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Is Volatility of Equity Markets a Volume Story? A Nonparametric Analysis

Christos I. Giannikos*
Department of Economics and Finance, *Baruch College*, *U.S.A*.

Hany Guirguis
Department of Economics and Finance, *Manhattan College*, *U.S.A*.

Deniz Ozenbas
Department of Economics and Finance, *Montclair State University*, *U.S.A*.

Abstract

In this paper we document and account for the non-normality of returns exhibited by the indices in our samples. Consequently we re-examine the relationship between volatility and volume while distinguishing between returns within a trading day and returns across trading days. Our results indicate that the volatility exhibited by both types of returns is positively and significantly related to volume. Hence the results provide an additional explanation for short-term volatility patterns, which is not necessarily within a strict price formation framework.

Key words: volatility; volume; multiple equation models; nonparametric methods
JEL classification: G10; G11; C14

1. Introduction

The relationship between volume of trade and the volatility of returns has been, for quite some time, at the center of microstructure research. Most empirical studies report a positive correlation between volatility and volume for a broad variety of portfolios and securities. Karpoff (1987) provides a detailed survey, which concludes that volume is positively related to the magnitude of the price change and, in equity markets, to the price change per se. Jones et al. (1994) show that the positive

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*Correspondence to: Department of Economics and Finance, Box B 10-225, Zicklin School of Business, Baruch College, One Bernard Baruch College, New York, NY 10010. Email: Christos_Giannikos@ baruch .cuny.edu. We are grateful to J. Harold Mulherin and Aditya Kaul for generously providing us with part of the data used in this paper. We would also like to thank the Editor, Pao-Long Chang, and especially the Managing Editor, Jong Shin Wei, and two anonymous referees for comments and suggestions that helped us significantly improve the paper.

volatility-volume relation actually reflects the positive relationship between volatility and the number of transactions.

Significant theoretical models that try to reconcile the empirical findings with the informational role of prices and volume have also been developed. O'Hara (1997) provides a broad review of the theoretical studies that relate trading volume to price adjustments.

Finally Gerety and Mulherin (1994), in their important paper, estimate transitory volatility throughout the trading day by using hourly Dow Jones 65 Composite price index data. They find that volatility steadily declines during the trading day, and they report that their findings are consistent with the hypothesis that trading aids price formation.

Our study re-examines the relationship between volatility and volume while distinguishing between returns within a trading day (intra-day) and returns across trading days (inter-day). Intra-day returns are half-hourly (or hourly) returns calculated at different times throughout the trading day. Inter-day returns are daily (24 hour) returns calculated at different times throughout the trading day. For example the return from 10:00am to 10:30am within the same trading day is an intra-day (half-hourly) return while the 24-hour return from 10:00am of day $t-1$ to 10:00am of day t is an inter-day return.

First, we document the non-normality of returns exhibited by the indices in our samples. We show that both intra- and inter-day returns exhibit highly non-normal behavior and we introduce a method to deal with this problem. Then, we try to answer the following simple question: Can intra- and inter-day volatility patterns be explained by volume?

Our results indicate that the volatility patterns could indeed be perceived as a volume story for the various time periods that we study. Furthermore, according to the work of Jones et al. (1994), replacing the volume with the number of transactions might be a legitimate and interesting venue of further research. But we leave this for the future.

The remainder of this paper is organized as follows. Section 1.1 discusses the dataset. The empirical methodology is presented in Section 2 while Section 3 outlays the results and Section 4 concludes.

1.1 Dataset

The first part of our data consists of hourly values of the Dow Jones 65 Composite Index for the 1971:01-1985:09 period and half-hourly values for the 1985:10-1990:12 period. This data set was used in Gerety and Muhlerin (1994). In addition, we wanted to check the robustness of our findings using more recent data. For this purpose we employ half-hourly values of the DIAMONDS, Trust Series 1, traded on the American Stock Exchange. We use data of this Exchange Traded Fund , which can be thought of as a proxy for Dow Jones 30 Industrials Index, for the period 1998:01-2001:11. Since DIAMONDS first started trading in 1998:01, we have to choose this date as our starting date.

By choosing indices we also make sure that firm-specific sources of transitory volatility cancel out as both Gerety and Mulherin (1994) and Amihud and Mendelson (1989) suggest.

2. Empirical Methodology

The first step in examining the causal relationship between the volatility of the returns and the volume is to estimate the volatility series during each trading period $\sigma^2_{n,t}$ as follows:

$$\sigma^2_{n,\,t} = \left[r_{n,\,t} - (\alpha_n + \sum_{m=1}^{12} \beta_{n,\,m} r_{n,\,t-m}) \right]^2 , \tag{1}$$

where $r_{n,t}$ is either the *inter-day* return or the *intra-day* return and α_n and $\beta_{n,m}$ are the estimated coefficients from regressing the returns on a constant and twelve of its own lags. In line with Jones et al. (1994) we also use twelve lags. This choice is additionally justified by using the Schwartz Bayesian Criterion (SBC) according to which the optimal number of lags never exceeds seven.

The *inter-day* return between day t and day $t-1$ that corresponds to the nth time period during the trading day is calculated as $\ln(p_{n,t} / p_{n,t-1})$ whereas the *intra-day* return that corresponds to the nth time period during the trading day and to day t is calculated as $\ln(p_{n,t} / p_{n-1,t})$.

Then, we regress the $\sigma^2_{n,t}$ series on a constant, twelve of its own lags (for the same reasons as above), and the natural log of the trading volume $v_{n,t}$ as follows:

$$\sigma^2_{n,\,t} = r_n + \sum_{m=1}^{12} p_{n,\,m} \sigma^2_{n,\,t-m} + \delta_n v_{n,\,t} + \varepsilon_{n,\,t} \,. \tag{2}$$

Naturally, volume that corresponds to the inter-day returns is volume transacted through the 24-hour interval whereas volume corresponding to intra-day returns is volume transacted during the half-hour (or one-hour) trading interval.

In estimating the above empirical model, we encounter two econometric problems. First, estimating each equation separately will not account for the dependence in the error terms across the different trading periods. Thus, we jointly estimate the two equations for each trading period by using a 2n-equation Seemingly Unrelated Regression (SUR).

The second problem stems from the fact that our returns are not normally distributed. Tables 1 and 2 confirm that both intra- and inter-day market returns in our sample do not conform to normality according to the skewness, kurtosis, and Jarque-Bera statistics. As a result, we use the percentile-t technique with double bootstrapping as our nonparametric approach to statistical inference. As pointed out by Hall (1988) and Hinckley (1988), the double-bootstrapping technique offers highly accurate confidence intervals especially in cases of small samples and severe non-normal probability distributions.

Table 1. Normality Tests of the Market Returns (Intra-Day)

	1971:01-1985:09			1985:10-1990:12			1998:01-2001:11		
	Skew	Kurt	JB	Skew	Kurt	JB	Skew	Kurt	JB
1	**−0.05**	3.25	1919.09	**0.06**	4.75	3499.39	**−0.03**	03.70	554.44
2	0.28	4.19	2767.90	−0.42	9.58	1.43e+04	**0.12**	01.23	63.55
3	0.16	2.60	1060.57	**−0.03**	2.59	1040.18	0.42	05.76	1371.19
4	**−0.04**	3.92	2388.85	−0.49	3.93	2536.13	−0.16	00.99	43.49
5	0.07	3.25	1638.58	−0.21	4.08	2602.99	−0.23	04.76	923.34
6	0.57	3.80	2431.70	0.42	6.72	7099.09	−0.13	02.95	355.82
7				−0.50	5.04	1423.12	−2.85	43.22	7.70e+04
8				−0.69	22.04	2.63e+04	4.18	51.42	1.10e+05
9				−1.02	7.37	3150.63	−1.51	15.68	1.02e+04
10				**−0.10**	6.15	2040.17	**0.03**	02.68	288.32
11				−0.20	3.10	0527.87	**0.07**	01.68	114.49
12				−0.36	7.43	3003.81	0.28	02.52	266.60
13				−0.94	9.66	5214.91	−0.80	07.69	2481.61

This procedure can be described as follows. First, draw 200 random samples without replacement from the returns (r). Second, run the SUR and calculate the coefficients at each draw (i). Then, calculate the standard deviation (σ'_n) of each (δ'_n) from the 200 regressions. Third, at each draw conduct another round of 100 bootstrapping using the draws of the series as your original series and run the SUR at each draw. Then calculate estimated coefficients (δ'_n) and their standard deviation $(\sigma''_n)_i$ from the 100 regressions.

Table 2. Normality Tests of the Market Returns (Intra-Day)

	1971:01-1985:09			1985:10-1990:12			1998:01-2001:11		
	Skew	Kurt	JB	Skew	Kurt	JB	Skew	Kurt	JB
1	0.31	2.76	1239.13	−40.29	2099.11	6.83e+08	−0.36	2.40	253.41
2	0.23	2.02	666.14	−40.97	2147.16	7.15e+08	−0.14	1.66	115.41
3	0.22	1.70	477.63	−40.80	2134.51	7.07e+08	**−0.01**	1.64	108.73
4	0.18	1.80	524.22	−40.98	2145.41	7.14e+08	**−0.06**	1.85	139.38
5	**0.05**	1.44	322.39	−41.96	2212.74	7.59e+08	**−0.01**	1.89	145.18
6	0.27	1.71	492.94	−42.05	2221.33	7.65e+08	**−0.10**	1.81	134.41
7				−08.89	195.33	2.07e+06	**−0.08**	1.74	123.04
8				−08.56	185.81	1.88e+06	−0.20	1.90	152.26
9				−08.63	187.67	1.91e+06	−0.26	2.16	196.59
10				−08.67	188.67	1.93e+06	−0.21	2.41	238.84
11				−09.30	207.45	2.33e+06	−0.22	2.83	324.97
12				−09.50	214.88	2.50e+06	−0.21	2.65	285.49
13				−09.31	205.37	2.29e+06	−0.37	2.99	376.68

Note: All the above reported skewness, kurtosis, and Jarque-Bera statistics are significant at the 10% level except for the numbers in bold, which indicate insignificance at the 10% level. Thus, one can reject the hypothesis that the inter-day returns are normally distributed.

The main advantage of using σ_n'' is to weight each δ_n' by its standard deviation. For more details see Effron and Tibshirani (1986) and Mooney and Duval (1993). Fourth, calculate *t*-statistics of the (δ_n') as follows:

$$t_i = \frac{(\delta_n')_i}{(\sigma_n'')_i}, \tag{3}$$

where i = 1, …, 200 and n = 1, …, 13 (1, …, 6 for the period 1971:01-1985:09). Fifth, order the 200 t_i and find the $a/2$ and $1-a/2$ percentile values of t, where a is the significance level (here 10%). Finally develop the 90% confidence interval around the estimated coefficients from the original data as follows:

$$P(\overline{\delta} + t_{.05}\,\sigma_n' \le \overline{\delta} \le \overline{\delta} + t_{.95}\,\sigma_n') = 0.95, \tag{4}$$

where

$$\overline{\delta} = \frac{1}{200} = \sum_{i=1}^{200} \delta_i\ . \tag{5}$$

If the zero line is located between the 5th and 95th percentile confidence intervals, the abnormal returns are statistically insignificant at the 10% level.

3. Results

Figure 1 deals with the intra-day volume and volatility relationship. It reveals that the coefficient on volume is positive and statistically significant for all intra-day trading periods during the time interval 1971:01-1985:09 (DJ-65 data), positive and statistically significant for all but three intra-day periods during the interval 1985:10-1990:12 (DJ-65 data), and positive and statistically significant for all but one intra-day trading period during the interval 1998:01-2001:11 (DIAMONDS ETF or DJ-30 data). Significance is always at the 10% level. Figure 2 presents results that also confirm a positive relationship between volume and volatility in the case of inter-day returns. The results are statistically significant at the 10% level for all different time periods for both indices and at any time during the days we examined.

4. Conclusion

Our results suggest that the patterns of volatility can be explained to a great extent by trading volume. This is an alternative to the information assimilation explanation that Gerety and Mulherin (1994) offer. The two alternatives do not appear to be *a priori* mutually exclusive. Also since volatility is very important for option prices, the above finding might be useful to option traders when coupled with out-of-sample forecasting. These issues and whether the occurrence of transactions is more important than their size in this framework are left to future research.

Figure 1: 90% Numerical Confidence Interval: Volume Coefficient in the Volatility Equation (Intra-Day)

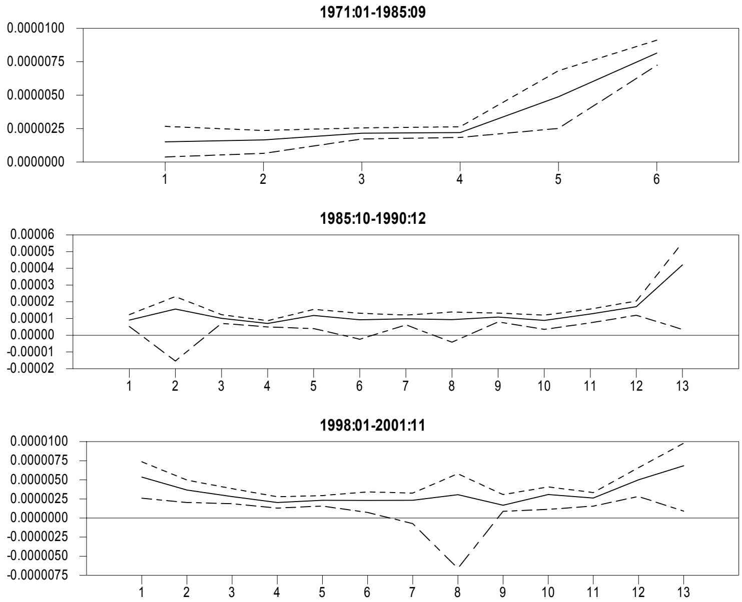
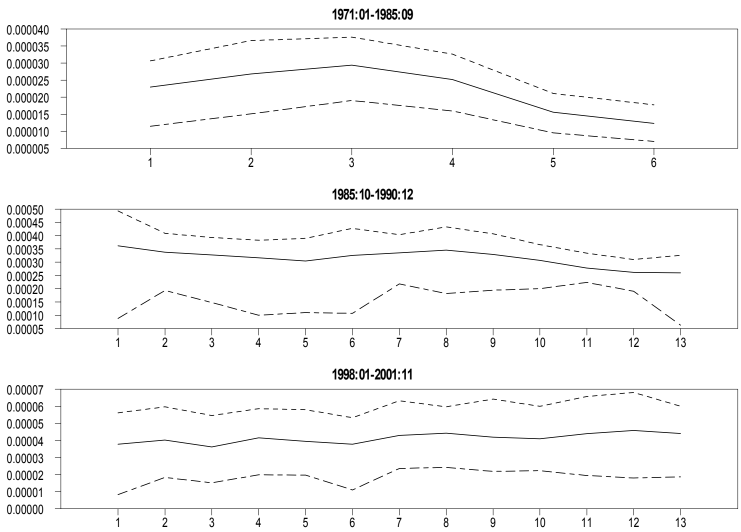


Figure 2: 90% Numerical Confidence Interval: Volume Coefficient in the Volatility Equation (Inter-Day)



Note: If the zero horizontal line is between the two dotted lines, then the volume coefficient is statistically insignificant at the 10% level for that trading inter-day interval.

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