Cournot Competition in a Multiproduct Duopoly: Specialization through Licensing

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Abstract

In a duopoly where both firms produce substitutes, we show that under process innovation, specialization is the equilibrium attained with cross-licensing. Each firm produces only the good for which it has an advantage, and social welfare may improve. Patent pool extension confirms the results.

Key words: cross-licensing; patent pool; specialization; process innovation

JEL classification: D45; O31

1. Introduction

Antitrust law historically has viewed cross-licensing or pooling agreements with suspicion because these mechanisms are potentially capable of promoting collusion in the product market. The literature on cross-licensing has in fact stressed that it facilitates collusion. Shapiro (1985, p. 26) states that: "two rivals (with or without innovations) alternately could design a cross-licensing agreement whereby each would pay the other a royalty per unit of output, ostensibly for the right to use the other's technology. By imposing a 'tax' on each other …, the firms could again achieve the fully collusive outcome. A cross-licensing contract may be required to achieve the fully collusive outcome if the firms produce different products or are otherwise heterogeneous."

Eswaran (1993) assumes that the firms license their technologies to each other but tacitly agree not to produce from the acquired technology as long as the contracting firm does not defect. In an infinitely repeated game it is shown that collusion can be sustained at a tacitly restricted level of production by credibly introducing the threat of increased rivalry in the market for each firm's product.

Ling (1996) is close to Eswaran's contribution as fixed fee licensing makes firms' costs symmetric and increases the licensee's scope for retaliation.

Fershtman et al. (1992) deals with cross-licensing of complementary technologies, which may be independently developed by different firms. Relevant to this note is the problem the firms face about how to design a cross-licensing agreement such that the resulting non-cooperative game yields equilibrium profits identical to the cooperative outcome.

This note studies product specialization in a duopoly where both firms produce two imperfect substitutes. We show that under process innovation, specialization is the equilibrium attained under optimal cross-licensing arrangements. The optimum licensing contracts are royalty contracts. Royalties are set so as to implement the joint profit maximization (monopoly) outcome as the unique Nash equilibrium of the competition game. The monopoly first-best optimum is attained: (i) each firm produces solely the good for which it has a technological advantage; (ii) the quantities of goods which are produced are the monopoly levels; (iii) firms' joint profits attain the first-best optimum, and social welfare may improve with respect to no licensing. We show that the same results are attained with a patent pool.

The plan of the paper is as follows. Section 2 describes the basic framework where the two firms diversify their production and considers the introduction of the process innovation that may lead to product specialization. Section 3 discusses the cross-licensing and the product specialization which results. Section 4 analyzes the welfare effects. Section 5 extends the analysis to the patent pool. A numerical example is provided in the Appendix.

2. Two Firms Diversifying Their Production

Let's consider a model of an industry composed of two symmetric firms that produce two imperfect substitutes: good 1 and good 2. Both firms can produce both goods. We assume linear demand functions:

$$p_1 = a - \theta(q_{21} + q_{22}) - (q_{11} + q_{12}) \tag{1}$$
$$p_2 = a - \theta(q_{11} + q_{12}) - (q_{21} + q_{22}),$$

where p_i is the price of good i, $i = 1,2$, q_{ij} the quantity of good i produced by firm j, and $\theta \in (0,1]$ represents the degree of product differentiation. These demands are derived from the maximization problem of a representative consumer (as shown by Singh and Vives, 1984) endowed with a utility function separable in money (denoted m) given by:

$$u(q_1, q_2) = a(q_1 + q_2) - q_1^2/2 - q_2^2/2 - \theta q_1 q_2 + m,$$

where $q_1 = q_{11} + q_{12}$ and $q_2 = q_{21} + q_{22}$.

Firm cost functions are linear and symmetric: each firm produces good i, $i = 1, 2$, at the constant marginal cost, c. We assume $c < a$ in order to avoid a corner solution. Firm profit functions are:

$$\Pi_1 = p_1 q_{11} + p_2 q_{21} - c q_{11} - c q_{21}$$
$$\Pi_2 = p_1 q_{12} + p_2 q_{22} - c q_{12} - c q_{22}.$$

We also assume Cournot competition. Firm 1 chooses its outputs:

$$\underset{q_{11}, q_{21}}{Max} \{ q_{11}(p_1 - c) + q_{21}(p_2 - c) \} \tag{2}$$
$$\text{s.t. (1)}$$
$$q_{11} \geq 0, q_{21} \geq 0,$$

and firm 2 chooses its outputs:

$$\underset{q_{21}, q_{22}}{Max} \{ q_{12}(p_1 - c) + q_{22}(p_2 - c) \}$$
$$\text{s.t. (1)}$$
$$q_{12} \geq 0, q_{22} \geq 0.$$

As profit functions are concave in output, first-order conditions are necessary and sufficient for a maximum.

Equilibrium outputs, prices and profits of system (2) are given by:

$$q_{11} = q_{12} = q_{21} = q_{22} = (a - c) / [3(1 + \theta)]$$
$$p_1 = p_2 = (a + 2c)/3.$$
$$\Pi_1 = \Pi_2 = 2(a - c)^2 / [9(1 + \theta)].$$

This lead to our first proposition.

***Proposition 1*:** In a duopoly composed of two symmetric firms that both produce two imperfect substitutes and linear demand functions as in (1), there exists a unique Nash equilibrium where both firms produce positive quantities for $c < a$.

Both firms are active in both markets and there exists limited specialization. Next we suppose that firm 1 discovers and patents a cost-reducing technology for good 1, and similarly firm 2 discovers and patents a cost-reducing technology for good 2, with both new technologies leading zero production costs.

The profit functions are:

$$\Pi_{1P} = p_1 q_{11} + p_2 q_{21} - c q_{21}$$
$$\Pi_{2P} = p_1 q_{12} + p_2 q_{22} - c q_{12},$$

where the subscript P denotes process innovation. In Cournot competition, firms 1 and 2 again choose their (individual) profit-maximizing outputs:

$$\underset{q_{11}, q_{21}}{Max}\{q_{11}p_1 + q_{21}(p_2 - c)\}$$

s.t. (1), $q_{11} \geq 0, q_{21} \geq 0$ (3)

$$\underset{q_{12}, q_{22}}{Max}\{q_{12}(p_1 - c) + q_{22}p_2\}$$

s.t. (1), $q_{12} \geq 0, q_{22} \geq 0$.

Solving system (3) leads to one of two cases. First, if

$$c < a(1-\theta)/(2+\theta),$$ (4)

then there is limited specialization (differentiation). Equilibrium outputs are strictly positive and are given by:

$$q_{11} = q_{22} = [a + c - a\theta + 2\theta c]/[3(1-\theta^2)]$$ (4.a)

$$q_{12} = q_{21} = [a - 2c - a\theta - \theta c]/[3(1-\theta^2)].$$ (4.b)

Prices and profits are:

$$p_1 = p_2 = (a+c)/3$$

$$\Pi_{1LC} = \Pi_{2LC} = [(a+c)^2 + (a-2c)^2 - 2\theta(a+c)(a-2c)]/[9(1-\theta^2)].$$ (4.c)

where the subscript LC denotes limited specialization and Cournot prices.

In contrast, if instead

$$c \geq a(1-\theta)/(2+\theta),$$ (5)

then there is full specialization. Equilibrium outputs, prices and profits are:

$$q_{12}^* = q_{21}^* = 0$$ (5.a)

$$q_{11}^* = q_{22}^* = a/(2+\theta)$$ (5.b)

$$p_1^* = p_2^* = a/(2+\theta)$$

$$\Pi_{1FM} = \Pi_{2FM} = [a/(2+\theta)]^2 .$$ (5.c)

where the subscript *FM* denotes full specialization and monopoly pricing.

Case 2 is the case of drastic innovation. It is an adaptation of the drastic and non-drastic innovation differences discussed by Arrow (1962). In fact a drastic innovation arises when the monopoly price despite the new technology does not exceed the competitive price under the old technology (Kamien et al., 1986, p. 472).

Clearly, if the innovation is drastic (i.e., inequality (5) holds), then there is specialization and firms earn monopoly profits. In this case, each firm produces exclusively the good for which it has a technological advantage, and monopoly levels of goods 1 and 2 are given by (5.b). When the innovation is non-drastic (i.e., inequality (4) holds), then both firms produce both goods, and the firms' profits fall

below monopoly levels by (4.c) and (5.c). This follows because the equilibria are symmetric—that is, in both cases each firm gains half of the industry profit—and because industry profits are higher when each segment of the market is monopolised by the firm that is more efficient at producing the corresponding good.

More formally, for all feasible c and all θ we have $\Pi_{FM} > \Pi_{LC}$. This is true because Π_{LC} is decreasing in c for $c < a(1-\theta)/(5+4\theta)$, it is increasing in c for $a(1-\theta)/(5+4\theta) < c < a(1-\theta)/(2+\theta)$, or $\Pi_{LC} < \Pi_{FM}$ for $c \in [0, a(1-\theta)/(5+4\theta))$. Therefore, $\Pi_{FM} > \Pi_{LC}$ for all θ and $\Pi_{FM} = \Pi_{LC}$ if $c = a(1-\theta)/(2+\theta)$. We summarize this result in the following proposition.

Proposition 2: The Nash-Cournot equilibrium entails:
- I) Full specialization in the case of drastic innovation, in which case:
    - i. each firm produces only the good for which it has a technological advantage;
    - ii. monopoly levels of goods 1 and 2 which are produced as in (5.b);
    - iii. firms' joint profits attain the first-best optimum;
- II) Limited specialization in the case of non-drastic innovation, in which case:
    - i. both firms produce both goods with output levels given by (4.a)-(4.b);
    - ii. firms' joint profits fall below the first-best optimum.

Clearly, in the case of non-drastic innovation, firms would be better off if they could commit to joint profit maximization. Indeed, let innovation be non-drastic and suppose firms can commit to joint profit maximization. Then each firm produces exclusively the good for which it has a technological advantage, monopoly levels of goods 1 and 2 are produced as in (5.b), and firms' joint profits attain the first-best optimum. This immediately follows from Proposition 2.

However, the only credible commitments are those that are incentive compatible, and $q_{12}^* = q_{21}^* = 0$ are not. Indeed, the unique Nash-Cournot equilibrium has (by Proposition 2):

$$q_{11} = q_{12} = q_{21} = q_{22} > 0.$$

We show below that there exists a cross-licensing scheme that implements the collusive outcome: the unique Nash-Cournot equilibrium entails full specialization, and firm profits attain the first-best optimum level.

3. Cross-Licensing

We now consider the possibility of a technology transfer from firm 1 to firm 2 for good 1 and similarly for firm 2 under licensing by means of a two part tariff (a fixed fee, F_i, and a royalty, r_i). We assume that the innovation is observable and verifiable, and similarly for output. Contracts of technology transfer from firm 1 to firm 2, say, are then enforceable and payments by the recipient can be conditioned on the recipient's output. We refer to technology transfer contracts as licensing contracts and name the party that makes the technology transfer the licensor and the

recipient the licensee. More specifically, a licensing contract states parties' obligations as follows: the licensor discloses the new technology to the licensee. The licensee pays the licensor a fixed fee and/or a royalty per unit of its output. A contract offer is made by one firm, and the other either rejects or accepts it. If the latter rejects it then it must use the old technology, if it accepts it and royalties are part of the contract, then royalty payment obligations are due independent of the technology used and therefore its profit-maximizing choice is necessarily to adopt the new cost-reducing technology.

The game played by the two firms is a non-cooperative two-stage game. In the first stage firm i chooses r_i and F_i given r_j and F_j, for $i = 1,2$ and $i \neq j$, simultaneously and independently so as to maximize its profit subject to its own and its rival's individual-rationality constraints—that is, the constraints assuring that the profit earned by each firm with cross-licensing is no less than that with no licensing (i.e., the profits given by (4.c)). In the second stage the firms engage in quantity Cournot competition as described in Section 2.

The profits functions are:

$$\Pi_{1Lic} = p_1 q_{11} + p_2 q_{21} + r_1 q_{12} - r_2 q_{21} + F_1 - F_2$$
$$\Pi_{2Lic} = p_1 q_{12} + p_2 q_{22} - r_1 q_{12} + r_2 q_{21} + F_2 - F_1 .$$

where the subscript *Lic* denotes licensing and outputs q_{11}, q_{21}, q_{12}, and q_{22} are the outcome of the Cournot competition stage given the royalty rates set in the first stage. Specifically, for any given royalty rates, equilibrium outputs, prices, and profits are:

$$q_{11} = \left[a + r_1 - a\theta + 2\theta r_2\right]/\left[3\left(1 - \theta^2\right)\right]$$
$$q_{12} = \left[a - 2r_1 - a\theta - \theta r_2\right]/\left[3\left(1 - \theta^2\right)\right]$$
$$q_{21} = \left[a - 2r_2 - a\theta - \theta r_1\right]/\left[3\left(1 - \theta^2\right)\right] \tag{6}$$
$$q_{22} = \left[a + r_2 - a\theta + 2\theta r_1\right]/\left[3\left(1 - \theta^2\right)\right]$$
$$p_1 = \left(a + r_1\right)/3$$
$$p_2 = \left(a + r_2\right)/3 .$$

In the first stage, each firm i chooses (r_i, F_i) in order to maximize its profits subject to its own and its rival's individual-rationality constraints and output non-negativity constraints as shown below.

Firm 1

$$\underset{r_1, F_1}{Max}\, \Pi_{1Lic}\left(r_1, r_2, \ldots\right)$$
$$\text{s.t. } \Pi_{2Lic}\left(r_1, r_2, \ldots\right) \geq \Pi_{2LC} ,$$
$$\Pi_{1Lic}\left(r_1, r_2, \ldots\right) \geq \Pi_{1LC} ,$$
$$q_{11} \geq 0, \ q_{21} \geq 0, \ q_{12} \geq 0, \ q_{22} \geq 0, \text{ and } r_1, F_1 \geq 0 ,$$

Firm 2

$$\underset{r_2, F_2}{Max} \Pi_{1Lic}(r_1, r_2, \ldots)$$

$$\text{s.t. } \Pi_{1Lic}(r_1, r_2, \ldots) \geq \Pi_{1LC},$$

$$\Pi_{2Lic}(r_1, r_2, \ldots) \geq \Pi_{2LC},$$

$$q_{11} \geq 0, \ q_{21} \geq 0, \ q_{12} \geq 0, \ q_{22} \geq 0, \text{ and } r_2, F_2 \geq 0,$$

where

$$\Pi_{1Lic} \equiv 1/3\{(a+r_1)[a+r_1-a\theta+2\theta r_2]/[3(1-\theta^2)] + (a+r_2)[a-2r_2-a\theta-\theta r_1]/[3(1-\theta^2)]\}$$

$$+ r_1[a-2r_1-a\theta-\theta r_2]/[3(1-\theta^2)] - r_2[a-2r_2-a\theta-\theta r_1]/[3(1-\theta^2)] + F_1 - F_2$$

$$\Pi_{2Lic} \equiv 1/3\{(a+r_1)[a-2r_1-a\theta-\theta r_2]/[3(1-\theta^2)] + (a+r_2)[a+r_2-a\theta+2\theta r_1]/[3(1-\theta^2)]\}$$

$$+ r_2[a-2r_2-a\theta-\theta r_1]/[3(1-\theta^2)] - r_1[a-2r_1-a\theta-\theta r_2]/[3(1-\theta^2)] + F_2 - F_1$$

In the unique Nash equilibrium, licensing contracts are:

$$r_1 = r_2 = a(1-\theta)/(2+\theta) \text{ and } F_1 = F_2 > 0.$$

These are payoffs equivalent to pure royalty contracts:

$$r_1 = r_2 = a(1-\theta)/(2+\theta) \text{ and } F_1 = F_2 = 0. \tag{7}$$

For any given c that satisfies inequality (4), the royalty rate exceeds the cost reduction by (7). Using (7) and solving for outputs, prices, and profits yields:

$$q_{11}^* = q_{22}^* = a/(2+\theta)$$

$$q_{12}^* = q_{21}^* = 0$$

$$p_1^* = p_2^* = a/(2+\theta)$$

$$\Pi_{1Lic} = \Pi_{2Lic} = a^2/(2+\theta)^2.$$

This leads to proposition 3.

***Proposition 3*:** The optimum licensing contracts are the royalty contracts defined by (7). These implement the monopoly first-best optimum: each firm produces solely the good for which it has a technological advantage, monopoly levels of goods 1 and 2 are produced as in (5.b), and firms' joint profits attain the first-best optimum.

Royalty licensing contracts act as an incentive-compatible commitment device for attaining joint profit maximization. The firm that has a technological cost advantage in the production of good j, say firm j, licenses its technology to its rival by means of a royalty contract. The royalty is set such that the licensee finds it optimal to abstain from producing good j, and in equilibrium royalties are not paid. Royalty licensing contracts are designed so as to act as off-equilibrium threats to

implement the joint profit maximization outcome as the unique Nash equilibrium of the Cournot competition game.

4. Welfare Effects

We now compare social welfare, defined as the sum of consumer surplus and profits, between cross-licensing and the process innovation status quo. The social welfare functions in the no-licensing case are:

$$W_{LC}(q_1, q_2) = u(q_1, q_2) - cq_{12} - cq_{21},$$

that is,

$$W_{LC} = \left(8a^2 - 8ac + 11c^2 - 8\theta a^2 + 8\theta ac + 7\theta c^2\right)/\left[9\left(1 - \theta^2\right)\right],$$

by (4.a) and (4.b). In contrast, in the cross-licensing case these are:

$$W_{Lic}(q_1^*, q_2^*) = u(q_1^*, q_2^*),$$

that is,

$$W_{Lic} = a^2(3 + \theta)/(2 + \theta)^2,$$

by (5.a) and (5.b). We conclude our final proposition.

Proposition 4: There exists ac^*, with $0 < c^* < 4a(1 - \theta)/(11 + 7\theta)$, such that social welfare in the cross-licensing case is greater than with no-licensing for all θ and $c \in [c^*, a(1 - \theta)/(2 + \theta)]$.

Proof: First notice that W_{LC} attains its minimum value when the cost, c, equals $c_{\min} = 4a(1 - \theta)/(11 + 7\theta)$ and when $W_{Lic} > W_{LC}(c_{\min})$. Further, note that $W_{Lic} < W_{LC}(c = 0)$ and $\lim W_{LC}(c \to [a(1 - \theta)/(2 + \theta)]) = W_{Lic}$. Social welfare then improves whenever $c \in [c^*, a(1 - \theta)/(2 + \theta)]$.

The economic intuition behind Proposition 4 is in that cross-licensing implements monopoly output and therefore induces an output decrease with respect to Cournot competition. However, it also gives rise to efficiency gains because production is obtained with the most efficient technology. Social welfare improves when the efficiency gain outweighs the output loss—that is, when the cost reduction obtained by adopting the innovation is sufficiently high—which is true when $c \geq c^*$.

5. Patent Pools

The patent pool game differs from the cross-licensing game above in that firms act cooperatively in the first stage. Firms 1 and 2 choose (r_i, F_i) that maximize

joints profits. In the second stage, firms again engage in quantity Cournot competition. The solution is again:

$$r_i = a(1-\theta)/(2+\theta) \text{ and } F_i = 0 .$$

That is, the royalty rate is identical to that derived for the cross-licensing case. The same holds for outputs, prices, and profits: each firm produces solely the good for which it has a technological advantage (full specialization), and monopoly levels of goods 1 and 2 are produced.

6. Conclusion

We study product specialization in a duopoly where each firm can produce two imperfect substitutes. We have shown that under process innovation, specialization is the equilibrium attained under optimal cross-licensing arrangements, as well as under patent pool. The optimum licensing contracts are royalty contracts. These are designed to implement the joint profit maximization outcome as the unique Nash equilibrium of the competition game. The monopoly first-best optimum is attained: each firm produces exclusively the good for which it has a technological advantage, the firms' joint profits attain the first-best optimum, and social welfare may improve with respect to no licensing.

Our result of collusive equilibrium resulting from an optimal licensing scheme with multiproduct firms that simultaneously engage in Cournot competition complements that attained by Filippini (2005) for single-product firms that strategically interact according to a Stackelberg (sequential) competition framework.

Appendix

A simple simulation exercise where $a = 1$ and $\theta = 0.5$ gives the following results.

	Limited specialization			Full specialization
	$c < a(1-\theta)/(2+\theta) = 0.2$			$c \geq a(1-\theta)/(2+\theta) = 0.2$
c	0	0.11	0.19	0.3
q_{ij}, $i = j$	0.22	0.32	0.391	0.4
q_{ij}, $i \neq j$	0.22	0.1	0.011	
p_i	0.33	0.37	0.396	0.4
Π_i	0.148	0.144	0.157	0.16
r_i	0.2	0.2	0.2	
F_i	≤ 0.148	≤ 0.144	≤ 0.157	
W_{LC}	0.592	0.5534	0.5575	
W_{Lie}	0.56	0.56	0.56	0.56

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