The Impact of Changing the Expected Time and Variance Equations of the Project Activities on The Completion Time and Cost of the Project in PERT Model

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Abstract

Scheduling is one of the most important tools used in project management, and Programming Evaluation Revision Technique (PERT) is one of the methods used in project scheduling. PERT depends on identifying three possible times for any activity: optimistic, pessimistic, and the most likely. Using these three times, the expected time to end the activity and then the critical path of the project, as well as the activity variance, and then the project standard deviation is calculated by offering different weights for these three times. Therefore, this study aimed to suggest the use of equal weights for these times in calculating both the expected time to complete the activity as well as its variance, in order to know the effect on the completion time of project, the standard deviation of the project, as well as the cost of project crashing. To do that the both models usual PERT model and the proposed model were applied on three real construction projects. The results revealed that the proposed model (equal time weights) has a positive impact on the project crashing cost if required, but the results also revealed a negative impact of applying the proposed model on the project completion time (longer critical path), and on the standard deviation of the project.

Keywords: PERT Model Critical Path, Variance, Project Standard Deviation, Project Cost, Project Crashing Cost

JEL Classification: L7, L74, N6

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1. Introduction

Projects have proliferated as a favorable environment for practicing commercial activities as modern organizations rely heavily on projects as the main organizational form to conduct their commercial operations. Projects can be described as strong strategic weapons that represent a central building block in the implementation of the organization's strategy (Filippov et al., 2012; Meskendahl, 2010; Shenhar et al., 2001); however, organizations, especially project-based organizations, may face some difficulties in delivering their programs and projects. Although they have identified the right projects and programs to invest in, the delivery will be too late or at the expense of excessive costs or high risks, and thus the organization does not achieve the strategic goals and directions it seeks.

Therefore, these projects and programs need attention when organizations seek to spend their capital wisely, with the required quality, and within the specified timetable and the framework of the strategic directions they seek to achieve.

Many researchers have emphasized the importance of studying project scheduling and costs (Behrendt & Wulke, 2004; Dell'Isola, 2002; Deng & Hung, 1998; Emhjellen et al. 2003; Khamooshi, 1996; Jayaraman, 2016; Lavingia, 2003; Liu, 2013). The project evaluation and review (PERT) method was developed to control the trade-off operations between time and cost (Al Nasseri et al., 2016). The PERT model is also considered as a convenient way to estimate the level of uncertainty through scheduling in project management (Hu, 2011; Zhu & Heady, 1994), Further, the PERT model is used to determine the time uncertainties, so the risks are identified and adjusted (Huang & Wang, 2009; Liu, 2013), On the other hand, the mathematical aspects of the critical path method (CPM) theory have been combined with the probabilistic concepts of PERT to provide a clear estimation of uncertainty because both methods adopt similar planning theories (Al Nasseri et al., 2016; Fulkerson, 1962; Hu, 2011; Kuklan et al., 1993; Mongalo & Lee, 1990; Wei et al., 2002). Hajdu and Bokor (2016) emphasized that instead of the time-consuming and costly process of selecting the proper activity during distribution, planners should devote much more effort to determine the activity durations adequately. On the other hand, Shankar et al. (2010) stated that researchers made numerous attempts to improve PERT analysis based on the subjective determination of a, m, and b over the past five decades (Azaron et al., 2006, Chen, 2007; Shankar et al., 2010). This study focuses on the methodology used in estimating the expected time and variance of the project activities and the consequent impact of the project. Using both the traditional and the proposed equations to calculate the expected activity time and variance, respectively, as steps used to calculate the project lifetime and the probability of ending it within a given time shows a high probability of success in completing the project within that given time. This schedule reduces the effect of uncertainty when managing a project, but when applying this schedule, managers are sometimes surprised there is a delay in the project completion time, resulting in a higher cost. Also, if the project delivery is promised within a specified time, project management has to speed up and, as a result, incur additional costs, possibly because of a defect in estimating the completion time accurately using the traditional equation, which depends on giving the most likely time *m* a weight equivalent to four times the optimistic time *a* and pessimistic time *b*, which weakens the effect of estimating pessimistic and optimistic times in determining the expected activity time. Therefore, this study seeks a more accurate estimate of the expected time in case of changing the three times' weights in the traditional model, taking the environment uncertainty into consideration more accurately. Thus, this study is implemented through three cases of projects, whose bids have already been referred through the Jordanian Ministry of Public Works and Housing during the year 2019.The three cases are presented to compare the results obtained using the proposed method as well as the traditional one.

Moreover, this study does not perform a quantitative evaluation of the PERT model. Therefore, the importance of this study arises from its attempt to reach a reliable scientific method that may be more accurate in determining the expected completion time for each project activity and for calculating the standard deviation of each activity; this can be done by using the normal arithmetic mean as in the proposed equation, ET =, to calculate the expected time instead of the weighted arithmetic mean as in the equation ET = $\frac{a+4m+b}{6}$ and by calculating the variance using the equation $\sigma^2 = \left(\frac{b-a}{3}\right)^2$ instead of the equation: $\sigma^2 = \left(\frac{b-a}{6}\right)^2$ used in the PERT model.

The proposal of this study depends on giving equal weight to the three times (*a* for the optimistic time, *m* for the most likely time, and *b* for the pessimistic time) and the resulting effect on the critical path length and the probability of project completion and cost within a given time, which is what all departments are seeking. The study's importance comes from its attempt to determine the interrelationships between the time, cost, and probability of achievement in managing the project as a result of changing the traditional model used in this field. This study includes the following variables and symbols: Expected time (ET), Variance (õ2), Critical path (CP), Project standard deviation (õp), Standard value (Z), Project normal completion cost (PNCC), Project crashing cost (PCC). Each of these variables are calculated in two ways: the first by using the traditional equations to calculate the expected time and variation used in the PERT method, and the second by using the proposed equations of this study but in the same order of solution steps.

2. Literature Review

The concept of project management refers to the planning, scheduling, and assessment processes of a group of activities that require a specific timeline and resources to be implemented. One of the most important technique is network analysis using PERT and CPM (Fulkerson, 1962; Hu, 2011; Wei et al., 2002). PERT and CPM were developed after World War I as tools and techniques to avoid misestimating project costs and completion time. These techniques, developed further in the 1950s within the Quantitative Management and Operations Research School, have achieved the desired accuracy and progress in the projects' planning and scheduling, which has led to a significant increase in applying and using these techniques since the last decades of the 20th century; more recently, software has been developed to enable project managers to quickly and easily estimate costs and completion times (Al-Ali, 2019; Webster, 1994). The applications of these tools have reached vast prospects in the field of management (Engwall, 2012). Many researchers have studied project management (Aretoulis, 2019; Bohnstedt & Wandahl, 2019; Kwofie et al., 2018; Luiz et al., 2019; Silvius & Schipper, 2015), and several software packages and computer systems have been developed to support and facilitate the use of these technologies (Burgelman & Vanhoucke, 2019; Luiz et al., 2019; San Cristobal et al., 2017), However, despite the significant role this technology is playing in the research process—specifically in helping project managers plan project times and schedule project activities—many projects are not completed within the predicted time but take longer than expected and exceed the planned cost (Rezvani & Khosravi, 2019; Stoy et al., 2012; Yang & Wei, 2010), Azaron et al. (2006), Chen (2007), and Walter et al. (2000) all agreed that project management is concerned with scheduling and monitoring activities so a project can be completed in the shortest possible time. Currently, although researchers have been able to identify reasons for delays in project completion time and increase in project costs (Rezvani & Khosravi, 2019; Silvius & Schipper, 2015), they have not yet reached a holistic and practical approach that can ensure the project implementation and completion as planned (Jayaraman, 2016). As Shankar et al. (2010) asserted, there are a few areas that are not yet open to sharp criticism as it is in BERT's applications. Nevertheless, some academics and researchers have criticized the BERT model (Clark; 1962; Golenko-Ginzburg, 1988; Grubbs, 1962; Hajdu & Bokor, 2016; Sasieni, 1986; Shankar et al., 2010; Shankar and Sireesha, 2009; Walter, et al., 2000). However, this study is not so much criticism as it is an attempt to open the door to propose, test, and modify the traditional PERT model so it becomes more accurate and thus better able to enable and assist project managers to plan and control projects. Yang (2007) indicated that the available scheduling methods' investigations help researchers avoid ambiguity in understanding these methods and thus raise their knowledge level concerning these methods.

3. Study Hypotheses

Researchers and scholars have identified reasons for delays in the project completion time as well as reasons for high costs of implementing projects (Rezvani & Khosravi, 2019; Silvius & Schipper, 2015). However, they have not yet been able to arrive at a comprehensive and pragmatic approach that can ensure that projects are implemented and completed as planned (Jayaraman, 2016). Because project management is concerned with monitoring and scheduling activities in such a way that projects can be completed in the shortest possible time (Azaron et al., 2006; Chen & Huang, 2007; Shankar et al., 2010), and based on the characteristics of the study problem, to achieve its objectives, these hypotheses are formulated:

First hypothesis H_1: Using the proposed equations to calculate the expected time and variance shortens the critical path of the project and thus improves project completion time.

Second hypothesis H_2: Using the proposed equations to calculate the expected time and variance reduces the project standard deviation.

Third hypothesis H_3: Using the proposed equations to calculate the expected time and variance reduces the total cost of project completion.

Fourth hypothesis H_4: Using the proposed equations to calculate the expected time and variance reduces the probability of a project completed within a specified time required by the project management below the critical path.

4. Study Methodology

Engwall (2012) asserted that scholars who focus on project management research need to keep themselves away from its tight historical bounds to project management textbooks and administrative project management techniques. Hence, the methodology of the study based on a real cases of three constructions projects, executed for the Ministry of Public Works and Housing by local contractors in 2019, the three projects were subjected to crashing process. All data including the crashing time of the three projects were obtained from the Jordanian Contractors Association.. Consequently, this study depended on using a proposed equation to calculate the activity duration: ET=$\frac{a+4m+b}{3}$, where a uniform weight was given to all of the times used to find the expected time. In other words, the normal arithmetic mean was used to find the expected time instead of the weighted arithmetic mean. This has resulted in using the equation of PERT model, $\sigma^2=\left(\frac{b-a}{3}\right)^2$, to calculate the activity variance instead of using the traditional equations ET=$\frac{a+4m+b}{6}$ and $\sigma^2=\left(\frac{b-a}{6}\right)^2$ to calculate the activity time and variance, respectively. Then the traditional and proposed equations of time and variance were applied and the results were obtained to conduct a comparison and test the study hypotheses.

4.1 Applying the Proposed Equation

This part will result in identifying the schedule using the PERT model and the proposed equations to extract the expected time (ET) and expected variance (õ2) for each activity in the project. Then, we will calculate the critical path (CP), project standard deviation, standard value (Z), and the probability of project completion within the required time and compare the achieved results with the results of the traditional equation used in the PERT model through the three study cases. The results achieved will then be used in testing the hypotheses.

The information of the elementary data presented in the tables of the three cases including the crashing time of each project was obtained from the Jordanian Contractors Association.

4.1.1 The First Case

The information of the first case are shown in Table 1.

Table 1. The elementary data of the project in the first case

Activity	Time Estimates/month			Crash Time	Activity Cost/JD	
	a	*m*	*b*	Month	Normal	Crash
A	5	7	15	5	700	1200
B	7	9	17	9	1600	2000
C	8	9	10	7	900	1500
D	3	6	9	3	500	900
E	2	4	6	2	500	700
F	4	5	6	4	500	800
G	0	0	0	0	0	0
H	4	6	14	5	700	1000
I	8	12	16	10	1800	2300
J	10	15	20	12	1400	2000
K	10	17	30	14	1400	3200
P	1	4	7	3	500	800
N	5	6	13	6	800	1400

We first apply the traditional equation: Using the traditional equation of the PERT model to calculate the activity time ET= $\frac{a+4m+b}{6}$ and activity variance $\sigma^2=\left(\frac{b-a}{6}\right)^2$ respectively, the values of ET and expected variance ($\tilde{o}^2$) are shown as in Table 2.

Table 2. Calculating ET and variance (õ2) in the first case using PERT model traditional equations

Activity	Time Estimate/Months			Expected Time ET	Variance $\tilde{o}^2$
	a	***m***	***b***		
A	5	7	15	8	2.777
B	7	9	17	10	2.777
C	8	9	10	9	0.333
D	3	6	9	6	1
E	2	4	6	4	0.444
F	4	5	6	5	0.111
G	0	0	0	0	0
H	4	6	14	7	2.777
I	8	12	16	12	1.777
J	10	15	20	15	2.777
K	10	17	30	18	11.111
P	1	4	7	4	1
N	5	6	13	7	1.777

Table 3. The available paths and critical path of the first case according to the traditional equation

Path/Activities	Path/Times	Path Duration
A - E – K	8 + 4 + 18	30
A - D - J – N	8 + 6 + 15 + 7	36
B - F - J – N	10 + 5 + 15 + 7	37
B - F - I - P – N	10 + 5 + 12 + 4 + 7	38
C - H - P – N	9 + 7 + 4 + 7	27

Paths are selected to determine the critical path as shown in Table 3.

Table 2 shows that the critical path is shown to be the path (B-F-I-P-N) and its duration is 38 months.

The project standard deviation is calculated by specifying the activities’ variation in the critical path, combining them and then taking the square root of the sum:

$\sigma = \sqrt{\sigma^2}$ And then the equation z = $\frac{X-\mu}{\sigma} = \frac{X-Cp}{\sigma}$.

The result will be the project standard deviation according to this equation:

$$= \sqrt{2.78 + .111 + 1.78 + 1 + 1.78} = \sqrt{7.451} = 2.73$$

$$Z = \frac{32-38}{2.73} = -2.1978$$
$$P = 14\%$$

This implies that the probability of completing the project before 6 months of the scheduled time is 14%.

We then calculate the cost of crashing the project to 32 months as required. The cost of crashing the project from 38 months to only 32 months is about 2050 JD, therefore the cost of completing the project will be 13,350 JD instead of 11,300 JD, and the results are shown in Tables 4 and 5.

Table 4. Times and cost of crashing project activities according to the traditional equation of the first case

	Activity Time months		**Activity Cost/JD**			
Activity	**ET**	**Crash**	**Normal**	**Crash**	**Max Crash Time**	**Crash Cost Per Month**
A	8	5	700	1200	3	166.67
B	10	9	1600	2000	1	400
C	9	7	900	1500	2	300
D	6	3	500	900	3	133.33
E	4	2	500	700	2	100
F	5	4	500	800	1	300
G	0	0	0	0	0	0
H	7	5	700	1000	2	150
I	12	10	1800	2300	2	250
J	15	12	1400	2000	3	200
K	18	14	1400	3200	4	450
P	4	3	500	800	1	300
N	7	6	800	1400	1	600

Table 5. Results of the crashing process according to the traditional equation of the first case

Iteration	Change	Complete Time Month	Project Cost JD	Critical Path
Original	-	38	11300	B-F-I-P-N
1	I (2)	37	11550	B-F-J-N
2	J (3)	36	11750	B-F-I-P-N
3	P (1)	35	12050	B-F-I-P-N
4	F (1)	34	12350	B-F-I-P-N
5	B (1)	33	12750	B-F-I-P-N
6	N (1)	32	13350	B-F-I-P-N

Second, we apply the proposed equations: Using the model's proposed equation in calculating the activity time, ET= $\frac{a+m+b}{3}$, and the traditional equation in calculating the activity variance: $\sigma^2 = \left(\frac{b-a}{3}\right)^2$, the values for both ET and expected variance ($\tilde{o}^2$) are shown in Table 6.

Table 6. Calculating ET and variance for the first case using the proposed equations of PERT model

Activity	**Time Estimate/Months**			**Expected Time ET**	**Variance õ^2**
	a	***m***	***b***		
A	5	7	15	9	11.111
B	7	9	17	11	11.111
C	8	9	10	9	0.444
D	3	6	9	6	4.000
E	2	4	6	4	1.777
F	4	5	6	5	0.444
G	0	0	0	0	0
H	4	6	14	8	11.111
I	8	12	16	12	7.111
J	10	15	20	15	11.111
K	10	17	30	19	44.444
P	1	4	7	4	4.000
N	5	6	13	8	7.111

We specify the critical path to be 40 months, as shown in Table 7.

Table 7. Calculating the available paths and selecting the critical path for the first case using the proposed equation

Path/Activities	Path/Times	Path Duration
A - E – K	9 + 4 + 19	32
A - D - J – N	9 + 6 + 15 + 8	38
B - F - J – N	11 + 5 + 15 + 8	39
B - F - I - P – N	11 + 5 + 12 + 4 + 8	40
C - H - P – N	9 + 8 + 4 + 8	29

The project standard deviation is calculated by specifying the activities' variation in the critical path, combining them, and then taking the square root of the sum. The result will be the project standard deviation according to the equation $\sigma = \sqrt{\sigma^2}$ and then the equation z = $\frac{X-\mu}{\sigma} = \frac{X-Cp}{\sigma}$

$$= \sqrt{11.111 + .444 + 7.111 + 4 + 7.111} = \sqrt{29.777} = 5.457$$

$$Z = \frac{32-40}{5.457} = -1.466$$

$$P = 7.1\%$$

This implies that the probability of completing the project 6 months before the scheduled time is 7.1%.

We calculate the cost of crashing the project to 32 months as required. The cost of crashing the project from 40 months to only 32 months is 1675 JD; therefore, the cost of completing the project will be 12,975 JD instead of 11,300 JD, and the results are shown in Tables 8 and 9.

This means that the project completion probability within a period of no more than 32 months is 7.1% instead of 14%, which means that the probability has decreased by nearly half (6.9%), reducing the uncertainty and increasing the probability that the project will not be completed within 32 months.

Table 8. Times and cost of crashing project activities of the first case according to the proposed equation

	Activity Time/Months		Activity Cost/JD			
Activity	**Ex.time**	**Crash**	**Normal**	**Crash**	**Max Crash Time**	**Crash Cost Per Month**
A	9	5	700	1200	4	125
B	11	9	1600	2000	2	200
C	9	7	900	1500	2	300
D	6	3	500	900	3	133.33
E	4	2	500	700	2	100
F	5	4	500	800	1	300
G	0	0	0	0	0	0
H	8	5	700	1000	3	100
I	12	10	1800	2300	2	250
J	15	12	1400	2000	3	200
K	19	14	1400	3200	5	360
P	4	3	500	800	1	300
N	8	6	800	1400	2	300

Table 9. Results of the crashing process of the first case according to the proposed equation

Iteration	Change	Complete Time Month	Project Cost JD	Critical Path
S	-	40	11300	B-F-I-P-N
1	B (2)	39	11500	B-F-J-N
2	A (4)	38	11625	B-F-I-P-N
3	I (2)	37	11875	B-F-I-P-N
4	J (3)	36	12075	B-F-I-P-N
5	N (2)	34	12375	B-F-I-P-N
6	P (1)	33	12675	B-F-I-P-N
7	F (1)	32	12975	B-F-I-P-N

A summary of the above calculations is presented in Table 10.

Table 10. Summary of case 1 calculations

Description	Traditional case	Proposed case
Completion time (CP)/week	38	40
Standard deviation (Ơp)	2.73	5.46
Probability in completion in 28 weeks(P)	14%	7.1%
Normal completion cost(PNCC)/JD	11300	11300
Crashed Cost (PCC)/JD	13350	12975
Time difference(Δ T)/week	6	8
Cost difference (Δ C)/JD	2050	1675
Crashing cost/unit time $\frac{\Delta C}{\Delta T}$ JD/Week	341.6	209.3

4.1.2 The Second Case

The information’s of the second case are shown in Table 11.

Table 11. The elementary project data of the second case

Activity	Time Estimate/Weeks			Crash	Activity Cost /JD	
	A	**m**	**b**	**Time /Weeks**	**Normal**	**Crash**
A	5	8	17	7	4800	6300
B	3	12	15	9	9100	15500
C	4	7	10	5	3000	4000
D	5	8	23	8	3600	5000
E	1	1	1	1	0	0
F	1	4	13	3	1500	2000
G	3	6	9	5	1800	2000
H	1	2.5	7	3	0	0
I	1	1	1	1	0	0
J	2	2	2	2	0	0
K	5	8	11	6	5000	7000

Applying the same procedure exactly as followed in case 1, the summary of the calculations has been shown in Table 12.

Table 12. Summary of case 2 calculations

Description	Traditional case	Proposed case
Completion time (CP)/week	35	38
Standard deviation (Ơp)	3.87	7.74
Probability of completion in 28 weeks(P)	3.55 %	9.58%
Normal completion cost(PNCC)/JD	28800	28800
Crashed Cost (PCC)/JD	31450	30850
Time difference(Δ T)/week	7	10
Cost difference (Δ C)/JD	2650	2050
Crashing cost/unit time $\frac{\Delta C}{\Delta T}$ JD/week	378.6	205

4.1.3 The Third Case

The information's of the third case are shown in Table 13.

Table 13. The elementary data for the project in the third case

	Time Estimate/weeks			Crash	Activity Cost /JD	
Activity	***A***	***m***	***b***	**Time Weeks/**	**Normal**	**Crash**
A	4	6	8	4	1000	1900
B	6	8	16	7	1000	1800
C	2	4	6	2	1500	2700
D	8	10	24	8	2000	3200
E	7	10	13	7	5000	8000
F	4	6	8	4	3000	4100
G	4	6	20	5	8000	10250
H	4	6	8	4	5000	6400
I	4	6	14	4	10000	12400
J	3	4	5	3	4000	4400
K	2	4	6	3	5000	5500

In the third case, we will also follow the same steps applied in the first and second cases on the traditional equation and proposed equation. Applying the same procedure exactly as followed in case 1 , the summery of the calculations has been shown in Table 14.

Table 14. Summary of case 3 calculations

Description	Traditional Case	Proposed Case
Completion time (CP)/week	43	44
Standard deviation (Ơp)	2.38	4.76
Probability of completion in 28 weeks (P)	5.9%	7.08%
Normal completion cost(PNCC)/JD	45500	45500
Crashed Cost (PCC)/JD	47350	47216
Time difference(Δ T)/week	6	7
Cost difference (Δ C)/JD	1850	1716
Crashing cost/unit time $\frac{\Delta C}{\Delta T}$ JD/week	308.33	245.14

5. Hypotheses testing and results

Testing hypotheses will depend on estimating percentage changes of the intended variables between the traditional equation model and the proposed equation model. Accepting or rejecting the hypotheses will be based on the amount and direction of this change. All the needed percentage changes and their directions are presented in Table 14.

Table 15. Changes of parameters between traditional and proposed equations

Description	1st Case	2nd Case	3rd Case
Completion time (CP)/week	+5.26%	+8.57%	+2.32%
Standard deviation (Ơp)	100%	100%	100%
Probability of completion in 28 weeks (P)	-49.2%	+177%	+32%
Normal completion cost(PNCC)/JD	0%	0%	0%
Crashed Cost (PCC)/JD	-2.8%	-1.9%	-2.8%
Time difference(Δ T)/week	+33%	+42.9%	+16.7%
Cost difference (Δ C)/JD	-18.3%	-22.6%	-7.2%
Crashing cost/unit time $\frac{\Delta C}{\Delta T}$ JD/week	-38.9%	-45.8%	-20.5%

Hypothesis H_1: In Table 14, the results of the three cases indicate that applying the proposed equation increases the length of the critical path. We therefore reject the first hypothesis and accept the alternative, that using the proposed equation would increase the critical path length and thus cause a delay in the project completion time.

Hypothesis H_2: The results in Table 14 indicate that the standard deviation is doubled when using the proposed equation. Therefore, we reject the second hypothesis and accept the alternative one, that using the proposed equation increases the project standard deviation.

Hypothesis H3: Note here that the traditional cost of the project during its original time (the critical path) and before any crashing is performed remains the same using both equations; the change is 0%, as shown in Table 14. Therefore, we reject the hypothesis stating that using the proposed equation reduces the cost of the project.

Hypothesis H4: Table 14 shows that the probability of completing the project in less than its original time (shorter than the critical path) when using the proposed equation increased in the second and third projects, although it was shortened in the first project. This also means rejecting the hypothesis that stated that using the proposed equation would reduce the probability of completing the project at a specific period that is less than the critical path.

5.1 Results

After testing these hypotheses, we reach the following results:

The results of the proposed model show that, compared with the conventional PERT model, the critical path length increases and the cost of completing the project remains the same. In such a case, the decision should depend on the priority between the completion time and its total cost. If the decision maker prefers the completion time, the crashing techniques should be implemented.

Using the proposed equation to calculate the project standard deviation increased the standard deviation by 100% in the three cases. Note that the standard deviation is one of the risk measurements in project management and that its doubling means there are significant risks of being unable to complete the project within the specified time and cost.

While the use of the proposed equation increases the probability of completing the project more quickly than the critical path in two of the three cases, we should take into account that the use of the proposed equation has led to increasing the original project time. This has made reducing the time period more likely than reducing it using the traditional equations.

The results of the proposed model show some reduction in the total cost of crashing the project's activity (s) per unit time. The equation of the crashing cost per unit of time is the same as in the following equation from Mantel (2019):

$$\text{Crashing Cost of one time unit} = \frac{\text{Crash cost - Normal cost}}{\text{Normal Time - Crash Time}} = \frac{\Delta C}{\Delta T}.$$

The results in Table 14 indicate that the cost of crashing the project in the proposed equation is lower than in the traditional equation.

6. Conclusion

This study's findings align with many studies on project scheduling and cost (e.g., Behrendt & Wulke, 2004; Dell'Isola, 2002; Deng & Hung, 1998; Emhjellen et al. 2003; Jayaraman, 2016; Khamooshi, 1996; Lavingia, 2003; Liu, 2013). This study has found there are advantages of using the proposed model in this study, which is to use the normal arithmetic mean equation instead of using the weighted arithmetic mean in calculating the activity time and standard deviation according to the equations shown in the body of this study; the most important of these advantages is that using the proposed equations reduces the cost rate of crashing the project if required. However, the use of these equations has significant drawbacks on project completion, especially in terms of increasing the critical path length, which means a delay in project completion and increasing the project standard deviation and its risks on achieving the project objectives. Meeting the objectives of a project means delivering the project to the customer on time without any delay and at the designated cost or price (bidding price). The results of this study are consistent with previous studies in asserting that project management is concerned with monitoring and scheduling activities in such a way that the project can be completed in the shortest possible time (Azaron et al., 2006; Chen & Huang, 2007; Shankar et al., 2010). In addition, this study does not recommend using the proposed equations to plan the project because of its associated risks on the project's completion time and cost (Aretoulis, 2019; Burgelman & Vanhoucke, 2019; Engwall, 2012). Despite these limitations accompanying the application of the proposed equations, this study remains open to academics and researchers to propose and present mathematical equations to help project managers estimate the time, variance, and cost of completing the project.

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