

## **Tournaments versus Piece Rates under Limited Liability**

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### **Abstract**

We discuss two incentive schemes that are frequently used in practice—tournaments and piece rates. The existing literature on the comparison of these two incentive schemes has focused on the case of unlimited liability. However, real workers' wealth is typically restricted. Therefore, this paper compares both schemes under the assumption of limited liability. The results show that piece rates dominate tournaments if idiosyncratic risk is sufficiently high despite the partial insurance effect of tournament compensation.

*Key words:* incentives; piece rates; rank-order tournaments

*JEL classification:* J31; J33; M5

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### **1. Introduction**

Since the seminal paper by Lazear and Rosen (1981) there has been a wide discussion of tournaments versus piece rates as alternative incentive schemes. In a tournament, at least two workers compete against each other for given winner and loser prizes. Under a piece-rate scheme, a worker's payment consists of a fixed payment and a certain percentage—the piece rate—of the worker's realized output in monetary terms.

There exist many examples for either incentive scheme in practice. Tournaments can be observed in sports (e.g., Ehrenberg and Bognanno, 1990), in broiler production (Knoeber and Thurman, 1994), and also in firms when people compete for job promotion (e.g., Baker et al., 1994). Basically, corporate tournaments will always be created if relative performance evaluation is linked to monetary consequences for the employees. Hence, forced-ranking or forced-distribution systems, in which supervisors have to rate their subordinates according to a given number of different grades, also belong to the class of tournament incentive schemes. Boyle (2001) reports that about 25% of Fortune 500

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companies utilize forced-ranking systems to tie pay to performance (e.g., Cisco Systems, Intel, and General Electric).

An extreme form of combining relative performance evaluation and tournament incentives is when layoffs are tied to the lowest grade. The most prominent advocate of such layoff tournaments is the former General Electric CEO Jack Welch, whose incentive philosophy demands dismissal of the bottom 10% of employees each year. A similar system was used by Enron. Enron employees were regularly rated on a scale of 1 to 5, and employees rated grade 5 were typically fired within six months.

There are also lots of examples of piece-rate schemes in practice; see among many others Lazear (2000) on the introduction of piece rates at the Safelite Glass Corporation and Freeman and Kleiner (1998) on the decline of piece-rate systems in the American shoe industry. For an earlier discussion of piece rates, see for example Lazear (1986), Gibbons (1987), and Fama (1991).

Lazear and Rosen (1981) show that both incentive schemes lead to first-best efforts given homogeneous and risk-neutral workers with unlimited liability. However, tournaments can dominate piece rates since tournaments only require an ordinal performance measure. Concerning risk-averse workers, there is no clear ranking between the two incentive schemes. On the one hand, tournaments provide a crude form of insurance since each agent receives at least the loser prize and at most the winner prize. On the other hand, tournaments have the drawback that in symmetric equilibrium the probability mass is distributed equally between the two prizes.

Green and Stokey (1983) emphasize that tournaments dominate piece rates if filtering of common noise is of major interest. In tournaments, common noise cancels out because of the relative comparison of worker performance. Piece rates use an absolute performance measure and, therefore, cannot serve as a risk filter in a static context.

Malcomson (1984) points to an important advantage of tournaments compared to piece rates. Since winner and loser prizes are fixed in advance (i.e., the employer commits himself to certain labor costs before the tournament starts), tournaments can create incentives even if the worker performance measure is unverifiable. However, an employer always needs a verifiable performance measure to utilize piece rates as an incentive scheme.

Although workers' liability is often limited in practice, the existing comparison of tournaments and piece rates has been restricted to the case of unlimited liability. By this assumption, loser prizes in tournaments and fixed payments in piece-rate schemes are allowed to be arbitrarily negative. Hence, not surprisingly given risk-neutral workers, first-best efforts are implemented under either incentive scheme. In tournaments, the optimal spread between winner and loser prizes can always be chosen to induce first-best incentives, whereas the (possibly negative) loser prize is used by the employer to extract the whole surplus, so that the workers' participation constraint always binds. In piece-rate schemes, optimal incentives are created by offering a piece rate of 100%. However, this selling-the-firm strategy which induces first-best incentives (i.e., maximizes welfare) is only optimal for the

employer if he or she is allowed to choose a negative fixed payment to extract all welfare gains from the workers.

Of course, if negative payments such as negative loser prizes or negative fixed payments are not feasible since workers are restricted in wealth (i.e., they are protected by limited liability), these standard solutions to the incentive problem under risk-neutral workers are not possible. But then the employer does not want to implement first-best efforts any longer because this would be too expensive. This paper addresses the open question of which incentive scheme, a tournament or piece rates, is preferable by the employer in situations with limited liability. The analysis of this paper offers several interesting results. In particular, it can be shown that if idiosyncratic risk is sufficiently high, piece rates will dominate tournaments. Of course, under limited liability, a high idiosyncratic risk is detrimental under either incentive scheme since there is a significant probability that a worker only realizes a very low output. However, the findings show that first-best efforts are less likely in tournaments than under a piece-rate scheme if such risk is large. Moreover, if in this situation only a second-best solution can be implemented and workers earn positive rents, efforts will be larger under the piece-rate than under the tournament scheme.

Note that the risk in the model is assumed to be idiosyncratic. In practice, this means that the risk is mainly determined by a worker's individual characteristics and by the special tasks that have been delegated to him or her. Another interpretation of the idiosyncratic error term is that there are talent or ability characteristics of the worker that are unknown to all players, including the worker, at a given moment (see Lazear and Rosen, 1981, p. 843). Given this interpretation of the error term, a tournament has perfect selection properties: workers are *ex ante* homogeneous (i.e., there is symmetric uncertainty about talent) and choose identical efforts in a symmetric equilibrium. Hence that worker whose unknown ability turns out to be highest will win. As this paper shows, from an incentive perspective, tournaments with idiosyncratic noise and limited liability may be problematic compared to piece rates if the idiosyncratic risk is large. However, if there is also a common error term (e.g., due to the economic situation of the firm or the industry, future working conditions to be chosen by the employer for the workforce, and the general monitoring ability of the supervisor if the common error term denotes measurement errors) and the impact of this error term dominates the influence of the idiosyncratic risk, then the employer will prefer tournaments to piece rates even under limited liability. This point is discussed more closely in Section 4.

The results of the paper offer a further implication for practice. It shows that the limited-liability problem should be less severe the higher a worker's reservation value. In practice, often the reservation value is determined by the employee's qualifications or position in the corporate hierarchy (see also Kim 1997, p. 910). Moreover, since the wealth of a manager is typically greater than that of a worker belonging to a lower hierarchical level, the relative disadvantages of tournaments due to limited liability should decrease for higher levels in the corporate hierarchy.

It is important to emphasize that we do not look for the optimal individual contract under limited liability in a principal-agent setting. Such a question has been

addressed by Sappington (1983) and Kim (1997) for example. The results of Kim (1997) show that the optimal incentive contract is a kind of bonus scheme. However, in this paper we compare two compensation schemes that are frequently used in practice in order to show under which circumstances tournaments dominate piece rates and vice versa. We also do not address the case of a principal who is characterized by limited liability, which is considered by Kahn and Scheinkman (1985), Tsoulouhas and Vukina (1999), and Tsoulouhas and Marinakis (2006). Lewis and Sappington (2000a, 2000b, 2001) extend the discussion of incentives for wealth-constrained agents by introducing private information about the agents' wealth and/or abilities. Optimal incentives under limited liability in organizations are discussed by Schmitz (2005a, 2005b).

The paper is organized as follows. The next section introduces the model. The results of the model are presented in Section 3. Section 4 concludes.

## 2. A Model

To compare tournaments with piece rates, a model with one employer and two workers is considered. All players are assumed to be risk neutral. The output of worker  $i$  ( $i = A, B$ ) is described by the production function  $q_i = e_i + \varepsilon_i$  with  $e_i$  denoting the worker's effort choice and  $\varepsilon_i$  idiosyncratic noise, which is distributed over  $[-\bar{\varepsilon}_L, \bar{\varepsilon}_H]$  with mean  $\hat{\varepsilon}$  and  $\bar{\varepsilon}_L, \bar{\varepsilon}_H > 0$ . As usual in tournament models,  $\varepsilon_A$  and  $\varepsilon_B$  are assumed to be identically and independently distributed (i.i.d.). Let  $G(\cdot)$  denote the cumulative distribution function and  $g(\cdot)$  the density of the composed random term  $\varepsilon_j - \varepsilon_i$  ( $i, j = A, B$  and  $i \neq j$ ). The output  $q_i$  is verifiable, whereas the employer observes neither  $e_i$  nor  $\varepsilon_i$ . The effort costs of worker  $i$  are described by the convex function  $c(e_i)$  with  $c(0) = 0$ ,  $c'(e_i) > 0$ ,  $c''(e_i) > 0$ , and  $c'''(e_i) \geq 0$ . Each worker is assumed to have a reservation value  $\bar{u} \geq 0$ , and, in any given case, the employer wants to hire the two workers (e.g., because of their human capital). The employer maximizes expected total output minus labor costs (i.e., wages), whereas each worker maximizes expected wages minus effort costs.

If the employer organizes a tournament between the two workers, at the first stage of the game he or she chooses a winner prize  $w_1$  to be given to the worker with the highest realized output and a loser prize  $w_2$  prior to the tournament to induce incentives. Let  $\Delta w = w_1 - w_2$  denote the prize spread. Then, given the tournament prizes, the two workers choose their optimal efforts at the second stage. In order to model limited liability, neither prize is allowed to become negative ( $w_1, w_2 \geq 0$ ). However, since positive incentives require  $w_1 > w_2$ , the limited-liability constraint actually reduces to  $w_2 \geq 0$ .

Under a piece-rate scheme, at the first stage of the game the employer uses a linear incentive formula  $w_i = \alpha + \beta q_i$  ( $i = A, B$ ) with  $q_i$  the realized output of worker  $i$ ,  $\alpha$  a fixed payment, and  $\beta \in [0, 1]$  the piece rate (see, e.g., Lazear and Rosen, 1981; Lazear, 1986; Gibbons, 1987; Fama, 1991). Again, the limited-liability assumption for the workers requires wages  $w_i$  to be non-negative ( $w_i \geq 0$ ). At the

second stage, each worker chooses his effort  $e_i$  for a given pair  $(\alpha, \beta)$ .

Most of the assumptions follow the standard tournament model by Lazear and Rosen (1981). However, there are two exceptions that should be highlighted. First, Lazear and Rosen do not address the problem of limited liability—the focus of this paper. Second, Lazear and Rosen do not assume that the distribution of the error term has a finite support. For this paper, the assumption of a finite support is important because limited liability is defined as guaranteeing the workers non-negative wages in any situation. Hence, in the case of an infinite support, piece rates would be clearly dominated by tournaments since it would be too expensive for the employer to compensate workers for possibly infinite losses. As an alternative, the employer could define a lower bound for possibly negative piece-rate payments  $\beta q_i$ . However, then we could also think about variants of the tournament model that may lead to further improvements. Altogether, in order to guarantee a “fair” comparison of piece rates and tournaments under realistic assumptions, a bounded influence of the error term is assumed throughout the paper.

### 3. Results

As a benchmark result, the first-best effort  $e^{FB}$  can be calculated. This effort maximizes  $E[q_i] - c(e_i)$ , which yields:

$$c'(e^{FB}) = 1, \quad i = A, B. \quad (1)$$

Under the tournament scheme, at the second stage of the game, worker  $i$  maximizes:

$$EU_i(e_i) = w_2 + \Delta w \cdot G(e_i - e_j) - c(e_i). \quad (2)$$

Hence, if an equilibrium in pure strategies exists at the tournament stage (for a model where existence is guaranteed see Kräkel, 2004), it will be unique and symmetric with each worker choosing effort implicitly defined by:

$$\Delta w g(0) = c'(e_T). \quad (3)$$

At the first stage, the employer chooses  $w_1$  and  $w_2$  to maximize net profit  $2e_T - w_1 - w_2$  subject to the workers' incentive constraint (3) and their participation constraint:

$$\frac{w_1 + w_2}{2} - c(e_T) \geq \bar{u}. \quad (4)$$

Since the employer's net profit and the workers' incentives (see (3)) strictly decrease with the value of the loser prize, the employer will choose that value of  $w_2$  that makes the worker participation constraint binding. Without restrictions on the loser

prize, the employer chooses  $\Delta w$  in order to maximize welfare (i.e., he or she implements  $e^{FB}$  for both workers) and extracts all rents from the workers by the optimally chosen loser prize. The incentive constraint and the binding participation constraint together lead to:

$$w_1^{FB} = c(e^{FB}) + \bar{u} + \frac{c'(e^{FB})}{2g(0)} \quad (5)$$

$$w_2^{FB} = c(e^{FB}) + \bar{u} - \frac{c'(e^{FB})}{2g(0)}. \quad (6)$$

Note, however, that due to the limited-liability assumption ( $w_2 \geq 0$ ) this solution will only be feasible if:

$$c(e^{FB}) + \bar{u} \geq \frac{c'(e^{FB})}{2g(0)} \Leftrightarrow c(e^{FB}) + \bar{u} \geq \frac{1}{2g(0)}.$$

Under the piece-rate scheme, the workers' incentive constraint is given by:

$$\beta = c'(e_{PR}), \quad (7)$$

and their participation constraint by:

$$\alpha + \beta(e_{PR} + \hat{\varepsilon}) - c(e_{PR}) \geq \bar{u}. \quad (8)$$

Therefore, the employer can implement  $e^{FB}$  and make the participation constraint binding by choosing:

$$\beta^{FB} = 1 \quad \text{and} \quad \alpha^{FB} = c(e^{FB}) + \bar{u} - e^{FB} - \hat{\varepsilon}.$$

Due to limited liability, the workers' wages have to be non-negative in the worst case (i.e.,  $w_i = \alpha + \beta(e_{PR} - \bar{\varepsilon}_L) \geq 0$ ). Hence, the first-best implementation will be feasible if:

$$c(e^{FB}) + \bar{u} \geq \hat{\varepsilon} + \bar{\varepsilon}_L.$$

Comparing tournaments with piece rates and using equation (1), we have the following proposition.

**Proposition 1:** (i) The higher the workers' reservation value,  $\bar{u}$ , the more likely  $e^{FB}$  is implemented under either incentive scheme. (ii) If  $1/2g(0)$  is greater (less) than  $\bar{\varepsilon}_L + \hat{\varepsilon}$ , implementation of  $e^{FB}$  will be more (less) likely under a piece-rate than under a tournament scheme.

Note that "more likely" means "under more parameter constellations." The intuition for result (i) comes from the fact that workers can be given stronger

incentives the higher their wealth. If workers have high reservation values, the employer must compensate workers for these foregone values by a large lump-sum payment when they sign the contract. By this, the workers' wealth increases significantly, so that it is more likely that the employer wants to create sufficiently high incentives that lead to first-best effort (for bonus schemes see Kim, 1997, p. 910). Hence in this context, large reservation values of the workers are strictly welfare enhancing.

Result (ii) shows that the larger  $\bar{\varepsilon}_L$  and the smaller  $1/g(0)$ , the more advantageous tournaments will be relative to piece rates. In particular, if:

$$\frac{1}{2g(0)} < \bar{\varepsilon}_L, \quad (9)$$

then the first-best effort  $e^{FB}$  will be more likely implemented under the tournament scheme. This result can also be explained intuitively:  $\bar{\varepsilon}_L$  characterizes the worst case under the piece-rate scheme, in which the workers' compensation must be still non-negative. Under the tournament scheme,  $w_2^{FB} = c(e^{FB}) + \bar{u} - c'(e^{FB})/2g(0) = c(e^{FB}) + \bar{u} - 1/2g(0)$ , which becomes negative if  $1/2g(0)$  is too large. Note that the marginal winning probability,  $g(\cdot)$ , determines incentives in the tournament and hence optimal prizes. If  $g(\cdot)$  is flat (i.e., the outcome of the tournament is mainly determined by luck)—and therefore  $g(0)$  is small—effort incentives will be rather low (see the incentive constraint (3)). Following Lazear (1995, p. 29), we can interpret  $1/g(0)$  as a measure of luck or risk in the tournament. Alternatively, we may interpret  $g(0)$  as a measure of the employer's monitoring precision. Hence, in a situation with considerable luck or a low monitoring precision, the employer has to choose a sufficiently high prize spread  $\Delta w$  to restore incentives. This means, however, that the loser prize  $w_2$  has to be rather small and that  $w_2^{FB}$  may become negative. Furthermore, the more biased  $q_i$  is as a signal for realized effort (i.e., the higher  $\hat{\varepsilon}$ ), the less likely first-best effort is implemented under piece rates. Altogether, small values of  $\bar{\varepsilon}_L$  and  $\hat{\varepsilon}$  relax the limited-liability constraint under the piece-rate scheme, whereas a small  $1/2g(0)$  relaxes the one under the tournament scheme, which drives result (ii) of Proposition 1.

Now we can examine whether risk harms tournament incentives more than piece-rate incentives. First note that—contrary to tournaments—risk will not influence piece-rate incentives given risk-neutral workers if there is unlimited liability. However, under limited liability, maximum bad luck clearly influences inequality (9): if  $\bar{\varepsilon}_L$ , and therefore risk, is large, piece rates will be disadvantageous. As mentioned above,  $1/2g(0)$  can also be used as a measure of risk. If  $1/2g(0)$  is large, tournaments will become disadvantageous, too. Hence, we have to examine which of these two effects is dominant. Of course, for  $\bar{\varepsilon}_L \rightarrow \infty$  (e.g., if the error terms are normally distributed) piece rates become prohibitively expensive for the employer, but tournaments still work. They offer workers a partial insurance since minimum and maximum income are determined by the loser and winner prizes, respectively.

When looking at less extreme cases, the comparison may lead to a different

result. Assume, for example, that the i.i.d. error terms  $\varepsilon_i$  and  $\varepsilon_j$  follow a normal distribution  $N(0, \sigma^2)$  that is truncated on the left at  $-\bar{\varepsilon}_L = -\bar{\varepsilon}$  and on the right at  $\bar{\varepsilon}_H = \bar{\varepsilon}$ . This implies that the convolution  $g(\cdot)$  for  $\varepsilon_j - \varepsilon_i$  is also a truncated normal distribution with mean zero. However, the variance of this new normal distribution is  $2\sigma^2$ , and the composed random variable  $\varepsilon_j - \varepsilon_i$  is distributed over the interval  $[-2\bar{\varepsilon}, 2\bar{\varepsilon}]$ . We obtain following result.

**Proposition 2:** Let  $\varepsilon_i$  and  $\varepsilon_j$  follow a normal distribution  $N(0, \sigma^2)$  truncated at  $-\bar{\varepsilon}$  and  $\bar{\varepsilon}$ . If  $\sigma^2$  is less (greater) than  $\bar{\varepsilon}^2/\ln 2$ , the left-hand side of (9) increases less (more) rapidly in  $\bar{\varepsilon}$  than the right-hand side. If  $\sigma^2 \rightarrow 0$  ( $\sigma^2 \rightarrow \infty$ ), inequality (9) always (never) holds.

**Proof:** See the Appendix.

Proposition 2 shows that for low variances of the initial normal distribution, inequality (9) is more likely to hold, whereas for high values of  $\sigma^2$  the opposite is true. If the variance tends to zero, the inequality is always satisfied, whereas for sufficiently high variances it will always be violated. Hence, tournaments only dominate piece rates if risk (i.e., the variance of  $\varepsilon_i$  and  $\varepsilon_j$ ) is not too large. Following Lazear and Rosen (1981) and Lazear (1995),  $\varepsilon_i$  and  $\varepsilon_j$  can be interpreted in different ways. For example, they can measure (a) exogenous risk of production technologies, (b) individual measurement errors when workers are evaluated, or (c) ex ante unknown abilities of the workers in the case of symmetric uncertainty. This means that, given limited liability, tournaments are only attractive to the employer compared to piece rates if workers use quite safe production technologies, the supervisor's monitoring precision is not too low, or initial uncertainty about the workers' talents is sufficiently reduced by introducing appropriate recruiting techniques.

Next, the employer's optimization problems under limited liability at the first stage are considered. When organizing a tournament the employer maximizes:

$$\pi_T = 2e_T - w_1 - w_2 \quad (10)$$

subject to incentive constraint (3), participation constraint (4), and the limited-liability constraint ( $w_2 \geq 0$ ). In the case of a piece-rate system, the employer maximizes:

$$\pi_{PR} = 2(1 - \beta)(e_{PR} + \hat{\varepsilon}) - 2\alpha \quad (11)$$

subject to incentive constraint (7), participation constraint (8), and the limited-liability constraint ( $w_i \geq 0$ ). Let  $e_T^*(e_{PR}^*)$  denote the workers' equilibrium effort under the tournament (piece-rate) scheme. The solution to the employer's optimization problems yields the following proposition.

**Proposition 3:** Let  $c'''(\cdot) > 0$ . In the employer's optimization problems, at least one constraint is binding. (i) If only the participation constraint is binding, we have



$e_T^* = e_{PR}^* = e^{FB}$ . (ii) If both constraints are binding, equilibrium efforts are described by:

$$\frac{c'(e_T^*)}{2g(0)} = \bar{u} + c(e_T^*) \quad \text{and} \quad c'(e_{PR}^*)(\bar{\varepsilon}_L + \hat{\varepsilon}) = c(e_{PR}^*) + \bar{u}. \quad (12)$$

(iii) If only the limited-liability constraint is binding, equilibrium efforts are characterized by:

$$2g(0) = c''(e_T^*) \quad \text{and} \quad \frac{1}{\bar{\varepsilon}_L + \hat{\varepsilon}} = c''(e_{PR}^*). \quad (13)$$

Let  $c(e_i) = \eta e_i^\delta$  with  $\delta > 2$  and  $\eta > 0$ . Given  $\bar{u} = 0$  and assuming the limited-liability constraint is binding, if  $1/2g(0)$  is greater (less) than  $\bar{\varepsilon}_L + \hat{\varepsilon}$ , workers are more (less) likely to receive a positive rent under the tournament than under the piece-rate scheme.

**Proof:** See the Appendix.

Case (i) is not surprising. As we know from the standard tournament model with risk-neutral workers, if there are no limited-liability problems (i.e., the limited-liability constraint is not binding), the first-best effort is implemented under either incentive scheme. However, if the limited-liability constraint is binding, we either have an interior solution provided  $\bar{u}$  is not too large, so that the participation constraint does not become binding (iii), or a corner solution (ii).

In principal-agent models with a wealth-constrained agent, typically the agent earns a positive rent. This case is described by Proposition 3 (iii). In this situation, the two workers exert more (less) effort under the piece-rate scheme than under the tournament scheme if  $1/2g(0)$  is greater (less) than  $\bar{\varepsilon}_L + \hat{\varepsilon}$ . Interestingly, this condition is identical with the one in Proposition 1 (ii). Together with Proposition 2, we conclude that tournaments are highly problematic if exogenous risk is large. Besides the fact that the first-best solution is less likely to be implemented, Proposition 3 (iii) shows that, in the case of a second-best solution, workers exert less effort compared to the piece-rate scheme. Moreover, according to the parametric example of Proposition 3 (iii), the inequality determines how likely the workers are to receive a positive rent under the tournament relative to the piece-rate scheme.

Recall that case (iii) corresponds to a situation in which workers earn positive rents. If the employer is able to control the exogenous risk to some extent (e.g., by increasing the monitoring precision), he or she can create additional incentives for the workers. Since these extra incentives only reduce the workers' rents, they are at first sight free for the employer. However, reducing risk often leads to additional costs for the employer so that we obtain a trade-off between incentives and risk-reducing costs though workers are risk neutral.

#### 4. Discussion

In this section, several points not yet discussed are addressed. First, one might question the realism of assuming limited liability for the workers. It is important to stress that the limited-liability assumption used in this paper simply supposes that wages are not allowed to be negative. Typically this assumption should be satisfied in practice. Consider, for example, the case of job-promotion tournaments. There the winner prize is the wage at the higher position in the hierarchy after being promoted, whereas the loser stays at his current job and receives his current wage as the loser prize. Of course, both prizes should be positive in this case. I can imagine only two situations where workers may receive negative wages. An example for a piece-rate scheme with a negative fixed payment may be the case of a lawyer or a physician who wants to buy into a partnership. An example for tournaments with negative loser prizes might be the layoff tournaments mentioned in the introduction. However, even in the last case tournament, losers often receive severance pay when laid off.

Another assumption that may be questioned is the existence of tournaments in the strict form as defined in Section 2. The main assumption here is that the employer fixes prizes in advance before the tournament starts. This assumption is crucial to provide incentives to the workers. Naturally, in sports contests, such as golf or tennis tournaments, prizes are set prior to the tournament. However, there are also many examples of pre-specified prizes in corporate tournaments. For example, in contests between salesmen, each contestant knows in advance how large the premium will be when being declared “salesperson of the month.” In large corporations, a considerable part of workers’ wages is attached to jobs, so that we typically have pre-specified prizes in job-promotion tournaments. Finally, the monetary consequences after being rated according to a forced-ranking system are usually known by the workers before the evaluation starts.

The production technology described by  $q_i = e_i + \varepsilon_i$  (with  $\varepsilon_A$  and  $\varepsilon_B$  being i.i.d. given an underlying probability distribution with variance  $\sigma^2$ ) in this paper is identical with the one used in most of Lazear and Rosen (1981): individual output is linear in effort and idiosyncratic noise, which are perfect substitutes in production. However, Lazear and Rosen also discuss (p. 856-857) the case in which a common error term, say  $\rho$ , is added to the production function, so that output is given by:

$$q_i = e_i + \varepsilon_i + \rho. \quad (14)$$

If we compare piece rates and tournaments under the new production function (14), now the employer may strictly prefer a tournament. By using a tournament, the common shock  $\rho$  is filtered out, which is not the case when using piece rates. Note that income risk under a piece-rate scheme is  $\beta^2 \sigma^2 + \beta^2 \text{Var}[\rho]$ , whereas for a tournament scheme the relative performance  $q_i - q_j$  becomes crucial, which has variance  $2\sigma^2$  (in other words, note that the support of the distribution for  $\varepsilon_i - \varepsilon_j$  is twice the support of the underlying distribution for  $\varepsilon_i$ ). Altogether, if there is common noise and if the impact of the common risk is sufficiently large relative to  $\sigma^2$ , tournaments always outperform piece rates given limited liability. This

important advantage of a tournament in the presence of a common shock was also highlighted in the seminal work of Holmstrom (1982).

However, tournaments may be advantageous even if there is no common error term. The results of this paper show under which conditions tournaments outperform piece rates when workers are protected by limited liability and there is only idiosyncratic noise. Tournaments are also preferable to piece rates from the employer's viewpoint if measurement costs are an issue since they only need an ordinal scale (Lazear and Rosen, 1981). Also important in practice is the point raised by Malcomson (1984): often workers' performances or performance signals are not verifiable by a third party (e.g., a court of law). In such situations, piece rates and other pay methods that are based on verifiable individual output do not work since the employer can save labor costs by claiming poor performance *ex post*. However, tournaments do work even in such settings.

## 5. Conclusion

In this paper, tournaments and piece rates are compared under the assumption of limited liability. The comparison shows that idiosyncratic risk or luck has a large impact on the profitability of both incentive schemes. While tournaments offer a partial insurance, piece rates may not work if potential losses become very large. However, if risk is sufficiently high, piece rates dominate tournaments because the first-best implementation is more likely under a piece-rate scheme and because efforts are greater under piece rates when workers earn positive rents.

The theoretical findings carry several practical implications. According to the results, an employer should prefer piece rates to tournaments if idiosyncratic risk is high. Therefore, the employer benefits from piece rates when (a) there is considerable symmetric uncertainty about workers' talents, (b) incentives should be generated for specific risky tasks, or (c) a worker's performance is highly volatile. Furthermore, the limited-liability constraint should bind more for workers on low hierarchy levels than for managers at higher levels of the corporate hierarchy. Hence, the relative dominance of piece rates over tournaments diminishes for higher levels of the corporate hierarchy.

## Appendix

**Proof of Proposition 2:** Defining  $z = \varepsilon_j - \varepsilon_i$ , the density of the truncated normal distribution can be written:

$$g(z) = \frac{1}{\sqrt{2\sigma^2}} \frac{\phi\left(\frac{z}{\sqrt{2\sigma^2}}\right)}{1 - 2\Phi\left(\frac{-2\bar{\varepsilon}}{\sqrt{2\sigma^2}}\right)},$$

with  $\phi(\cdot)$  denoting the density and  $\Phi(\cdot)$  the cumulative distribution function of the standardized normal distribution. We obtain:

$$\frac{1}{2g(0)} = \frac{\sqrt{2\sigma^2}}{2} \frac{1 - 2\Phi\left(\frac{-2\bar{\varepsilon}}{\sqrt{2\sigma^2}}\right)}{\phi(0)} = \sigma\sqrt{\pi} \left(1 - 2 \int_{-\infty}^{\frac{-2\bar{\varepsilon}}{\sqrt{2\sigma^2}}} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{x^2}{2}\right\} dx\right),$$

which we denote  $\Psi(\sigma^2, \bar{\varepsilon})$ . Differentiating this expression (and therefore the left-hand side of (9)) with respect to  $\bar{\varepsilon}$  yields:

$$\frac{\partial \Psi(\sigma^2, \bar{\varepsilon})}{\partial \bar{\varepsilon}} = 2 \exp\left\{-\frac{\bar{\varepsilon}^2}{\sigma^2}\right\},$$

Hence, the left-hand side of (9) increases less rapidly in  $\bar{\varepsilon}$  than the right-hand side if:

$$2 \exp\left\{-\frac{\bar{\varepsilon}^2}{\sigma^2}\right\} < 1 \Leftrightarrow \sigma^2 < \frac{\bar{\varepsilon}^2}{\ln 2}.$$

The second result of Proposition 2 becomes obvious upon inspection of  $\Psi(\sigma^2, \bar{\varepsilon})$ . As  $\sigma^2 \rightarrow 0$ , the upper limit of the integral tends to  $-\infty$  and the whole integral tends to 0, so that  $\Psi(\sigma^2, \bar{\varepsilon})$  goes to zero. As  $\sigma^2 \rightarrow \infty$ , the upper limit of the interval tends to 0, so that the whole interval approaches  $1/2$  and, therefore, the term in brackets to 0. However, the expression  $\sigma\sqrt{\pi}$  grows more rapidly to infinity.

**Proof of Proposition 3:** Let  $e_T^*(\Delta w)$  ( $e_{PR}^*(\beta)$ ) denote a worker's optimal effort choice characterized by incentive constraint (3) ((7)). Then the employer's optimization problems are described by the two Lagrangians:

$$L_T(w_1, w_2) = 2e_T^*(\Delta w) - w_1 - w_2 + \lambda_1 \left[ \frac{w_1 + w_2}{2} - c(e_T^*(\Delta w)) - \bar{u} \right] + \lambda_2 w_2 \quad (\text{A1})$$

and

$$L_{PR}(\alpha, \beta) = 2(1 - \beta)(e_{PR}^*(\beta) + \hat{\varepsilon}) - 2\alpha + \lambda_1 [\alpha + \beta(e_{PR}^*(\beta) + \hat{\varepsilon}) - c(e_{PR}^*(\beta)) - \bar{u}] + \lambda_2 [\alpha + \beta(e_{PR}^*(\beta)) - \bar{\varepsilon}_L]. \quad (\text{A2})$$

We obtain the following optimality conditions for  $w_1$  and  $w_2$  (for brevity, the conditions for the multipliers and the restrictions are omitted):

$$2e_T^{*'} - 1 + \lambda_1 \left[ \frac{1}{2} - c'(e_T^*) e_T^{*'} \right] = 0 \quad (\text{A3})$$

$$-2e_T^* - 1 + \lambda_1 \left[ \frac{1}{2} + c'(e_T^*)e_T^* \right] + \lambda_2 = 0, \quad (\text{A4})$$

with  $e_T^* = e_T^*(\Delta w)$ . From (A3) and (A4) we get  $\lambda_1 + \lambda_2 = 2$ . Hence, at least one constraint must be binding in equilibrium. Concerning the piece-rate scheme, the optimality conditions for  $\alpha$  and  $\beta$  are:

$$-2 + \lambda_1 + \lambda_2 = 0 \quad (\text{A5})$$

$$\begin{aligned} & -2(e_{PR}^* + \hat{\varepsilon}) + 2(1 - \beta)e_{PR}^* + \lambda_1 \left[ e_{PR}^* + \hat{\varepsilon} + \beta e_{PR}^* - c'(e_{PR}^*)e_{PR}^* \right] \\ & + \lambda_2 \left[ e_{PR}^* - \bar{\varepsilon}_L + \beta e_{PR}^* \right] = 0. \end{aligned} \quad (\text{A6})$$

According to (A5), again at least one constraint must be binding. Note that from (3) and (7) we have:

$$e_T^* = \frac{g(0)}{c''(e_T^*)} \quad \text{and} \quad e_{PR}^* = \frac{1}{c''(e_{PR}^*)}. \quad (\text{A7})$$

(i) Substituting  $\lambda_1 = 2$  and  $\lambda_2 = 0$  into the optimality conditions immediately replicates the benchmark result of Lazear and Rosen (1981) for the case of unlimited liability. (ii) Combining the binding limited-liability constraints, the binding participation constraints, and the incentive constraints (3) and (7) leads to (12). (iii) Inserting  $\lambda_1 = 0$  into (A3) together with (A7) yields  $e_T^*$ . Using  $\lambda_1 = 0$  and  $\lambda_2 = 2$  in (A6) together with (A7) gives  $e_{PR}^*$ .

Finally, consider the case of a parameterized cost function  $c(e_i) = \eta e_i^\delta$  with  $\delta > 2$  and  $\eta > 0$ . Given a binding limited-liability constraint and  $\bar{u} = 0$ , workers receive a positive rent under the tournament scheme if  $w_1/2 - c(e_T^*) > 0$  with  $e_T^*$  described by  $2g(0) = c''(e_T^*)$ . Substituting for  $w_1$  according to the incentive constraint  $w_1 g(0) = c'(e_T^*)$  leads to the equivalence  $c'(e_T^*)/2g(0) - c(e_T^*) > 0 \Leftrightarrow 1/2g(0) > c(e_T^*)/c'(e_T^*)$ . Using the specific form of the cost function yields:

$$e_T^* = c''^{-1}(2g(0)) = \left( \frac{2g(0)}{\eta\delta(\delta-1)} \right)^{\frac{1}{\delta-2}},$$

and the inequality becomes

$$\frac{\delta}{2g(0)} > \left( \frac{2g(0)}{\eta\delta(\delta-1)} \right)^{\frac{1}{\delta-2}}.$$

Under the piece-rate scheme, workers get a positive rent if  $\alpha + \beta(e_{PR}^* + \hat{\varepsilon}) > c(e_{PR}^*)$  with  $e_{PR}^*$  characterized by  $1/(\bar{\varepsilon}_L + \hat{\varepsilon}) = c''(e_{PR}^*)$ —that is:

$$e_{PR}^* = \left( \frac{1}{\eta\delta(\delta-1)(\bar{e}_L + \hat{e})} \right)^{\frac{1}{\delta-2}}.$$

Because of the binding limited-liability constraint  $\alpha + \beta(e_{PR}^* - \bar{e}_L) = 0$  and the incentive constraint  $\beta = c'(e_{PR}^*)$ , the inequality can be written  $\bar{e}_L + \hat{e} > c(e_T^*)/c'(e_T^*)$ . Using the parametric form of the cost function and the concrete expression for the equilibrium effort  $e_{PR}^*$ , we obtain:

$$(\bar{e}_L + \hat{e})\delta > \left( \frac{1}{\eta\delta(\delta-1)(\bar{e}_L + \hat{e})} \right)^{\frac{1}{\delta-2}}. \quad (A9)$$

Comparing (A8) with (A9) completes the proof.

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