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# Macrodynamic and Financial Effects of a Large-Scale Technology Change

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**Abstract**

We examine the implications of technological change which results in large-scale capital depreciation for the macrodynamic and financial properties of a dynamic general equilibrium model. In an economy where investors fear a capital-devaluing change in technology, the introduction of the possibility of such an event helps to resolve the equity premium and risk-free rate puzzles.

*Key words*: general equilibrium; asset pricing; technology shocks

*JEL classification*: E23; E32; G12

## 1. Introduction

This paper examines the impact of a large-scale technological change on the behavior of macroeconomic and financial time series in a dynamic general equilibrium model with a growing production sector. By a large-scale change we mean a switch in a techno-economic regime created around a general-purpose technology (such as the microprocessor). Our basic premise is that at the onset of epochs characterized by major technological innovations, investors devalue (depreciate) a portion of the capital stock inherited from the previous technological regime because it is poorly suited to new economic conditions. Devaluation results in a portion of the capital stock effectively becoming worthless (unproductive). Since the value of the firm's securities measures the value of its accumulated capital, asset prices will reflect the probable arrival of a new technological regime. We demonstrate that the mere possibility of such an event has a major impact on equity prices and returns.

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Data seem to support the hypothesis of capital devaluation at the start of a new technological regime. Hall (2001) finds a greater than 50% drop in the value of securities in the U.S. stock market in 1973-74. In fact, during 1973–74, the value of securities in the U.S. market fell below the replacement cost of plant and equipment. This fact suggests that a significant portion of existing capital stock was deemed obsolete by the stock market. Wei (2003) inquires whether the effective scrapping of a substantial fraction of the capital stock was due to higher energy prices. He concludes that the energy price shock of the 1970s alone cannot explain the market decline in 1974. Greenwood and Jovanovic (1999) suggest that in 1974 the U.S. economy first realized the implications of a techno-economic regime switch based on the information technology (IT) for incumbent firms with large investments in old technologies. Figure 5 in Hobijn and Jovanovic (2001) shows the ratio of market value of incumbent firms to GDP. This ratio declined by more than 50% around 1974 and never recovered to its pre-1974 level. The value of the market relative to GDP has tripled over the same period.

Investors with equity holdings in incumbent firms consider the arrival of new technology to be a "catastrophe." Rietz (1988) finds that the introduction of rare catastrophic events into a dynamic general equilibrium exchange economy produces a realistic equity premium. Danthine and Donaldson (1999) show, however, that in the production economies, Rietz's catastrophic events, when actually experienced, would make the macroeconomic series excessively volatile and would not plausibly resolve the puzzle because of the agents' ability to smooth their consumption streams through investment and labor-leisure choices.

Historically, large-scale technological innovations are infrequent and the data sample under observation might not contain the actual, feared capital-devaluing technological change. Yet investors may rationally attach a very small positive probability to its occurrence. These circumstances are generally referred to as the "peso phenomenon" (see Evans, 1996, for a literature survey). Danthine and Donaldson (1999) find that in a production economy, the effect of a feared catastrophic state on financial returns is especially strong in the "peso" samples where a disaster is not actually observed. Campbell (1999) points out two difficulties with the plausibility of the peso argument in earlier research. The peso explanation for the equity premium requires a potential catastrophe, which affects stock market investors more seriously than investors in short-term debt instruments. Additionally, the robustness of the equity premium across countries suggests investors in all countries are concerned about catastrophes. We define a disaster as a state where agents in the economy become aware of a technological regime shift. We think that our definition of a "catastrophic event" satisfies Campbell's requirements.

We find that the samples that actually experienced capital-devaluing technological change provide a very good description of the financial statistics, but the macroeconomic series in these samples are unrealistically volatile. By contrast, in the peso samples, where the feared event does not materialize, the macroeconomic description of the economy remains satisfactory, yet the equity premium puzzle disappears. In the peso setting in our model, the risk premium widens mainly because of the decrease in the risk-free rate and not because of the increase in the return on equity. Hence, we do not encounter the risk-free rate puzzle of Weil (1989). In previous successful attempts to resolve the puzzle, the equity premium increased because of the increase in the equity return while the risk-free rate remained high (Jermann, 1998, and Avalos, 1999, among many others). Our findings suggest that in times of great uncertainty, policy makers cannot infer sufficient information about concerns of the investors by looking at the macro-aggregates. More information about investors' expectations is revealed by the security markets, especially by the low risk-free rate.

## 2. The Model

We construct a parsimonious general equilibrium model with a growing production sector, the certainty version of which has its origins in Cass (1965) and Koopmans (1965) and the generalization to uncertainty can be found in Brock and Mirman (1972), Kydland and Prescott (1982), Hansen (1985), and others. We use an approach proposed by Danthine and Donaldson (2002) to decentralize the model and then alter the previous specifications to allow for the shock to the capital stock, which captures the idea of a major shift in technology.

Our economy is populated by an infinite number of identical households indexed by [0,1]. These households are simultaneously consumers, workers, and investors. There is also a single firm, which behaves competitively. Claims to output of the firm are traded in the stock market. Investors can save by owning shares or by buying a risk-free asset—a one-period discount bond which pays one unit of consumption at maturity in every state. We denote the period $t$ prices of the equity security and the risk-free asset by $q_t^e$ and $q_t^b$ respectively. Let $Z_t^e$ represent the fraction of the single equity share and $Z_t^b$ the number of risk-free bonds held by a household in period $t$. Each household maximizes its lifetime utility over consumption and leisure by choosing its asset allocation ( $Z_t^e$ and $Z_t^b$ ) and work time ( $N_t^h$ ) supplied to solve:

$$Max_{\{Z_{t+1}^e, Z_{t+1}^b, N_t^h\}} E\left(\sum_{t=0}^{\infty} \beta^t U\left(C_t^h, 1-N_t^h\right)\right)$$

subject to

$$C_t^h + q_t^e Z_{t+1}^e + q_t^b Z_{t+1}^b \leq \left(q_t^e + D_t\right) Z_t^e + Z_t^b + W_t N_t^h, \tag{1}$$

where $\beta$ is the subjective discount factor, $C_t^h$ is per capita consumption, and $W_t$ denotes the competitively determined wage rate. The period $t$ dividend, $D_t$, will be defined shortly. The period preference ordering of the representative household is assumed to be of the standard CES form:

$$U\left(C_t^h, 1-N_t^h\right)=\frac{\left(\left(C_t^h\right)^{\gamma}\left(1-N_t^h\right)^{1-\gamma}\right)^{1-\xi}}{1-\xi}. \tag{2}$$

The representative firm begins period *t* with the stock of capital $K_t^f$ carried over from the previous period and one equity share outstanding; i.e., $Z_t^f=1$. The evolution of the capital stock is given by:

$$K_t^f=\left((1-\Omega)K_{t-1}^f+I_{t-1}\right)\Theta_t, \tag{3}$$

where $I_t$ is period *t* investment in the new capital and $\Omega$ is the normal (i.e., "wear-and-tear") depreciation rate. $\Theta_t$ is a "catastrophic" shock to existing capital stock; it is equal to 1 if there is no large-scale technological change in period *t*. If a new technology arrives in period *t*, $\Theta_t$ is equal to $\overline{\Theta}$, which is positive but less than 1.

Occasionally, agents in the economy learn of major technological developments and realize that after the arrival of the new technology a portion of the existing capital stock, including the last period's investment, will be replaced with new productive assets. Under this interpretation, $\overline{\Theta}$ represents the fraction of existing capital stock still considered valuable for production after a major change in technology.

After observing the realization of shocks, the firm hires labor $N_t^f$ taking the period equilibrium wage as given and produces and sells its output $Y_t$. The production technology is described by a standard Cobb-Douglas production function of the form:

$$Y_t=\widetilde{\Lambda}_t f\left(K_t^f, N_t^f, \widetilde{P}_t\right)=\widetilde{\Lambda}_t\left(K_t^f\right)^{\alpha}\left(N_t^f\widetilde{P}_t\right)^{1-\alpha}, \tag{4}$$

where $\widetilde{\Lambda}_t$ is total factor productivity in period *t* and $\widetilde{P}_t$ denotes labor productivity in period *t*, which grows according to the following exogenous process:

$$P_t=\widetilde{X}_t P_{t-1}. \tag{5}$$

In equation (5), $\widetilde{X}_t$ stands for the stationary random growth rate of labor productivity. The proceeds of the output sale are used to pay the wage bill $W_tN_t$, to finance investments $I_t$ (under the knowledge of the equation of motion on capital stock (3)), and, residually, to pay dividends:

$$D_t=Y_t-W_tN_t^f-I_t. \tag{6}$$

The firm's objective is to maximize its pre-dividend stock market value on a period-by-period basis:

$$Max_{\left\{I_{t+j}, N_{t+j}^f\right\}}\left(D_t+q_t^e\right)$$

subject to

$$
\begin{aligned}
q_t^e &= E\left(\sum_{j=1}^{\infty}\beta^j MRS_{t,t+j}^h D_{t+j}\right)\\
K_{t+j}^f &= \left((1-\Omega)K_{t+j-1}^f + I_{t+j-1}\right)\Theta_{t+j}\\
Y_{t+j} &= \Lambda_{t+j}\left(K_{t+j}^f\right)^{\alpha}\left(N_{t+j}^f P_{t+j}\right)^{1-\alpha}\\
D_{t+j} &= Y_{t+j} - W_{t+j}N_{t+j}^f - I_{t+j},
\end{aligned}
\tag{7}
$$

where $MRS_{t,t+j}^h$ is the marginal rate of substitution in consumption of the worker-shareholder between periods *t* and *t+j*:

$$
MRS_{t,t+j}^h = \frac{U_1\left(C_{t+j}^h, 1-N_{t+j}^h\right)}{U_1\left(C_t^h, 1-N_t^h\right)} = \frac{\left(C_{t+j}^h\right)^{\gamma(1-\xi)-1}\left(1-N_{t+j}^h\right)^{(1-\gamma)(1-\xi)}}{\left(C_t^h\right)^{\gamma(1-\xi)-1}\left(1-N_t^h\right)^{(1-\gamma)(1-\xi)}}. \tag{8}
$$

## 2.1 Model Analysis

In its original form, our growth model is not level-stationary in its optimal macroeconomic and price series; all of them, with the exception of labor services, grow as labor productivity $P_t$ grows. As noted by Hansen (1985), the model's output can be made stationary under the appropriate change of variables to allow use of standard stochastic dynamic programming techniques. We normalize consumption, wage, output, dividend, investment, capital stock, and asset price series by the productivity parameter. For example, transformed in this manner, the period *t* output $Y_t$ is given by $\hat{Y}_t = Y_t / P_t$ .

The transformed household's maximization problem is thus:

$$
\underset{\left\{Z_{t+1}^e, Z_{t+1}^b, N_t^h\right\}}{Max} E\left(\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}\beta\tilde{X}_s^{\gamma(1-\xi)}\right)U\left(\hat{C}_t^h, 1-N_t^h\right)\right) \tag{9}
$$

subject to the normalized household's budget constraint:

$$
\hat{C}_t^h + \hat{q}_t^e Z_{t+1}^e + \hat{q}_t^b Z_{t+1}^b \le \left(\hat{q}_t^e + \hat{D}_t\right)Z_t^e + \frac{1}{P_t}Z_t^b + \hat{W}_t N_t^h . \tag{10}
$$

Under normalization, the uncertainty in the growth rate of labor productivity $\tilde{X}$ manifests itself in the uncertainty of the representative household's subjective discount factor between periods 0 and *t*, which is given by $\prod_{s=0}^{t}\beta\tilde{X}_s^{\gamma(1-\xi)}$ in the transformed objective function (9).

The transformed maximization problem of the firm is:

$$Max_{\{I_{t+j},N^f_{t+j}\}}\left(\hat{D}_t + \hat{q}^e_t\right) \tag{11}$$

subject to the normalized constraints:

$$\hat{q}^e_t = E\left(\sum_{j=1}^{\infty}\left(\prod_{s=0}^{t}\beta\widetilde{X}_s^{\gamma(1-\xi)-1}\right)M\hat{R}S^h_{t,t+j}\hat{D}_{t+j}\right) \tag{12}$$

$$X_{t+j}\hat{K}^f_{t+j} = \left((1-\Omega)\hat{K}^f_{t+j-1} + \hat{I}_{t+j-1}\right)\Theta_{t+j} \tag{13}$$

$$\hat{Y}_{t+j} = \Lambda_{t+j}\left(\hat{K}^f_{t+j}\right)^{\alpha}\left(N^f_{t+j}\right)^{1-\alpha} \tag{14}$$

$$\hat{D}_{t+j} = \hat{Y}_{t+j} - \hat{W}_{t+j}N^f_{t+j} - \hat{I}_{t+j}\,. \tag{15}$$

By construction, the newly transformed optimization problems of the representative household and firm can be expressed as stationary dynamic programs. Let vector $\widetilde{S}_t = [\,\widetilde{\Lambda}_t, \widetilde{X}_t, \widetilde{\Theta}_t\,]$denote the vector of exogenous state variables. The value function $V^h$ $(\hat{K}, \widetilde{S})$ represents a solution to the normalized stationary problem (9) starting from some initial conditions $(\hat{K}, \widetilde{S})$:

$$V^h\left(\hat{K},\widetilde{S}\right) = \underset{\{Z^e,Z^b,N^h\}}{Max}\left\{U\left(\hat{C}^h, 1-N^h\right) + \beta\int\left(\widetilde{X}'\right)^{\gamma(1-\xi)}V^h\left(\hat{K}',\widetilde{S}'\right)dH_M\left(\widetilde{S}',\widetilde{S}\right)\right\} \tag{16}$$

subject to the normalized constraint (10). (Primes denote next period values.) At all times, the above expectation is computed using the conditional shock distribution $dH_M\,(\widetilde{S}_{t+1}, \widetilde{S}_t)$.

The first order conditions for the representative household's equity and risk-free asset holdings are:

$$\hat{q}^e_t = \beta\int \widetilde{X}_{t+1}^{\gamma(1-\xi)-1}M\hat{R}S^h_{t,t+1}\left(\hat{q}^e_{t+1} + \hat{D}_{t+1}\right)dH_M\left(\widetilde{S}_{t+1},\widetilde{S}_t\right) \tag{17}$$

$$\hat{q}^b_t = \beta\int \widetilde{X}_{t+1}^{\gamma(1-\xi)-1}M\hat{R}S^h_{t,t+1}\frac{1}{P_{t+1}}dH_M\left(\widetilde{S}_{t+1},\widetilde{S}_t\right), \tag{18}$$

while the first order condition for the labor decision is:

$$\hat{W}_t = \frac{\hat{C}^h_t(1-\gamma)}{\left(1-N^h_t\right)\gamma}\,. \tag{19}$$

The firm's transformed maximization problem (11) subject to (12), (13), (14), and (15) admits an equivalent recursive formulation:

$$V^f\left(\hat{K},\tilde{S}\right)=\underset{\{\hat{I},N^f\}}{Max}\left\{\frac{U_1\left(\hat{C}^h,1-N^h\right)}{U_1\left(\hat{C}^h,1-N^h\right)}\left(\Lambda f\left(\hat{K}^f N^f\right)-\hat{W}N^f-\hat{I}\right)\right.$$
$$\left.+\beta\int\left(\tilde{X}'\right)^{\gamma(1-\xi)-1}V^f\left(\hat{K}',\tilde{S}'\right)dH_M\left(\tilde{S}',\tilde{S}\right)\right\} \tag{20}$$

subject to constraint (13). The necessary and sufficient conditions for the firm's problem are its optimal labor hiring decision and the Euler equation describing its optimal investment choice:

$$\left(1-\alpha\right)\frac{\hat{Y}_t}{N_t^f}=\hat{W} \tag{21}$$

$$-1+E_t\left\{\beta X_{t+1}^{\gamma(1-\xi)-1}M\hat{R}S_{t,t+1}^h\left(f_K\left(\hat{K}_{t+1}^f,N_{t+1}^f\right)\Lambda_{t+1}+1-\Omega\right)\Theta_{t+1}\right\}=0\,. \tag{22}$$

***Definition***: The market equilibrium in this economy is a wage function $\hat{W}_t=w(\hat{K}_t,\tilde{S}_t)$, a share price function $\hat{q}_t^e=q^e(\hat{K}_t,\tilde{S}_t)$, and a risk-free security price function $\hat{q}_t^b=q^b(\hat{K}_t,\tilde{S}_t)$ such that (17), (18), (19), (21), and (22) are satisfied along with the market-clearing conditions:

$$\begin{aligned}
N_t^f &= \int_0^1 N_t^h d\varphi = N_t^h = N_t\\
\hat{K}_t^f &= \hat{K}_t\\
Z_t^f &= \int_0^1 Z_t^e d\varphi = Z_t^e = 1\\
Z_t^b &= \int_0^1 Z_t^b d\varphi = 0\\
\hat{C}_t^h &= \int_0^1 \hat{C}_t^h d\varphi = \hat{C}_t,
\end{aligned} \tag{23}$$

where $\varphi$ represents the measure of households. By an application of Blackwell's theorem the equilibrium exists for this economy.

### 2.2 Rates of Return on Financial Securities

Upon obtaining the equilibrium share price function, we compute the time series of normalized gross equity returns using the following definition:

$$\hat{R}_{t,t+1}^e\left(\hat{K}_t,\tilde{S}_t\right)=\frac{\hat{q}_{t+1}^e\left(\hat{K}_{t+1},\tilde{S}_{t+1}\right)+\hat{D}_{t+1}\left(\hat{K}_{t+1},\tilde{S}_{t+1}\right)}{\hat{q}_t^e\left(\hat{K}_t,\tilde{S}_t\right)}\,. \tag{24}$$

The actual rate of return on the equity security is given by:

$$R^e_{t,t+1} = \hat{R}^e_{t,t+1} X_{t+1} .$$

Using the equilibrium bond price function, the period-by-period normalized gross risk-free rate is computed as:

$$\hat{R}^b_{t,t+1}\left(\hat{K}_t, \widetilde{S}_t\right) = \frac{1/P_{t+1}}{\hat{q}^b_t\left(\hat{K}_t, \widetilde{S}_t\right)} \tag{25}$$

with the actual risk-free rate given by:

$$R^b_{t,t+1} = \hat{R}^b_{t,t+1} X_{t+1} .$$

Note that all asset prices are calculated using the conditional shock distribution $dH_M$ ( $\widetilde{S}_{t+1}, \widetilde{S}_t$ ), meaning that investors take into account the possibility of the arrival of the new technology when pricing assets.

### 3. Model Solution

We solve the model using the standard nonlinear value function iteration technique. We approximate shock processes with a coarse state partition consisting of two carefully chosen states for each shock process ($\lambda_1$, $\lambda_2$, $\Theta_1 = 1$, $\Theta_2 = \overline{\Theta}$ ) and allow the conditional distribution of all shock processes to follow a finite-state, discrete Markov chain. The growth rate of labor productivity is constant. (In unreported simulations, we estimated the implications of a variable growth rate of labor productivity. Our findings indicate that the substitution of a stochastic growth rate for the constant growth rate of labor productivity has almost no effect on the macrodynamic and financial properties of the model in either the samples where the actual disaster state occurs or in the peso samples.) The transition matrix $M$ for this economy is given below:

$$M = \begin{array}{c} \begin{matrix} (\lambda_1,1) & (\lambda_2,1) & (\lambda_2,\overline{\Theta}) & (\lambda_2,\overline{\Theta}) \end{matrix} \\ \begin{bmatrix} \Psi - \eta_{13} & \pi - \eta_{14} & \eta_{13} & \eta_{14} \\ \pi - \eta_{23} & \Psi - \eta_{24} & \eta_{23} & \eta_{24} \\ \sigma - \eta_{33} & \Gamma - \eta_{34} & \eta_{33} & \eta_{34} \\ \Gamma - \eta_{43} & \sigma - \eta_{44} & \eta_{43} & \eta_{44} \end{bmatrix} \end{array} \begin{matrix} (\lambda_1,1) \\ (\lambda_2,1) \\ (\lambda_2,\overline{\Theta}) \\ (\lambda_2,\overline{\Theta}) \end{matrix} . \tag{26}$$

Parameters $\eta_{ij}$ determine the likelihood of entering a disaster state $j$ ($j$ = 3,4) from state $i$ ($i$ = 1,2,3,4). Parameters $\eta_{jj}$ determine the average number of periods remaining in the disaster state. Given the specifications of $E\lambda$, $E\Theta$, $\sigma_\lambda$, $\sigma_\Theta$, $\rho_{\lambda\lambda}$, $\rho_{\Theta\Theta}$, $\rho_{\lambda\Theta}$, we can express each of these quantities in terms of the unknowns {$\Psi$, $\pi$, $\sigma$, $\Gamma$, $\eta_{13}$, $\eta_{14}$, $\eta_{23}$, $\eta_{24}$, $\eta_{33}$, $\eta_{34}$, $\eta_{43}$, $\eta_{44}$}. Together with the requirements that probabilities in each row of $M$ sum up to 1, these equations constitute a system of 11 equations and 12 unknowns, allowing us to regard one of the entries in $M$ as a free parameter. This parameter can be varied until all entries of the matrix are positive and disaster states 3 and 4 are highly unlikely. If no such parameter can be found, this is evidence that the specifications of correlations between shocks are inconsistent.

### 3.1 Calibration

Following previous dynamic equilibrium literature (see Hansen, 1985, Mehra and Prescott, 1985, and King and Rebelo, 1999), we choose parameter values as follows: the quarterly discount factor $\beta = 0.99$; $\Omega = 0.025$; $\gamma = 0.33$, which fixes the average time worked at approximately 0.3, and the coefficient of relative risk aversion $\xi = 3$. The production function parameter is fixed at $\alpha = 0.36$, which guarantees an income share of capital in the model economy to be 36%—the income share of capital in the U.S. economy. The growth rate of labor productivity is set at 0.7% per quarter (chosen to approximate the average quarterly growth rate of the U.S. economy in the postwar period). Motivated by Hall's (2001) estimate of the 1974 losses in the value of capital stock, we choose $\overline{\Theta} = 0.5$ as our benchmark case. The stationary probability of $\Theta = \overline{\Theta}$ is set to be 1.34%. We set $\lambda_1 = 1.021$, $\lambda_2 = 0.979$ to replicate the standard deviation of output in the U.S., where $\sigma_y$ is equal to 1.82%. Sensitivity analysis for different values of $\overline{\Theta}$ and various stationary probabilities is presented in Tables 3 and 4.

### 3.2 Peso Effect

Technological "revolutions" result in severe capital stock losses for incumbent firms. Investors rationally attach a positive probability to such events, which means that they view the matrix $M$ in (26) as the true Markov process, objectively and subjectively anticipated, for this economy in their decision making. The probability that the capital depreciating technological change would be observed in any given period is very small and it is possible that disaster states may never materialize in the sample period under observation all the while the investors were rationally expecting these events to occur. We call such samples the "peso samples."

## 4. Results

Table 1 summarizes the macroeconomic behavior of the model economy. For purposes of comparison, row one contains the statistics derived from the U.S. data. Rows two through four present the average statistics for a complete set of 500 samples of 200 quarterly observations each. In all simulations, we normalize shocks to produce $\sigma_y$ comparable to the standard deviation of output observed in the U.S. data. In row two, we report results from the variant of the model with $\Theta \equiv 1$ (disaster states are not anticipated and not present in the data samples), driven by persistent technology shocks. In this case, the model replicates Hansen's (1985) indivisible labor economy, a standard benchmark in the real business cycle literature. Row three shows statistics for the stationary economy, where the disaster state actually occurs and is fully and rationally anticipated by the agents. Row four presents results for the peso samples, where capital devaluation is rationally anticipated by the agents but never materializes in the samples under observation.

Statistics in row three reveal that capital devaluation, actually experienced, makes the standard model economy ex-post substantially more variable. The effect of the Θ shock on capital stock is especially dramatic, which is not surprising since capital stock is directly affected by it. Relative to the standard model, output is twice as variable. When combined with slightly less variable investment, the increase in output variability results in more than a six-fold jump in the variability of consumption, which is three times its observed value. In summary, our results suggest that an introduction of a capital-devaluing technological change substantially compromises the ability of the standard model to replicate macroeconomic behavior of the U.S. economy.

The last row of Table 1 demonstrates that the pure possibility of capital destruction has almost no effect on macroeconomic properties of our model. Therefore, the perceived possibility of capital devaluation does not in itself alter the ability of the standard model to explain the stylized facts of the business cycle.

**Table 1. Matching Macro-Aggregate Moments**

| | Std. Dev. | | | | | Corr. with Output | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\sigma_y$ | $\sigma_c$ | $\sigma_n$ | $\sigma_i$ | $\sigma_k$ | $\rho_{cy}$ | $\rho_{ny}$ | $\rho_{iy}$ | $\rho_{ky}$ |
| U.S. Economy | 1.81 | 1.35 | 1.79 | 5.3 | 0.63 | 0.88 | 0.88 | 0.8 | 0.04 |
| Capital devaluation is not anticipated and not present in the data | 1.81 | 0.69 | 0.95 | 5.82 | 0.53 | 0.77 | 0.95 | 0.97 | 0.04 |
| Capital devaluation is anticipated and present in the data | 3.66 | 4.41 | 1.32 | 5.56 | 9.64 | 0.88 | −0.13 | 0.52 | 0.75 |
| Capital devaluation is anticipated but not present in the data | 1.82 | 0.73 | 0.97 | 6.03 | 0.58 | 0.72 | 0.93 | 0.96 | 0.05 |

Notes: Business cycle statistics for the U.S. economy are from King and Rebelo (1999), whose data set is from Stock and Watson (1999) and covers the period from 1947 (first quarter) to 1996 (fourth quarter). Data sources are described in Stock and Watson. Standard deviations are given in percents. All statistics are per quarter. ( $\xi = 3, \overline{\Theta} = 0.5, p\left(\Theta = \overline{\Theta}\right) = 1.34\%$)

Table 2 presents a financial summary. The second row (Θ = 1) clearly replicates the classic equity premium puzzle of Mehra and Prescott (1985): A very high risk-free rate and a trivially small (0.02%) equity premium. The volatility puzzle is present as well: The return on equity, the risk-free rate, and the equity risk premium are all too smooth when compared to their U.S. counterparts.

When capital devaluation is introduced into the model and it is observed with its anticipated relative frequency, the return on equity increases from 6.91% to 7.76%; the risk-free rate decreases from 6.89% to 5.71%. We are able to produce a

2.05% premium. Second moments are much improved as well: The volatility of the equity return is now 11.68% and the volatilities of the risk-free rate and the equity premium are 3.42% and 11.96% respectively, which is very close to their empirical values. The economy, which actually experiences capital devaluation, is a "high aggregate risk economy." Investors facing higher consumption uncertainty are insuring themselves by buying risk-free assets. This effect increases the demand for the safe assets driving down their returns. The equity security is the asset afflicted by capital destruction. It is less desirable for consumption-smoothing purposes and commands a higher return.

The corresponding financial statistics from the peso samples look quite different: The return on the equity security increases further to 8.22%, and the risk-free rate goes down to 2.91%. These effects together give rise to a 5.31% equity premium. The variability of asset returns, however, remains low. Cecchetti et al. (1998) propose a way to increase the second moments of the asset returns distribution in a similar setting. Letting the transition matrix $M$ in (26) exhibit stochastic variation would allow matching the standard deviations of returns.

**Table 2. Selected Financial Statistics**

| | Mean Values | | | Std. Dev. | | | Corr. with Growth Rate of Output | |
|---|---|---|---|---|---|---|---|---|
| | $r^e$ | $r^b$ | $r^p$ | $\sigma_{r^e}$ | $\sigma_{r^b}$ | $\sigma_{r^p}$ | $\rho_{r^e \Delta y}$ | $\rho_{r^b \Delta y}$ |
| U.S. Economy | 6.98 | 0.8 | 6.18 | 16.54 | 5.67 | 16.76 | | |
| Capital devaluation is not anticipated and not present in the data | 6.91 | 6.89 | 0.02 | 1.26 | 0.89 | 1.00 | 0.35 | 0.03 |
| Capital devaluation is anticipated and present in the data | 7.76 | 5.71 | 2.05 | 11.68 | 3.42 | 11.96 | 0.58 | 0.14 |
| Capital devaluation is anticipated but not present in the data | 8.22 | 2.91 | 5.31 | 1.5 | 1.33 | 1.13 | 0.32 | 0.03 |

Notes: Financial statistics for the U.S. economy are from Mehra and Prescott (1985). Financial returns and their standard deviations are given in percents and annualized. ( $\xi = 3, \overline{\Theta} = 0.5, p(\Theta = \overline{\Theta}) = 1.34\%$)

To interpret the above results, we note that when the economy enters the actual capital devaluation state the return on the equity security is negative because a portion of the asset's value disappears. In a disaster state, output of the consumption good decreases as part of the capital used in production is depreciated. The capital depreciation states are the states where consumption is low. In these states, agents don't want to save because they expect higher consumption in the periods to come, given the positive probability of better shocks in the future and zero probability of worse shock realization. The marginal utility of consumption is very high in disaster states, and expected marginal utility of consumption in future periods is low. This fact translates into a low demand for the risk-free asset (as it is the main instrument for saving in disaster states) and the high risk-free rate. In the peso samples, we exclude capital depreciation states where the return on the equity security is the lowest and return on the risk-free security is the highest. Accordingly, the expected return on equity slightly rises and the expected risk-free rate is substantially reduced. The elimination of both the lowest tail of the equity return distribution and the highest tail of the risk-free return distribution from peso samples also accounts for the substantially lower standard deviation of returns to both securities.

**Changing the Magnitude of Capital Destruction**: The quantity $\overline{\Theta}$ represents the fraction of capital stock left in productive use after the arrival of a new technology; the larger the value of $\overline{\Theta}$, the less severe the consequences of the technology change (see Table 3).

**Table 3. Changing the Magnitude of Capital Devaluation**

| | Capital devaluation is anticipated and present in the sample | | | Capital devaluation is anticipated but not present in the sample | | |
|---|---|---|---|---|---|---|
| | $\overline{\Theta}=0.5$ | $\overline{\Theta}=0.7$ | $\overline{\Theta}=0.9$ | $\overline{\Theta}=0.5$ | $\overline{\Theta}=0.7$ | $\overline{\Theta}=0.9$ |
| $r^e$ | 7.76 | 7.14 | 6.91 | 8.22 | 7.64 | 7.13 |
| $r^b$ | 5.71 | 6.52 | 6.81 | 2.91 | 5.12 | 6.40 |
| $r^p = r^e - r^b$ | 2.05 | 0.62 | 0.10 | 5.31 | 2.52 | 0.73 |
| Standard Deviations | | | | | | |
| $r^e$ | 11.68 | 6.96 | 2.73 | 1.50 | 1.34 | 1.28 |
| $r^b$ | 3.42 | 1.88 | 1.11 | 1.33 | 1.03 | 0.94 |
| $r^p = r^e - r^b$ | 11.96 | 7.03 | 2.61 | 1.13 | 1.05 | 1.02 |
| Output | 3.66 | 2.51 | 1.94 | 1.82 | 1.81 | 1.81 |
| Consumption | 4.41 | 2.45 | 1.06 | 0.73 | 0.73 | 0.71 |
| Employment | 1.32 | 1.06 | 0.97 | 0.97 | 0.95 | 0.95 |
| Investment | 5.56 | 5.56 | 5.81 | 6.03 | 5.88 | 5.83 |
| Capital Stock | 9.64 | 5.04 | 1.62 | 0.58 | 0.55 | 0.53 |
| Contemporaneous Correlation with Output | | | | | | |
| Consumption | 0.88 | 0.81 | 0.72 | 0.72 | 0.74 | 0.76 |
| Employment | −0.13 | 0.33 | 0.85 | 0.93 | 0.93 | 0.94 |
| Investment | 0.52 | 0.70 | 0.93 | 0.96 | 0.97 | 0.97 |
| Capital Stock | 0.75 | 0.60 | 0.29 | 0.05 | 0.05 | 0.04 |
| Contemporaneous Correlation with Growth Rate of Output | | | | | | |
| $r^e$ | 0.58 | 0.50 | 0.35 | 0.32 | 0.35 | 0.35 |
| $r^b$ | 0.14 | 0.14 | 0.08 | 0.03 | 0.03 | 0.03 |

Notes: In all cases presented in this table $\xi = 3, p\left(\Theta = \overline{\Theta}\right) = 1.34\%$ .

In an actual disaster scenario, as the magnitude of $\overline{\Theta}$ increases, the volatility of the macro-series decreases. The effect is especially apparent for the capital stock. In the peso cases, the perceived magnitude of capital destruction has almost no effect on the real side of the economy.

On the financial side, whether the capital depreciation actually takes place or it is merely perceived by the agents, the asset returns are quite sensitive to Θ modifications: The return on equity decreases and the risk-free rate increases. In the economy where destruction actually takes place, the risk-free rate climbs from 5.71% (if 50% of the capital stock is depreciated) to 6.81% (if only 10% depreciation of the capital stock occurs). The premium shrinks from 2.05% to 0.1%. In the peso setting, the effect is similar, but the risk-free rate grows much faster. In the peso samples, the change in $\overline{\Theta}$ from 0.5 to 0.9 increases the risk-free rate from 2.91% to 6.4%. The results mean that the decrease in the magnitude of a disaster (experienced or perceived) makes the equity security more attractive; at the same time, the advantage to holding the risk-free asset diminishes.

**Changing the Probability of a Major Change in Technology**: In the samples where capital depreciation is actually present, the volatility of macroeconomic aggregates increases rapidly with the increase in the likelihood of the disaster states (see Table 4).

**Table 4. Changing the Probability of Capital Devaluation**

| | Capital devaluation is anticipated and present in the sample | | | Capital devaluation is anticipated but not present in the sample | | |
|---|---|---|---|---|---|---|
| $p(\Theta = \overline{\Theta})$ | 0.44 | 1.34 | 1.74 | 0.44 | 1.34 | 1.74 |
| $r^e$ | 7.19 | 7.76 | 7.98 | 7.36 | 8.22 | 8.58 |
| $r^b$ | 6.39 | 5.71 | 5.30 | 5.47 | 2.91 | 1.48 |
| $r^p = r^e - r^b$ | 0.80 | 2.05 | 2.68 | 1.89 | 5.31 | 7.10 |
| Standard Deviation | | | | | | |
| $r^e$ | 5.69 | 11.68 | 13.89 | 1.35 | 1.50 | 1.54 |
| $r^b$ | 1.95 | 3.42 | 4.20 | 1.05 | 1.33 | 1.30 |
| $r^p = r^e - r^b$ | 5.64 | 11.96 | 14.44 | 1.00 | 1.13 | 1.20 |
| Output | 2.55 | 3.66 | 4.13 | 1.82 | 1.82 | 1.79 |
| Consumption | 2.30 | 4.41 | 5.20 | 0.72 | 0.73 | 0.76 |
| Employment | 1.12 | 1.32 | 1.40 | 0.98 | 0.97 | 0.93 |
| Investment | 5.86 | 5.56 | 5.36 | 5.99 | 6.03 | 5.86 |
| Capital Stock | 4.51 | 9.64 | 11.47 | 0.54 | 0.58 | 0.57 |
| Contemporaneous Correlation with Output | | | | | | |
| Consumption | 0.79 | 0.88 | 0.92 | 0.71 | 0.72 | 0.74 |
| Employment | 0.44 | −0.13 | −0.31 | 0.93 | 0.93 | 0.92 |
| Investment | 0.75 | 0.52 | 0.46 | 0.97 | 0.96 | 0.96 |
| Capital Stock | 0.40 | 0.75 | 0.83 | 0.03 | 0.05 | 0.06 |
| Contemporaneous Correlation with Growth Rate Output | | | | | | |
| $r^e$ | 0.42 | 0.58 | 0.62 | 0.29 | 0.32 | 0.31 |
| $r^b$ | 0.07 | 0.14 | 0.23 | −0.02 | 0.03 | 0.17 |

Notes: In all cases presented in this table $\xi = 3$, $\overline{\Theta} = 0.5$ .

Capital and consumption are especially responsive to changes in this parameter. The increase in the macro uncertainty results in the upward movement of the equity return and the downward movement of the risk-free rate.

In the peso setting, changes in a perceived probability of the disaster have a negligible effect on the volatility of the macroeconomic variables but they affect the first moments of the financial returns. The risk-free rate is especially sensitive. An increase in the perceived probability of capital depreciation from 0.44% to 1.74% drives down the risk-free rate from 5.47% to 1.48% producing the 7.1% equity premium. These findings can be explained in light of our earlier argument.

## 5. Conclusion

In the context of a dynamic general equilibrium model with a growing production sector, we introduce the possibility of a technological regime switch which results in the depreciation of a significant portion of existing capital stock. Investors in the incumbent firms view this event as a "catastrophe." Defined in this way, the disaster state has a potentially more profound impact on the aggregate behavior of the economy than a disaster state characterized by low output realization because the capital depreciation affects not only the output but also other macro-aggregates, which are highly correlated with output, for many future periods. Consequently, it has greater implications for asset pricing.

We have shown that in the samples with the actually experienced capital depreciation, the first and the second moments of financial returns come close to their counterparts in the data; however, in these samples, the volatility of the macroeconomic series is unrealistically high.

By contrast, if capital devaluation is merely a possibility but is not observed in the sample data (i.e., the peso samples), our model adequately describes the basic facts of the observed business cycle. A perceived possibility of capital depreciation further reduces the mean risk-free rate, even relative to the actual capital devaluation scenario, and slightly increases the equity return. As a result, in the model that contains no channel for shock amplification, such as indivisible labor, variable capital utilization, or adjustment costs, without any market imperfections or leverage, we are able to achieve a realistic equity premium of 5.31% in the benchmark case.

## References

Avalos, F., (1999), "Time to Build, Aggregate Fluctuations and Asset Pricing," Ph.D. Dissertation, University of Chicago.

Brock, W. A. and L. J. Mirman, (1972), "Optimal Economic Growth and Uncertainty: The Discounted Case," *Journal of Economic Theory*, 4, 479-513.

Campbell, J. Y., (1999), "Asset Prices, Consumption and the Business Cycle," in *Handbook of Macroeconomics*, J. Taylor and M. Woodford eds., Amsterdam: Elsevier, Vol. 1C, 1231-1303.

Cass, D., (1965), "Optimum Growth in an Aggregative Model of Capital Accumulation," *Review of Economic Studies*, 32, 223-240.

Cecchetti, S. G., P. Lam, and N. C. Mark, (1998), "Asset Pricing with Distorted Beliefs: Are Equity Returns Too Good to Be True?" *National Bureau of Economic Research Working Paper* 6354.

Danthine, J. P. and J. B. Donaldson, (1999), "Non-Falsified Expectations and General Equilibrium Asset Pricing: the Power of the Peso," *The Economic Journal*, 109, 607-635.

Danthine, J. P. and J. B. Donaldson, (2002), "Decentralizing the Stochastic Growth Model," *CEPR Discussion Paper* 3348.

Evans, D. D., (1996), "Peso Problems: Their Theoretical and Empirical Implications," in *Handbook of Statistics*, G. S. Maddala and C. R. Rao eds., Amsterdam: Elsevier, Vol. 14, 613-646.

Greenwood, J. and B. Jovanovic, (1999), "The Information-Technology Revolution and the Stock Market," *American Economic Review*, 89(2), 116-122.

Hall, R. E., (2001), "The Stock Market and Capital Accumulation," *American Economic Review*, 91, 1185-1202.

Hansen, G. D., (1985), "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics*, 16, 309-327.

Hobijn, B. and B. Jovanovic, (2001), "The Information-Technology Revolution and the Stock Market: Evidence," *American Economic Review*, 91(5), 1203-1220.

Jermann, U., (1998), "Asset Pricing in Production Economies," *Journal of Monetary Economics,* 41, 257-275.

King, R. G. and S. T. Rebelo, (1999), "Resuscitating Real Business Cycles," in *Handbook of Macroeconomics*, J. Taylor and M. Woodford eds., Amsterdam: Elsevier, Vol. 1B, 927-1007.

Koopmans, T., (1965), "On the Concept of Optimal Economic Growth," in *The Econometric Approach to Development Planning*, Amsterdam: North-Holland and Chicago: Rand-McNally.

Kydland, F. and E. Prescott, (1982), "Time to Build and Aggregate Fluctuations," *Econometrica,* 50, 1345-1370.

Mehra, R. and E. Prescott, (1985), "The Equity Premium Puzzle," *Journal of Monetary Economics*, 15, 145-161.

Rietz, T. A., (1988), "The Equity Premium: a Solution." *Journal of Monetary Economics*, 22, 117-131.

Stock, J. and M. Watson, (1999), "Business Cycle Fluctuations in U.S. Macroeconomic Time Series," in *Handbook of Macroeconomics*, J. Taylor and M. Woodford eds., Amsterdam: Elsevier, Vol. 1A, 3-64.

Wei, C., (2003), "Energy, the Stock Market and the Putty-Clay Investment Model," *American Economic Review*, 93(1), 311-324.

Weil, P., (1989), "The Equity Premium Puzzle and the Risk-Free Rate Puzzle," *Journal of Monetary Economics*, 24(3), 401-422.