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Output Subsidies and Quotas under Uncertainty and Firm Heterogeneity

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Abstract

This paper studies the relative efficiency of two kinds of regulations, quantity restrictions (quotas) and output subsidies, in an imperfectly competitive market in the presence of two sources of uncertainty, costs and prices. We find that when these two sources of uncertainty are independently distributed, the output subsidy instrument has a comparative advantage over the quantity instrument. However, when we take into account the possibility of correlation between the random components and across firms' marginal costs, we find that a positive (negative) correlation tends to favor the quantity (subsidy) instrument. Finally, we show that when the correlation is positive, it is possible to find situations in which the quantity instrument has comparative advantages over the subsidy instrument.

Key words: cost uncertainty; demand uncertainty; firm heterogeneity; output subsidy; quota

JEL classification: D8; L51

1. Introduction

In this paper we study how to drive the Cournot equilibrium allocations to the optimal ones in an imperfectly competitive market in the presence of firm heterogeneity and uncertainty. We consider two types of instruments: output subsidies and quantity restrictions (quotas). The question we want to answer is which type of instrument should be used when there are two sources of uncertainty: uncertainty in marginal costs and uncertainty in prices. Under certainty, the two policies, output subsidies and quantity restrictions, yield the same result. However, this is not always the case given the existence of uncertainty in prices (imperfect information about future demand) and/or uncertainty in marginal costs. In addition,

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due to firm heterogeneity, marginal costs may vary across firms. This may create a situation in which output subsidy policies are more efficient for some firms and quantity restrictions are more efficient for others. Finally, when both sources of uncertainty are present, it should be asked whether there is some degree of statistical dependence between them. Additionally, given firm heterogeneity, some degree of statistical dependence across firms' marginal costs is possible.

Since the seminal work of Weitzman (1974), much research has focused on the role of different policy instruments in a context of uncertainty. In this sense, Weitzman (1974) derives a condition for the relative efficiency between tax and quantity instruments under uncertainty when firms are price-takers, that is, in the case of non-strategic interaction between firms. He obtains that the relative efficiency depends on the relative slopes of the marginal benefit and cost functions and that benefit uncertainty is irrelevant for choosing between price and quantity instruments.

Within the pollution control literature, following Weitzman's (1974) model, several researchers have analyzed the design of regulatory policies in the presence of uncertainty. This literature mainly studies the use of two instruments: prices and quantities. The key question is which type of instrument should be used. Stavins (1996) extends Weitzman's model by considering correlations between benefit and cost uncertainty. He finds that the correlation effect is likely to overwhelm the usual result that benefit uncertainty is irrelevant for choosing between price and quantity instruments and that for some cases the quantity instrument can be more efficient than the price instrument. Choi and Johnson (1987), in a model with price uncertainty, showed that the ex ante equivalent variation and the expected equivalent variation are equal for income risk neutral consumers and that expected equivalent variation provides a lower bound for ex ante equivalent variation when income risk aversion is presumed.

On the other hand, Wu (2000) extends the analysis to study the relative efficiency of taxes and quantities in the presence of input substitution and firm heterogeneity. He shows that input substitution possibilities are very important for determining the relative efficiency of each instrument when input use is regulated. Additionally, firm heterogeneity implies that the planner would have to apply different instruments to achieve the first-best solution, but the planner may not have firm-level information. Blair et al. (1995) introduce uncertainty into the demand function, showing that the imposition of minimum sales levels and maximum consumption levels can provide significant welfare gains relative to the case where the regulator can dictate only a single price for a single quantity of the regulated product. More recently, Hoel and Karp (2001) analyzed the case of multiplicative uncertainty, extending previous studies in which uncertainty enters additively, that is, it affects the level but not the slope of the firm's marginal costs.

The effects of uncertainty have also been widely studied in the context of international strategic trade policy. The third-market model developed by Brander and Spencer (1985) has been extended in several directions to account for uncertainty in the demand function. Cooper and Riezman (1989) expand the instrument set considered by Brander and Spencer to include quantity controls and allow for a multiple number of firms in each country. Cooper and Riezman (1989) consider a model in which the countries choose the type of policy (an export subsidy or a strict quantity control) in the first stage and a level for policy in the second stage. The random intercept of demand is then revealed, and thereafter firms in each country compete. With small noise, countries choose quantity controls because each country is able to immunize its firms from the profit-shifting policies of rivals. However, with large noise, countries choose export subsidies because firms are given the flexibility to respond. Arvan (1991) considers a similar model and finds that a country with a relative small number of firms acts like a Stackelberg leader while a country with a relatively large number of firms acts like a Stackelberg follower. Hwang and Schulman (1993) extend the previous analyses by considering non-intervention as another policy instrument. They show that the subsidy policy is the best response if the two countries have the same number of firms.

In this paper we extend the previous analysis to an imperfect market structure and focus on two regulatory instruments: output subsidies and quantity restrictions. First, we consider an imperfect market with uncertainty in both marginal costs and prices. Second, we consider that marginal costs are not identical across firms, i.e., there is firm heterogeneity. Finally, we take into account that the random components of demand and marginal costs may be not independently distributed.

Previous analyses of the relative efficiency of different regulatory instruments under uncertainty only focus on situations in which firms are price-takers. However, imperfect market structures are very frequent in practice, and therefore it is important to study how the relative efficiency of each instrument given is affected by the existence of strategic interactions among firms.

In our model, we assume that the planner does not have firm-level information to implement differential output subsidies or quantity restrictions. In this respect, we compute the social surplus for each instrument, then the comparative advantage of subsidies over quantities is defined as the difference in the expected social surplus. We also assume the existence of a limited degree of uncertainty in order to justify the use of a second-order approximation.

The comparative advantage of one instrument over the other depends on the number of firms, the market size, the degree of firm heterogeneity, the degrees of demand and cost uncertainty, and the correlations between both sources of uncertainty and across firms. Assuming no correlation between perturbations, we find that the output subsidy instrument always has a comparative advantage over the quantity instrument. The intuition behind this result is the following: in the case of the quantity instrument, the government selects the level of output for each firm, and then firms are required to produce this level of output regardless of the state of nature. Therefore, with the quantity instrument, firms have no flexibility in choosing their output. However, in the case of output subsidies, firms can choose their output, given the subsidy and the reaction of other firms. The subsidy instrument allows firms to adjust their output decisions to demand shocks, taking the subsidy level as given. This asymmetry will lead to a relative advantage of the subsidy instrument over the quantity instrument.

Proceeding further, we consider the possibility of correlation between the random components. When both sources of uncertainty are present, some degree of statistical dependence between them may be possible. It may also be possible to find some degree of statistical dependence across firms' marginal costs. By incorporating the correlation effects in the analysis, we find that a positive (negative) correlation tends to favor the quantity (subsidy) instrument. In fact, it is possible to find situations in which, in the case of a positive correlation, the quantity instrument has comparative advantages over the subsidy instrument. The explanation of this result is as follows: when the correlation is positive, the marginal costs of the firms are lower or higher than the average marginal cost of the industry estimated by the planner. In this case the subsidy instrument will lead to situations of underproduction or overproduction. If the marginal costs of the firms are lower than the estimated average marginal costs, the subsidy instrument causes the total output to be higher than the expected optimal level. On the other hand, if the marginal costs of the firms are higher than the estimated average marginal cost, then the subsidy instrument will lead to a total output lower than the expected optimal level. This will reduce the relative advantage of the subsidy instrument versus the quantity instrument.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3 we study the comparative advantage of output subsidies over quantity restrictions. We finish with our main conclusions presented in Section 4.

2. The Model

We concentrate on a quantity-setter model with n firms, which are distinguished by their type (a set of characteristics) θ^i, $i = 1, \ldots, n$. It is assumed that although individual firms know their own characteristics, the planner does not. The planner views each θ^i as a random variable with strictly positive density on the interval $[\underline{\theta}, \overline{\theta}]$.

There is a homogenous good produced by firms, where p is the market price of the good and x is the total quantity produced. The consumer surplus is given by:

$$U^0(x,\eta) = V(x,\eta) - px , \tag{1}$$

where $V(x,\eta)$ is a strictly increasing function of x and η is a random variable that can be observed by the firms but not by the planner. As in the case of the firm type, we assume that the planner has only limited information about the demand function. Thus, the planner views η as a random variable with strictly positive density on the interval $[\underline{\eta}, \overline{\eta}]$.

Firm $i = 1, \ldots, n$ has a cost function denoted $C(x^i, \theta^i)$ such that $C(0, \theta^i) = 0$, where x^i is the output of firm i. Let firm i 's payoff function be:

$$U^i(x^i, x, \theta^i, \eta) = V_1(x,\eta)x^i - C(x^i, \theta^i) , \tag{2}$$

where $V_1(x,\eta)$ is the derivative of $V(x,\eta)$ representing the inverse demand function mapping aggregate output into prices and x is total output.

It is easy to show that the resulting market equilibrium is not optimal (see Corchón, 2001). For a homogeneous product, firms underproduce in relation to the optimum, and aggregate output is less than the optimal level. In this context, a policy that increases output is generally welfare-enhancing. We focus on output subsidies and quantity restriction policies because these are two simple instruments that are traditionally employed and frequently contrasted. In a context of perfect certainty, there is a formal identity between the use of subsidies and the use of quantities as planning instruments.

However, under uncertainty, both instruments have different effects. In the case of the quantity instrument, the government selects the level of output for each firm. Firms are required to produce this level of output regardless of the state of nature. Therefore, with the quantity instrument, firms have no flexibility in choosing their output. In the case of output subsidies, firms can choose their output taking into account the subsidy and the reaction of the other firms. The subsidy instrument allows firms to adjust their output decisions to demand shocks, taking the subsidy level as given. Because of the asymmetry of information between firms and governments, there is a trade-off between the quantity instrument (which prevents firms from adjusting to demand shocks) and the output subsidy instrument (which allows for adjustment).

We assume that the planner knows the distributions of η and θ across firms and can use this information in policy formulation. The planner has to choose the instruments before observing realizations of the random components. We also make the following standard assumption in order to ensure the existence of an interior solution (see Corchón, 2001).

***Assumption A.1*:** For all $i = 1,..,n$ and for all values of θ and η, we have that $V_{11}(x,\eta) + V_{111}(x,\eta)x^i < 0$ and $V_{11}(x,\eta) - C_{11}(x^i,\theta^i) < 0$.

Assumption A.1 simply states that the inverse demand function is strictly decreasing and that V is strictly concave.

Since firms are not necessarily identical, their marginal costs may vary. However, the planner may not have firm-level information to implement differential output subsidies or differential quantity restrictions. Furthermore, it may also be politically infeasible and technically difficult to apply differential regulations to firms. Therefore, the problem of knowing the exact value of θ^i, which can be different across firms, implies that the level of output subsidies and quantity restrictions are the same for all firms. We now examine the firms' and the planner's decisions under such uniform instruments.

Quantities

Under the quantity instrument, a particular output level is imposed on all firms. The optimal quantity instrument under uncertainty and firm heterogeneity determines those target outputs, $\hat{x}^i$, that maximize expected total surplus, so that:

$$\max_{\hat{x}^i} E\left[V(\hat{x},\eta) - \sum_{i=1}^{n} C(x^i,\theta^i)\right],$$

where $E[\cdot]$ is the expected value operator. The solution $\hat{x}^i$ must satisfy the following first-order conditions:

$$E[V_1(\hat{x},\eta)] = E\left[C_1(x^i,\theta^i)\right] \text{ for } i = 1,\ldots,n. \tag{3}$$

This implies that the quantity should be set at the level where the mean price equals the mean marginal cost.

Output subsidy

When a subsidy instrument s is announced, each firm chooses the output level that maximizes profits:

$$\max_{x^i} V_1(x,\eta)x^i - C(x^i,\theta^i) + sx^i \ .$$

The first order condition for this maximization problem implies that:

$$V_1(x,\eta) + V_{11}(x,\eta)x^i + s = C_1(x^i,\theta^i) \ . \tag{4}$$

The planner will choose the level of subsidies s that maximizes the expected total surplus given the reaction functions $x^i(s,\eta,\theta)$, where $\theta = (\theta^1,\ldots,\theta^n)$:

$$\max_s E\left[V(x(s,\eta,\theta),\eta) - \sum_{i=1}^n C(x^i(s,\eta,\theta),\theta^i)\right].$$

The solution must satisfy the first-order condition:

$$E\left[V_1(x(s,\eta,\theta),\eta)x_1(s,\eta,\theta)\right] = E\left[\sum_{i=1}^n C_1(x^i(s,\eta,\theta),\theta^i)\,x_1^i(s,\eta,\theta)\right],$$

where x_1 is defined as $\sum_i \partial x^i / \partial s$ and x_1^i is defined as $\partial x^i / \partial s$. The above expression can be rewritten using (4) as follows:

$$E\left[V_1(x(s,\eta,\theta),\eta)x_1(s,\eta,\theta)\right] =$$
$$E\left[\sum_{i=1}^n \left(\left[s + V_{11}(x(s,\eta,\theta),\eta)x^i(s,\eta,\theta) + V_1(x(s,\eta,\theta),\eta)\right]x_1^i(s,\eta,\theta)\right)\right].$$

Given that $x_1 = \sum_{i=1}^n x_1^i$, and cancelling out the term $E\left[V_1(x(s,\eta,\theta),\eta)x_1(s,\eta,\theta)\right]$, we obtain that:

$$sE\left[x_1(s,\eta,\theta)\right] =$$
$$-E\left[V_{11}(x(s,\eta,\theta),\eta)(\sum_{i=1}^n x^i(s,\eta,\theta)x_1^i(s,\eta,\theta))\right].$$

Finally, the output subsidy can be defined as follows:

$$s = -\frac{E\left[V_{11}(x(s,\eta,\theta),\eta)(\sum_{i=1}^n x^i(s,\eta,\theta)x_1^i(s,\eta,\theta))\right]}{E\left[x_1(s,\eta,\theta)\right]} \ . \tag{5}$$

The ex-post profit maximizing output of each firm, $\tilde{x}^i$, expressed as a function of (η,θ) corresponds to the optimal ex ante subsidy $\tilde{s}$, which can be defined as:

$$\tilde{x}^i(\eta,\theta) = x^i(\tilde{s},\eta,\theta)\,, \tag{6}$$

where $\tilde{x}(\eta,\theta) = \sum_{i=1}^{n} \tilde{x}^i(\eta,\theta)$ and $\tilde{x}^{-i}(\eta,\theta) = \sum_{j\neq i} \tilde{x}^j(\eta,\theta)$.

Following Weitzman (1974), we assume that the amount of uncertainty with respect to the cost functions and the inverse demand function is taken as sufficiently small to justify a second-order approximation of $C(x^i,\theta^i)$ and $V(x,\eta)$ within the small range of $\tilde{x}(\eta,\theta)$ as it varies around $\hat{x}$, where $\hat{x}$ is the output level chosen when the quantity instrument is used. However, as Malcomson (1978) points out, the use of second-order approximation has limitations and problems that, in some cases, can lead to a wrong result about the more efficient instrument.

Let the symbol "$\cong$" denote an "accurate local approximation" within an appropriate neighborhood of $x = \hat{x}$. Then:

$$C(x^i,\theta^i) \cong C(\hat{x}^i,\theta^i) + (C' + \alpha(\theta^i))(x^i - \hat{x}^i) + \frac{C''}{2}(x^i - \hat{x}^i)^2\,, \tag{7}$$

$$V(x,\eta) \cong V(\hat{x},\eta) + (V' + \upsilon(\eta))(x - \hat{x}) + \frac{V''}{2}(x - \hat{x})^2\,. \tag{8}$$

In the above equations, $V(\hat{x},\eta)$, $C(\hat{x}^i,\theta^i)$, $\alpha(\theta^i)$, and $\upsilon(\eta)$ are stochastic functions and V', V'', C', and C'' are fixed coefficients. $\alpha(\theta^i)$ is a pure unbiased shift of the marginal cost function, and $\upsilon(\eta)$ is a shift of the inverse demand function. Note that Assumption A.1 means that $V'' < 0$ and $V'' - C'' < 0$. Without loss of generality, we also make the following assumption.

***Assumption A.2*:** $E[\alpha(\theta^i)] = E[\upsilon(\eta)] = 0$ for $i = 1,\ldots,n$.

Assumption A.2 simply implies that $\alpha(\theta^i)$ and $\upsilon(\eta)$ are standardized, so that their expected values are zero.

Differentiating (7) and (8) with respect to x^i and x, yields:

$$C_1(x^i,\theta^i) \cong (C' + \alpha(\theta^i)) + C''(x^i - \hat{x}^i)\,, \tag{9}$$

$$V_1(x,\eta) \cong (V' + \upsilon(\eta)) + V''(x - \hat{x})\,, \tag{10}$$

and applying the expected value operator we obtain the following expressions for the fixed coefficients of (7) and (8):

$$E[C_1(\hat{x}^i,\theta^i)] \cong C'\,,$$

$$E[V_1(\hat{x},\eta)] \cong V'\,,$$

$$C_{11}(x^i,\theta^i) \cong C''\,,$$

$$V_{11}(x,\eta) \cong V''\,.$$

From (3) we obtain that

$$V' = C' . \tag{11}$$

From expressions (4), (9), and (10), for $i = 1,\ldots,n$, we can rewrite:

$$V' + \upsilon(\eta) + V''(x(\tilde{s},\eta,\theta) - \hat{x}) + V''x^i(\tilde{s},\eta,\theta) + \tilde{s} \cong C' + \alpha(\theta^i) + C''(x^i(\tilde{s},\eta,\theta) - \hat{x}^i)$$

as:

$$x^i(\tilde{s},\eta,\theta) \cong \frac{C' + \alpha(\theta^i) + (V'' - C'')\hat{x}^i - \tilde{s} - V' - \upsilon(\eta) - V''(x^{-i}(\tilde{s},\eta,\theta) - \hat{x}^{-i})}{2V'' - C''} \tag{12}$$

and solving the above system of equations, x^i and x can be defined as:

$$x^i(\tilde{s},\eta,\theta) \cong \frac{C' - \tilde{s} - V' - \upsilon(\eta)}{(n+1)V'' - C''} + \frac{\alpha(\theta^i)(nV'' - C'') - V''\sum_{j\neq i}\alpha(\theta^j) + (V'')^2\hat{x}^{-i} + [(V'')^2 - (n+1)V''C'' + (C'')^2]\hat{x}^i}{((n+1)V'' - C'')(V'' - C'')} \tag{13}$$

and

$$x(\tilde{s},\eta,\theta) \cong \frac{n(C' - \tilde{s} - V' - \upsilon(\eta)) + \sum_{i=1}^{n}\alpha(\theta^i) + (nV'' - C'')\hat{x}}{(n+1)V'' - C''}, \tag{14}$$

implying that:

$$x_1(\tilde{s},\eta,\theta) \cong -\frac{n}{(n+1)V'' - C''} \tag{15}$$

and

$$x_1^i(\tilde{s},\eta,\theta) \cong -\frac{1}{(n+1)V'' - C''} . \tag{16}$$

Substituting (15) and (16) into (5) and cancelling out $(n+1)V'' - C''$ yields:

$$\tilde{s} \cong \frac{1}{n}E[V_{11}(x(\tilde{s},\eta,\theta))x(\tilde{s},\eta,\theta)] . \tag{17}$$

Next, we obtain the expression for the optimal ex ante level of output subsidy. Replacing x in (8) by the expression for $x(\tilde{s},\eta,\theta)$ from (11) and (14), plugging this into (17), using Assumption A.2, the ex ante output subsidy is obtained as:

$$\tilde{s} \cong -\frac{V''\hat{x}}{n}, \tag{18}$$

where the ex ante output subsidy depends on the curvature of the inverse demand function, on the output level fixed by the quantity instrument, and on the number of firms.

Finally, combining (6), (14), and (18), the ex ante total subsidy output is:

$$\tilde{x} \cong \hat{x} + \frac{\sum_{i=1}^{n} \alpha(\theta^i) - n\upsilon(\eta)}{(n+1)V'' - C''}, \tag{19}$$

and combining (6), (13), and (19), the ex ante subsidy output of firm i is:

$$\tilde{x}^i \cong \frac{\hat{x}}{n} + \frac{\alpha(\theta^i)(nV'' - C'') - V'' \sum_{j \neq i} \alpha(\theta^j) - \upsilon(\eta)(V'' - C'')}{((n+1)V'' - C'')(V'' - C'')}. \tag{20}$$

3. Output Subsidies versus Quantities

Next, we compare social welfare under the two instruments; that is, we compare their relative efficiency in the presence of uncertainty and firm heterogeneity. Following Weitzman (1974), we define the comparative advantage of subsidies over quantities as the expected net difference in gains obtained under the two instruments:

$$\Delta \equiv E\left[(V(\tilde{x}(\eta,\theta),\eta) - \sum_{i=1}^{n} C(\tilde{x}^i(\eta,\theta),\theta^i) - (V(\hat{x},\eta) + \sum_{i=1}^{n} C(x^i,\theta^i)) \right]. \tag{21}$$

If this expression is positive, the output subsidies instrument has a comparative advantage over the quantities instrument. Alternatively, substituting $x = \hat{x}$, $x^i = \hat{x}^i$, $x = \tilde{x}(\eta,\theta)$, and $x^i = \tilde{x}^i(\eta,\theta)$ from (20) into (7) and (8) and plugging the resulting values into (21), using Assumption A.2, and collecting terms, the comparative advantage of output subsidies over quantities in the presence of uncertainty and firm heterogeneity can be written as follows (see the Appendix):

$$\Delta \cong \left[(a_2 - a_1)\sigma_\theta^2 + a_2 n \sigma_\eta^2 + a_1 n \mu_\theta - 2a_2 \mu_{\theta,\eta} \right], \tag{22}$$

where a_1 and a_2 are functions of V'', C'', and the number of firms:

$$a_1 = \frac{n(n-1)V''\left[(2n+3)(V'')^2 - (n+4)V''C'' + (C'')^2\right]}{2((n+1)V'' - C'')^2 (V'' - C'')^2}, \tag{23}$$

$$a_2 = -\frac{n[(n+2)V'' - C'']}{2((n+1)V'' - C'')^2}, \tag{24}$$

and where σ_θ^2 is the mean square error in marginal costs:

$$\sigma_\theta^2 \equiv E\left[(C_1(x^i,\theta^i) - E[C_1(x^i,\theta^i)])^2 \right] \cong E[(\alpha(\theta^i))^2] \text{ for } i = 1,\ldots,n,$$

and σ_η^2 is the mean square error in prices:

$$\sigma_\eta^2 \equiv E\left[(V_1(x,\eta) - E[V_1(x,\eta)])^2\right] \cong E[(\upsilon(\eta))^2].$$

Here $\mu_\theta = E[\alpha(\theta^i)\alpha(\theta^j)]$, $i, j = 1,\ldots,n$, $i \neq j$, is the covariance between marginal costs due to firm heterogeneity. The correlation coefficient between marginal costs across firms, $\rho_\theta = \mu_\theta / \sigma_\theta^2$, can be positive or negative. For example, a positive correlation can be generated by a general improvement in technology, a shift in the price of a factor, or a tax paid by all the firms. A negative correlation can be generated by an improvement in the technology used by one firm that decreases its marginal costs and increases the marginal costs of its competitors. For example, assume there are two firms, $i = 1,2$, that catch fish in a common sea. Each firm i 's cost function is linear and is given by cx^i. Suppose that the per-unit cost of production for firm 1 decreases due to an improvement in technology, so that $c^1 < c$. It is not unreasonable to assume that $c < c^2$, since firm 2's production will be rushed by the competition.

The value $\mu_{\eta,\theta} = E[\upsilon(\eta)\alpha(\theta^i)]$ is the covariance between marginal cost uncertainty and price uncertainty. As we have already pointed out, the correlation coefficient between marginal costs and price uncertainty, $\rho_{\eta,\theta} = \mu_{\eta,\theta} / \sigma_\theta \sigma_\eta$, can be positive or negative. Substitution effects can generate a negative correlation. This way, an increase in marginal costs in the production of some goods can lead consumers to substitute these goods for other similar goods. Therefore, an increase in marginal costs can generate a shift to the left of the inverse demand function and hence a negative correlation. Investments in cleaner technologies, for instance, can generate a shift to the right of the inverse demand functions, representing a positive correlation. Currently, consumers are demanding those goods produced by means of an environmentally friendly technology.

From Assumption A.1 and equations (7) and (8), we have that $V'' < 0$ and $V'' - C'' < 0$. Therefore, parameter a_2 is always positive but a_1 is always negative.

First, we study the case when there are no correlation effects, that is, $\mu_\theta = \mu_{\eta,\theta} = 0$. In this case, we find that the output subsidy instrument has a comparative advantage over the quantity instrument. Intuitions about this result can be found in the different reactions of firms. Given the uncertainty about the cost function, the planner has to use a uniform level for the instruments, in spite of firm heterogeneity. The quantity instrument does not permit firms to adjust output based on the reactions of rival firms. However, this will be possible with the use of output subsidies. With this second instrument, firms have flexibility to respond to the level of the subsidy and to the reactions of rival firms. In this context, a small miscalculation of the quantity results in a larger deviation from the optimal outcome than with a small miscalculation of the subsidy. Consequently, uniform output subsidies are the best instrument to maximize expected social welfare in a context of uncertainty. In other words, the error the planner incurs is larger under the quantity instrument than under the subsidy instrument. In both cases, the final output is the expected optimal one, but the distribution of the output across firms is different. Whereas in the first case all firms produce the same quantity, in the second case the more efficient firms produce more than the less efficient firms.

Next, we consider the possibility of correlation between the random components and across firms' marginal costs. Inspection of expression (22) reveals that a positive (negative) correlation favors the quantity (subsidy) instrument. Nonetheless, the question now is whether the correlation effect is really likely to reverse the instrument choice, that is, under what condition does a positive correlation give the quantity instrument a comparative advantage over the subsidy instrument. We explore the most advantageous case for the quantity instrument, that is, when $\mu_\theta = \sigma_\theta^2$ and $\mu_{\eta,\theta} = \sigma_\theta \sigma_\eta$ (i.e., $\rho_\theta = \rho_{\theta,\eta} = 1$). In this case, expression (22) can be written as:

$$\Delta \cong a_2 \sigma_\theta^2 \left[(n-1)\frac{a_1}{a_2} + 1 + n\left(\frac{\sigma_\eta}{\sigma_\theta}\right)^2 - 2\frac{\sigma_\eta}{\sigma_\theta} \right]. \qquad (25)$$

Figure 1 presents the locus of $\Delta = 0$ for a fixed number of firms in terms of the ratios C''/V'' and $\sigma_\eta/\sigma_\theta$. In the region above this locus, the quantity instrument has a relative advantage over the subsidy instrument, i.e., $\Delta < 0$. In the region below this locus, the subsidy instrument has a relative advantage over the quantity instrument, i.e., $\Delta > 0$. As we increase the number of firms, the locus $\Delta = 0$ shifts to the right. The relative advantage of subsidies over quantities is increasing in the ratio $\sigma_\eta/\sigma_\theta$ and decreasing in the ratio C''/V'' and the number of firms. Therefore, it is possible to find situations in which the quantity instrument has comparative advantage over the subsidy instrument when correlation among firms cost is positive.

The intuition behind why positive correlation favors the quantity instrument is as follows: when the correlation is positive, the marginal costs of the firms are below or above the marginal cost of the industry estimated by the planner. This means that the level of the subsidy will be higher or lower than the optimal subsidy level, which increases the error that the planner incurs with the subsidy instrument given that the reaction of the firms will lead to an output level different than the expected optimal level. In fact, if the marginal costs of the firms are below the average marginal cost of the industry estimated by the planner, firms will overproduce. In this case, the total output will be larger than the expected optimal level due to an overestimation of the optimal subsidy level. In contrast, if the marginal costs of the firms are larger than the average cost of the industry estimated by the planner, this will lead to an underestimation of the optimal subsidy level and hence to an underproduction situation. A simple numerical example illustrates this fact. Assume that the inverse demand function is $p(x) = 50 - x$ and that the average marginal cost of the industry estimated by the planner is $\overline{\theta} = 20$. Therefore, the expected optimal output is $x = 50 - \overline{\theta} = 30$. Considering the existence of two firms (firms 1 and 2), the output of each firm under the quantity instrument will be $\hat{x}_1 = \hat{x}_2 = 15$. Assume now that the correlation of costs is positive and this leads to a marginal cost of each firm of $\theta_1 = 30$ and $\theta_2 = 25$, that is, the marginal costs of both firms are larger than the estimated average cost. Using expression (20) in the text, the output produced by each firm under the subsidy instrument will be $\widetilde{x}_1 = 15 - 5/3$ and $\widetilde{x}_2 = 15$, that is, a total output lower than the expected optimal level.

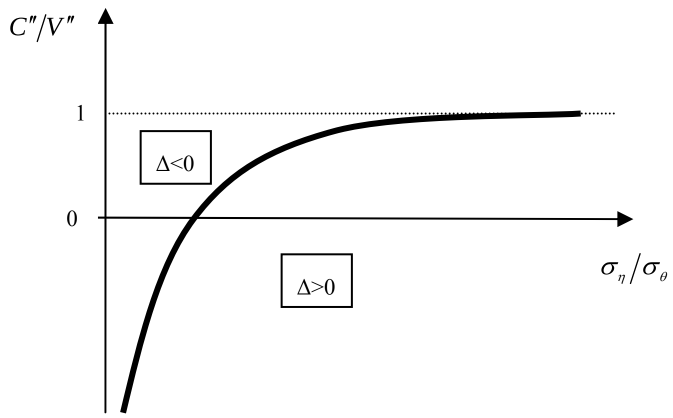
4. Conclusions

In this paper we analyze, in an imperfectly competitive market, the effects of the presence of simultaneous uncertainty in both cost and demand functions and firm heterogeneity on the application of two instruments: output subsidies and quotas.

In general, we find that, under cost uncertainty only, under demand uncertainty only, or under both sources of uncertainty when they are independently distributed, the output subsidy instrument has a comparative advantage over the quantity instrument. The explanation for this result is that the quantity instrument does not allow the firms to adjust their output to the reactions of rival firms, whereas with the subsidy instrument, firms have flexibility to respond to the level of the subsidy and to the reactions of rival firms. In this context, a small miscalculation in the quantity results in a larger deviation from the optimal level than with a small miscalculation of the subsidy.

However, when we allow for correlation due to both uncertainty and heterogeneity, this correlation effect is likely to reverse the instrument choice. We find that a positive (negative) correlation tends to favor the quantity (subsidy) instrument. In particular, it is possible to find situations in which the quantity instrument has a comparative advantage over the subsidy instrument. Therefore, this analysis shows that, in identifying the efficient policy instrument in an imperfectly competitive market, we have to pay attention to the correlation effects.

Figure 1. Relative Advantage of the Subsidy Instrument over the Quantity Instrument



Appendix

In this appendix we describe the derivation of the main result of the paper, i.e., the comparative advantage of output subsidies over quantities as shown by expression (22). The expected difference in gains under the two instruments is given by:

$$\Delta \equiv E\left[V(\tilde{x}(\eta,\theta),\eta) - V(\hat{x},\eta)\right] - \sum_{i=1}^{n} E\left[C(\tilde{x}^i(\eta,\theta),\theta^i) - C(\hat{x}^i,\theta^i)\right]$$

We now examine the two right-hand side terms defining the comparative advantage of the output subsidy instrument over the quantity instrument.

(i) $E\left[V(\tilde{x}(\eta,\theta),\eta)-V(\hat{x},\eta)\right]$.

From expression (8) we obtain:

$$E\left[V(\tilde{x}(\eta,\theta),\eta)-V(\hat{x},\eta)\right]=E\left[V'(\tilde{x}-\hat{x})+\frac{V''}{2}(\tilde{x}-\hat{x})^2\right],$$

and plugging (20) into the two terms of this equation and using Assumption A.2 we obtain that:

$$E\left[(V'+\upsilon(\eta))(\tilde{x}-\hat{x})\right]=\frac{-n\sigma_\eta^2+n\rho_{\eta,\theta}}{(n+1)V''-C''}$$

and

$$E\left[\frac{V''}{2}(\tilde{x}-\hat{x})^2\right]=\frac{V''}{2((n+1)V''-C'')^2}(n\sigma_\theta^2+n^2\sigma_\eta^2+n(n-1)\rho_\theta-2n^2\rho_{\eta,\theta}).$$

(ii) $E\left[C(\tilde{x}^i(\eta,\theta),\theta^i)-C(\hat{x}^i,\theta^i)\right]$.

From expression (7), we obtain that:

$$E\left[C(\tilde{x}^i(\eta,\theta),\theta^i)-C(\hat{x}^i,\theta^i)\right]=E\left[(C'+\alpha(\theta^i))(\tilde{x}^i-\hat{x}^i)+\frac{C''}{2}(\tilde{x}^i-\hat{x}^i)^2\right],$$

and plugging (20) into the two terms in this equation and using Assumption A.2 we obtain:

$$\sum_{i=1}^{n}E\left[(C'+\alpha(\theta^i))(\tilde{x}^i-\hat{x}^i)\right]=$$
$$\frac{n}{((n+1)V''-C'')(V''-C'')}\left[(nV''-C'')\sigma_\theta^2-(n-1)V''\rho_\theta-(V''-C'')\rho_{\eta,\theta}\right]$$

and

$$\sum_{i=1}^{n}E\left[\frac{C''}{2}(\tilde{x}^i-\hat{x}^i)^2\right]=$$
$$\frac{nC''((n^2+n-1)(V'')^2-2nV''C''+(C'')^2)}{2((n+1)V''-C'')^2(V''-C'')^2}\sigma_\theta^2+\frac{nC''}{2((n+1)V''-C'')^2}\sigma_\eta^2$$
$$-\frac{(V'')^2(n-1)(n-2)-2(n-1)(nV''-C'')V''}{2((n+1)V''-C'')^2(V''-C'')^2}\rho_\theta-\frac{nC''}{((n+1)V''-C'')^2}\rho_{\eta,\theta}.$$

Finally, collecting terms, we obtain:

$$\Delta \cong n(a_1\mu_\theta + a_2\sigma_\eta^2 + a_3\sigma_\theta^2 + a_4\mu_{\eta,\theta}) ,$$

where a_1, a_2, a_3, and a_4 are functions of V'', C'', and the number of firms:

$$a_1 = \frac{n(n-1)V''\left[(2n+3)(V'')^2 - (n+4)V''C'' + (C'')^2\right]}{2((n+1)V'' - C'')^2(V''-C'')^2} ,$$

$$a_2 = -\frac{n[(n+2)V'' - C'']}{2((n+1)V'' - C'')^2} ,$$

$$a_3 = -\frac{(2n^2+2n-1)(V'')^3 - (n^2+5n+1)(V'')^2C'' + (3+2n)V''(C'')^2 - (C'')^3}{2((n+1)V'' - C'')^2(V''-C'')^2} ,$$

$$a_4 = \frac{(n+1)V'' + (V''-C'')}{((n+1)V'' - C'')^2} .$$

Given that $a_3 = a_2 - a_1/n$ and $a_2 = -(n/2)a_4$, we obtain expression (22).

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