables but not identical as transactions can be of different sizes. A similar analysis with independent n positions of a portfolio is presented by Dembo *et al.* (2004). The main result of the above paper is an application of Bahadur and Rao's Theorem. Bahadur and Rao's work involved ordinary sums of i.i.d. random variables. The LD argument that is used in our study could be considered as an extension of the theorem of Bahadur and Rao to the case of weighted sums of i.i.d. continuous random variables. This is a well-known LD theorem developed by Book (1972).

3. Large Deviations

The seller always provides to the broker the volatility of the buy basket, the volatility of the sell basket and the volatility of the entire basket as well [see Giannikos *et al.* (2012)]. Suppose that the volatility of the total dollar value of the basket is represented by σ. The distribution of the weights in the basket is commonly provided to the brokers. Therefore, the following analysis could be easily used by one of the participating brokers before the auction. Although our LD approach could be used for a number of distributions with finite moment generating functions, we assume, for simplicity, that the implementation shortfall is normally distributed around an expected value as modelled by Almgren *et al.* (2005).

Suppose R_i is normally distributed with mean $s_i + l_i$ and variance $w_i\sigma^2$ then $\sum_{i=1}^{n} w_i = 1$. Define $y_i = \frac{R_i - s_i - l_i}{\sqrt{w_i}\sigma} = \frac{x_i - l_i}{\sqrt{w_i}\sigma}$ and $z_i = -y_i$. Thus

$z_i \sim N(0,1)$. Obviously,

$\mathrm{P}(b_k \mathrm{N} + \mathrm{n}\,\bar{x} > 0) = \mathrm{P}(b_k \mathrm{N} + \sigma \sum_{i=1}^{n} \sqrt{w_i}\, y_i + T > 0) = 1 - \mathrm{P}(S_n > c \sum_{i=1}^{n} \alpha_{ni})$,

where $c = \frac{b_k N + T}{\sigma \sum_{i=1}^{n} \alpha_{ni}}$, $a_{ni} = \sqrt{w_i}, i = 1, \dots, n$, and $S_n = \sum_{i=1}^{n} \alpha_{ni}\, z_i$.

Therefore, $\mathrm{P}(\mathrm{bN} + \mathrm{n}\,\bar{x} > 0) = 1 - \mathrm{P}(S_n > c \sum_{i=1}^{n} \alpha_{ni}) \geq \delta$. Now we use a theorem developed by Book (1972) with the following conditions: Assume a sequence $Y = \{Y_i : 1 < i < n\}$ of i.i.d. random variables, $E[Y] = 0$ and $E[Y^2] = 1$. Let $\{a_{ni} : 1 < \mathrm{i} < n, 1 < i < \infty\}$ be double array of nonnegative real numbers such that, $\sum_{i=1}^{n} a_{ni}^2 = 1$. We work with an approximation of $\mathrm{P}(S_n > c \sum_{i=1}^{n} \alpha_i)$ as $\mathrm{n} \to \infty$. Let $\varphi(t)$ to be the moment-generating function of y_i. Suppose $Q(t) = \frac{\varphi\prime(t)}{\varphi(t)}$ and, $\bar{\sigma}_n^2 = Var(\bar{S}_n) = \sum_{i=1}^{n} a_{ni}^2\, Q'(h_n a_{ni})$, where h_n is a solution $\sum_{i=1}^{n} a_{ni}\, Q(h a_{ni}) = \mathrm{c} \sum_{i=1}^{n} a_{ni}$. Then Book (1972) proves that

$$\mathrm{P}(S_n > c\ \textstyle\sum_{i=1}^{n}\alpha_{ni}) \\ =(2\pi)^{-\frac{1}{2}}(\bar{\sigma}_n h_n)^{-1}\exp(-h_n c\textstyle\sum_{i=1}^{n}\alpha_{nk})\big(\prod_{i=1}^{n}\varphi(h_n\alpha_{nk})\,(1+O(\sigma_n)\big), \qquad (2)$$

where

$\mathrm{O}(\sigma_n) \to 0$ as n→ ∞

The main result of this study is established under the same conditions and given below.

Theorem

If implementation shortfall is normally distributed for the assets of the portfolio, an LD approximation for the optimal bid b* is given by max $\left\{\frac{\theta}{K-1}, \frac{\sigma\,\lambda-T}{N}\right\}$, where λ is the infimum of set L, L is the solution set of the inequality $\ln(x)+\frac{1}{2}x^2+q\geq 0$, and q= $\ln\big((1-\delta)\sqrt{2\pi}\big)$. δ represents the maximum acceptable probability to experience a loss, π is the irrational number 3.14…, T expresses the expected execution cost from portfolios' slippage, σ is the volatility of the total dollar value of the basket, and N is the total number of shares.

Proof

Fix n and define $A_n=\sum_{i=1}^{n}a_{ni}$. Thus $c=\frac{b_kN+T}{\sigma A_n}$is a constant. Since $z_i\sim N(0,1)$, then

φ(t) = $e^{\frac{1}{2}t^2}$ and Q(t) = t, Then solving equation $\sum_{i=1}^{n}a_{ni}\,Q(ha_{ni})$ = c , we get $h_n=\frac{b_kN+T}{\sigma}$. Therefore (2) is equivalent to

$$\mathrm{P}(S_n > c\ \textstyle\sum_{i=1}^{n}\alpha_{ni}) \ = \frac{1}{\sqrt{2\pi}}\frac{\sigma}{b_kN+T}\exp\left(-\frac{1}{2}\left(\frac{b_kN+T}{\sigma}\right)^2\right).$$

And the constraint of the optimization problem (1) is written

$$1-\frac{1}{\sqrt{2\pi}}\frac{\sigma}{b_kN+T}\exp\left(-\frac{1}{2}\left(\frac{b_kN+T}{\sigma}\right)^2\right)\geq\delta$$

Simple algebraic calculations show that b should be a solution of the below inequality

$$\ln\left(\frac{b_k N + T}{\sigma}\right) + \frac{1}{2}\left(\frac{b_k N + T}{\sigma}\right)^2 + ln\left((1-\delta)\sqrt{2\pi}\right) \geq 0 \qquad (3)$$

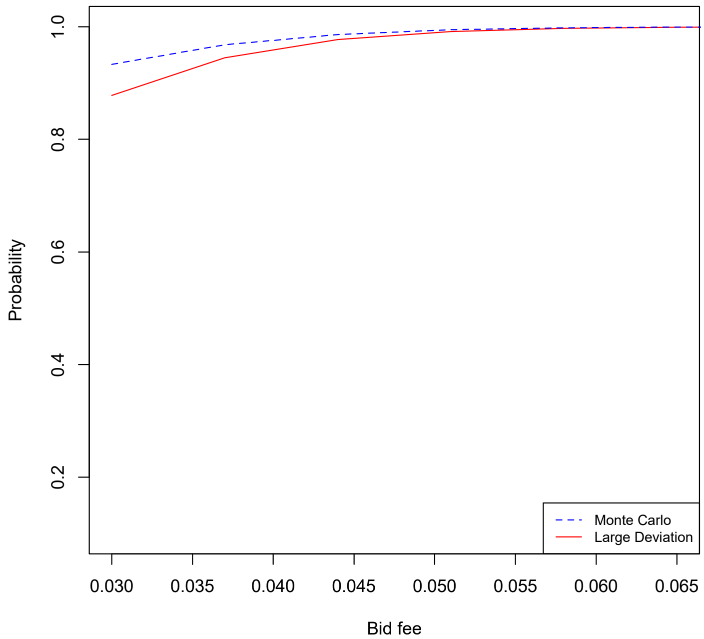
A solution set for b exists. Let L be the solution set of the above inequality (3). Since the bids in the blind auction are uniformly distributed, the probability of win for the submitted bid is given as $\left(\frac{\theta - b_k}{\theta}\right)^{K-1}$. The expected revenue for the participating broker is

$E(Revenue) = b_k \mathrm{N}\left(\frac{\theta - b_k}{\theta}\right)^{K-1}$. The solution for $\frac{\partial E(Revenue)}{\partial \theta} = 0$ is $b_k = \frac{\theta}{K-1}$. Since $E(Revenue)$ is a decreasing function of b in the interval $\left(\frac{\theta}{K-1}, \theta\right)$ then the solution b* of the problem (1) is given by $b^* = max\left\{\frac{\theta}{K-1}, \frac{\sigma\,\lambda - T}{N}\right\}$, where λ is the infimum of set L, L is the solution set of the inequality $\ln(x) + \frac{1}{2}x^2 + q \geq 0$ and $q = \ln\left((1-\delta)\sqrt{2\pi}\right)$.

4. **Numerical Example**

In this section, we develop a numerical example that compares the large deviation (LD) theoretical model and an approximation by Monte Carlo simulation. Giannikos *et al.* (2012) use a sample of 140 baskets. Based on their summary statistics, the mean number of stocks in a basket was 234, the average total trade size was $352 million and the mean number of shares per basket was 12 million while the average winning bid cost was 45 basis points. The cost of principal bid is expressed in basis points, as the ratio of total execution cost to the basket's market value.

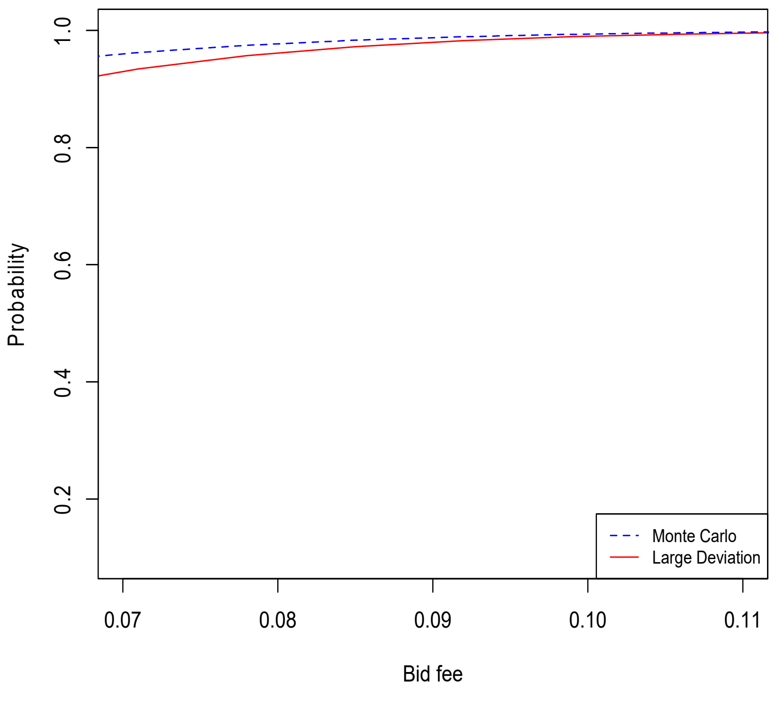
Figure 1. LD vs Monte Carlo (Basket Volatility σ =100,000)



Kavajecz and Keim (2005) use a sample of 48 completed basket and 35 passed baskets. The mean number of stocks in a basket was 163, the average total trade size was $ 88.97 million and the mean number of shares per basket was approximately 4 million. By selecting parameters within the range presented in the literature, we assume that the client wants to sell a portfolio with 163 assets and a total of 5 million shares, i.e. N=5,000,000 and n=163. The weights a_i of each asset in this basket were produced by a uniform distribution such that $\sum_{i=1}^{n} a_{ni}^2 = 1$. For this numerical example $A_n = \sum_{i=1}^{n} a_{ni} = 12.1$. Let the dollar value of the portfolio to be 90 million. Figure 1 shows how the probability of a broker observing a positive profit depends on the level of the winning bid fee. Monte Carlo simulations (with 1 million scenarios) are compared with the LD method developed in this paper. Based on the Figure 1, as the bid fee increases, the risk of a loss is mitigated, and the probability of positive profits tends to be 1. For example, a broker with a submitted winning bid fee of 6 cents per share knows that the probability to observe a loss is approximately 0.0021, based on the LD model. A winning bid fee of 6 cents sets out the cost of principal bid to 33 basis points.

Also, we have assumed a volatility σ =100,000 for this basket. This assumed value for the volatility depends on the broker's expectations regarding crossing shares with his own inventory which will mitigate the execution cost from slippage. Thus, the variability in the submitted bids reflects the different expectations of the brokers regarding the volatility of the basket which is related to their estimate of their own trading abilities and circumstances (for instance their current inventory). Naturally, a riskier basket will increase the cost of principal bid. As we observe in Figure 2, for a basket with the same characteristics and double volatility, the original graph is shifted to the right reflecting this increase in the cost of the principal bid. Not surprisingly, similarly consistent with intuition shifts would be observed when other characteristics are modified, such as the expected execution cost from slippage.

Figure 2. LD vs Monte Carlo (Basket Volatility σ =200,000)



5. Conclusion

Since a significant percentage of the daily trading volume in major stock exchanges involves principal bid transactions, BPB is considered as one of the most important trading mechanisms. This paper extends the literature on this topic by introducing a model that formulates the optimal bid that should be submitted by the broker. Although the model has been created from the broker's perspective, it may be beneficial for the clients as well. This LD model approximates precisely the probability of a harmful rare event, allows to develop a closed-form solution, and helps the broker to connect his potential bids with the probability of observing a positive profit. The brokers' inability to model the uncertainty they face, as a result of the blind process of these transactions, is the main reason why the observed submitted bids are very high and often leads clients to choose an agency trade after receiving the bids. A numerical example illustrates how close the LD solution is to the Monte Carlo approximation. Finally, the development of a closed formula for the optimal bid in the case of more involved market impact models could be the subject of further research.

References

Almgren, R. and N. Chriss, (2003), "Bidding principles," *Risk Magazine*, (June 1) 97-102.

Almgren, R., C. Thum, E. Hauptmann, and H. Li, (2005), "Equity Market Impact," *Risk*, 18, 57-62.

Bollen, N. P. B., T. Smith, and R. E. Whaley, (2004), "Modeling the bid/ask spread: measuring the inventory-holding premium," *Journal of Financial Economics*, 72, 97-141.

Book, S. A., (1972), "Large deviation probabilities for weighted sums," *Annals of Mathematical Statistics*, 43(4), 1221-1234.

Dembo, A., J. D. Deushel, and D. Duffie, (2004), "Large portfolio losses," *Finance and Stochastics*, 8, 3-16.

Giannikos, C. and M. Suen, (2007), "Estimating two structural spread models for trading blind principal bids," *White paper*, *European Financial Management Association*, Norfolk, VA.

Giannikos, C. and M. Suen, (2007), "Pricing determinants of blind principal bidding and liquidity provider behavior," *White paper*, *European Financial Management Association*, Norfolk, VA.

Giannikos, C., H. Guirguis, and T.S. Suen, (2012), "Modeling the blind principal bid basket trading cost," *European Financial Management*, 18, 271-302

Glasserman, P., (2005), "Tail approximations for portfolio credit risk," *Journal of Computational Finance*, 9, 1-41.

Kavajecz, K., and D. Keim, (2005), "Packaging liquidity: Blind auctions and transaction efficiencies," *Journal of Financial and Quantitative Analysis*, 40, 465-492.

Padilla, M., and B. Van Roy, (2011), "Intermediated Blind Portfolio Auctions," *Management Science*, 58, 1747-1760.

Stoll, H. R., (1978), "The supply of dealer services: An empirical study of NASDAQ stocks," *Journal of Finance*, 33, 1153-1172.

Stoll, H.R., (1978), "The supply of dealer services in securities markets," *Journal of Finance*, 33, 1133-1151.

Stutzer, M., (2003), "Portfolio choice with endogenous utility: A large deviation approach," *Journal of Econometrics*, 116, 65-386.