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A Note on Deteriorating Jobs and Learning in Single-Machine Scheduling Problems

Wen-Chiung Lee*
Department of Statistics, *Feng Chia University*, *Taiwan*

Abstract

In this note, we investigate the effects of deterioration and learning in single-machine scheduling problems. Although the learning effect and the concept of deteriorating jobs have been extensively studied, they have never been considered simultaneously. It is shown in several examples that the optimal schedule of the problem may be different from that of the classical one. Nevertheless, the makespan and the total flow time minimization problems remain polynomial solvable.

Key words: scheduling; single machine; learning effect; deteriorating jobs

JEL classification: C69

1. Introduction

Pinedo (2002) pointed out that processing time distributions may be subject to change due to learning or deterioration phenomena. For example, when the distributions correspond to a manual operation, the possibility of learning exists. Biskup (1999) indicated that the learning effect has been observed in numerous practical situations in different branches of industry and for a variety of corporate activities. In a separate vein of literature, it has been noticed that jobs may deteriorate as they wait to be processed. Kunnathur and Gupta (1990) and Mosheiov (1995) presented several real-life situations where deteriorating jobs might occur. However, it is surprising that the effects of deterioration and learning have never been considered concurrently. In this paper, we investigate the implications of these phenomena occurring simultaneously for single-machine scheduling problems.

Analysis of scheduling problems in which the processing time of a job is a function of its starting time was introduced by Browne and Yechiali (1990). Mosheiov (1991) considered the flow time minimization problem under the assumption that basic processing times remain the same in the linear deterioration model. The motivation for analyzing identical basic processing times arises not only from

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*Correspondence to: Department of Statistics, Feng Chia University, 100 Wenhua Road, Taichung 407, Taiwan. E-mail: wclee@fcu.edu.tw. The author is grateful to a referee for valuable comments.

its intrinsic interest, but it also serves as a good approximation to the general case. Later, Mosheiov (1994) further simplified the model to a simple linear deterioration model in which jobs have a fixed job-dependent growth rate but no basic processing times. This follows from the fact that as the number of jobs increases, the starting times of many jobs are postponed and their basic processing times become irrelevant.

Bachman and Janiak (2000) showed that the maximum lateness minimization problem under the linear deterioration assumption is NP-complete, and two heuristic algorithms were presented as a consequence. Bachman et al. (2002) proved that minimizing the total weighted completion time is NP-complete. Ng et al. (2002) studied three different decreasing linear functions of deterioration, showed that two of these problems could be solved in $O(n\log n)$ time, and presented a dynamic programming method for the third problem. Other types of deterioration have also been discussed. For instance, Kunnathur and Gupta (1990), Kubiak and van de Velde (1998), and Mosheiov (1995) considered piecewise linear deteriorating functions, while Voutsinas and Pappis (2002) utilized the exponential deterioration function. An extensive review of research related to scheduling deteriorating jobs was provided by Alidaee and Womer (1999).

Biskup (1999) was among the pioneers that brought the concept of learning into the field of scheduling, although it has been widely employed in management science since its discovery by Wright (1936). Biskup (1999) proved that single-machine scheduling with a learning effect remains polynomial solvable if the objective is to minimize the deviation from a common due date or to minimize the sum of flow time. Mosheiov (2001) continued to explore the Biskup (1999) model and showed that the single-machine makespan minimization problem remains polynomial solvable. Moreover, he provided counterexamples to show that the optimal schedule may be different from that of the classical problem. Mosheiov and Sidney (2003) considered a case of the job-dependent learning curve where the learning in the production process of some jobs is faster than that of others.

The remainder of this note is organized as follows. In Section 2, we formulate the models. In Section 3, we show that some standard performance measures remain polynomial solvable. In addition, we show by counterexamples that the optimal solution for the classical version no longer holds in the first model. In Section 4, we give two examples to demonstrate that problems are even more complicated in the second model. The last section concludes.

2. Model Formulation

The focus of this paper is to study the effects of deterioration and learning simultaneously. The learning effect model provided by Biskup (1999) is combined with the simple linear deterioration model to form the first model. We then combine the learning effect model and the linear deterioration model with constant basic processing time to yield the second model. The models are described as follows.

There are n jobs to be scheduled on a single machine. All jobs are available for processing at time $t_0 > 0$. The machine can handle one job at a time and preemption is not allowed. Each job J_i has a deterioration rate α_i, and jobs are numbered according to the smallest deteriorating rate (SDR) rule—i.e., $\alpha_1 \le \alpha_2 \le ... \le \alpha_n$. Let $p_{i,r}$ be the processing time of job J_i if it is scheduled in position r in a sequence. In the first model,

$$p_{i,r} = \alpha_i t r^a, \tag{1}$$

where $t \ge t_0$ is the starting time for job J_i and $a \le 0$ is the learning index, given as the (base 2) logarithm of the learning rate. In the second model,

$$p_{i,r} = (p_0 + \alpha_i t) r^a, \tag{2}$$

where p_0 is the common basic processing time.

3. The First Model

In this section, we examine several well-known classical single-machine scheduling problems under the assumption that the actual processing time has the form of the first model. Although these classical problems have polynomial-time optimal schedules, we find in most cases that solving the problem requires more computational effort if the effects of deterioration and learning are present.

3.1 Makespan Minimization

In the classical makespan minimization problem, the value of the makespan is sequence independent. However, the optimal solution is obtained by the smallest deteriorating rate (SDR) principle if both effects are considered in conjunction.

Theorem 1: If the actual processing times follow the form of equation (1), then the makespan is minimized by the SDR sequence.

Proof: Suppose $\alpha_i \le \alpha_j$. Let S_1 and S_2 be two job schedules where the difference between S_1 and S_2 is a pairwise interchange of two adjacent jobs J_i and J_j; that is,

$$S_1 = <\pi\, J_i\, J_j\, \pi'>$$

and

$$S_2 = <\pi\, J_j\, J_i\, \pi'>,$$

where π and π' are partial sequences. Furthermore, we assume that there are $r-1$ jobs in π. Thus, J_i and J_j are the rth and the $(r+1)$th jobs, respectively, in S_1. Likewise, J_j and J_i are scheduled in the rth and the $(r+1)$th positions in S_2. To further simplify the notation, let A denote the completion time of the last job in π. To show S_1 dominates S_2, it suffices to show that the $(r+1)$th jobs in S_1 and S_2 satisfy the condition that

$$C_j(S_1) \le C_i(S_2) .$$

The actual processing time for J_i is $p_{i,r} = \alpha_i A r^a$ and its completion time is

$$\begin{aligned} C_i(S_1) &= A + p_{i,r} \\ &= A(1+\alpha_i r^a). \end{aligned}$$

Thus, the actual processing time for J_j is $p_{j,r+1} = \alpha_j C_i(S_1)(r+1)^a$ and the completion time is

$$\begin{aligned} C_j(S_1) &= C_i(S_1) + p_{j,r+1} \\ &= A(1+\alpha_i r^a)(1+\alpha_j(r+1)^a). \end{aligned} \tag{3}$$

Similarly, it is easy to derive the completion times of J_j and J_i in S_2 as

$$C_j(S_2) = A(1+\alpha_j r^a)$$

and

$$C_i(S_2) = A(1+\alpha_j r^a)(1+\alpha_i(r+1)^a) . \tag{4}$$

Based on equations (3) and (4), we find

$$C_i(S_2) - C_j(S_1) = A[(\alpha_j - \alpha_i)(r^a - (r+1)^a)] \ge 0 .$$

Thus, S_1 dominates S_2.

3.2 Flow Time Minimization

By the standard pairwise interchange argument as in Theorem 1, it can be shown that the optimal schedule for minimizing the flow time is obtained by the SDR rule. This result coincides with the shortest processing time (SPT) rule for the classical problem.

Theorem 2: If the actual processing times follow the form of equation (1), then the sum of flow times is minimized by the SDR sequence.

3.3 Sum of Weighted Flow Time Minimization

Smith (1956) showed that the weighted smallest processing time (WSPT) policy produces the optimal schedule for minimizing the sum of weighted flow time. That is, jobs are scheduled in a non-decreasing order of p_i / W_i values. In the following, we provide an example to demonstrate that the weighted smallest deteriorating rate (WSDR) policy does not optimally solve the problem.

Example 1: Suppose $n = 2$, $t_0 = 1$, $\alpha_1 = 2$, $\alpha_2 = 3$, $W_1 = 3$, and $W_2 = 4$. As in Biskup (1999), learning takes place by the 80% learning curve, which yields $a = -0.322$. The sequence obtained by the WSDR rule is <1, 2>, and the total weighted flow time is 49.8. However, the total weighted flow time of sequence <2, 1> is 47.2.

3.4 Total Lateness Minimization

Let d_i be the due date of job i, $i = 1, \ldots, n$. Recall that the lateness of job i is defined as $L_i = C_i - d_i$. In the classical scheduling problem, the total lateness is minimized by the SPT rule when the processing times are known and constant. Since $\sum L_j = \sum(C_i - d_i) = \sum C_i - \sum d_i$, the first term is minimized by the SDR rule as described in Subsection 3.2, while the second term is a constant, independent of the schedule. Thus, we obtain the following result.

Theorem 3: If the actual processing times follow the form of equation (1), then the sum of the lateness is minimized by the SDR sequence.

3.5 Maximum Lateness Minimization

Recall that the maximum lateness is defined as $L_{\max} = \max\{L_i, i = 1, \ldots, n\}$. In the classical scheduling problem, it is well known that the earliest due date (EDD) policy produces the optimal schedule. In the following example, we show that this is no longer true when deterioration and learning effects are incorporated.

Example 2: Suppose $n = 2$, $t_0 = 1$, $\alpha_1 = 2$, $\alpha_2 = 0.1$, $d_1 = 5$, and $d_2 = 6$. The maximum lateness of the sequence obtained by the EDD rule is –2. The optimal schedule is <2, 1> with maximum lateness –2.14.

3.6 Number of Tardy Jobs Minimization

Moore (1968) provided an algorithm to solve the classical form of this problem. Jackson (1955) pointed out that if a schedule with no tardy jobs exists, then the EDD sequence contains no tardy jobs. Here is an example showing that Jackson's theorem does not hold if the effects of learning and deterioration are taken into account.

Example 3: Suppose $n = 2$, $t_0 = 1$, $\alpha_1 = 10$, $\alpha_2 = 0.1$, $d_1 = 10$, and $d_2 = 11$. The EDD sequence <1, 2> contains two tardy jobs, while the alternative sequence <2, 1> contains no tardy jobs.

4. The Second Model

Although the makespan and the flow time minimization problems are optimally solved by the SDR policy in the first model, we show in the next two examples that the SDR policy no longer provides the optimal solution for these criteria in the second model.

Example 4: (Makespan minimization) Suppose $n = 2$, $t_0 = 1$, $p_0 = 1$, $\alpha_1 = 1$, and $\alpha_2 = 2$. The makespan of the SDR sequence is 8.6, while the makespan of the alternative sequence is 8.

Example 5: (Flow time minimization) Suppose $n = 2$, $t_0 = 1$, $p_0 =1$, $\alpha_1 = 1$, and $\alpha_2 = 3$. The total flow time obtained by the SDR sequence is 9.6, but the optimal sequence is <2, 1> with a total flow time of 9.2.

5. Conclusion

In this note, the effects of learning and deterioration are considered in conjunction. It is shown that the makespan, the flow time, and the total lateness minimization problems all can be optimally solved by the smallest deteriorating rate policy in the first model, but these results are no longer true in the second model. We proceed to show, even in the first model, that (1) the weighted smallest deteriorating rate rule does not provide the optimal solution for minimizing the sum of weighted flow time, (2) the earliest due date rule does not produce the optimal schedule for the maximum lateness minimization problem, and (3) Moore's algorithm does not yield the optimal sequence for minimizing the number of tardy jobs.

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