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Having Fun with Organized Kissing: A Pedagogical Note

Jong-Shin Wei*
Department of International Business, Wenzao Ursuline College of Languages, Taiwan

Abstract

Harbaugh's daughter went to a French school at which every day before class, everyone in the class must kiss each other. It is easy to see that a class of 35 kids must engage in 595 kisses. Organizing the kissing for the purpose of time-saving and predicting how much time it would take are proposed in this pedagogical note inspired by the entertaining puzzle made by Harbaugh. We also illustrate how this problem-solving case study, requiring no mathematics but careful reasoning, might be incorporated in teaching undergraduate students majoring in economics or business.

Key words: problem-solving; efficiency; instructional method; pairing up

JEL classification: A22; A12; B40; C65; D02

1. Inspiration and Introduction

According to Harbaugh (2003), his daughter went to a French school at which every day before class, everyone in the class must kiss each other. Here is a good point made by Harbaugh: *How many kisses will it take?* Although people can kiss each other from two times up to as many as four or even five times, for simplicity we shall just assume that only one kiss occurs when two kids kiss each other.

As the law in France set the enrollment limit at 35, Harbaugh found that a class of 35 must engage in 595 kisses. Our students who know a little bit probability or statistics should have no problem in applying the combination formula $C(n,r)$, read "n choose r", to see $C(35,2)=(35)(35-1)/2=595$. For those who do not have such background, we can use dots to form an "n by n" array. Number rows by $1,2,\ldots,n-1$, and n; likewise for columns. There are n dots sitting on the main diagonal (northwest and southeast direction), representing kids kissing himself/herself. With help from this array, we begin with n^2 dots, next remove dots on the main diagonal, and finally disregard dots in the lower triangle, yielding the number of kisses for the case of n kids as $(n^2-n)/2=n(n-1)/2$, demonstrating

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*Correspondence to: Department of International Business, Wenzao Ursuline College of Languages, 900 Mintsu 1st Road, Kaohsiung 807, Taiwan. E-mail: jsw12011958@gmail.com. Helpful comments from a reviewer are gratefully acknowledged. The usual disclaimer applies.

the combination formula.

Professor Harbaugh ended with an interesting question: "*Class break is only for 10 minutes. Take a class of 35 kids as an example and assume that each kiss takes 5 seconds, how can we organize the kissing for the sake of efficiency?*"

In this pedagogical note, we propose in Section 2 a systematic way to organize the kissing activities for any natural number n (no smaller than 2). Once implemented, this organized kissing will take 335 seconds (about 5.6 minutes) for $n = 35$. Of course, if each kiss takes 10 seconds, then we need 670 seconds (about 11 minutes). In either case, both the 10-minute class break and the enrollment limit (set at 35) seem to be well justified. At each point in time, the number of kisses taking place is referred to as "load". We can figure out the maximal load as: $n/2$ if n is even; $(n-1)/2$ if n is odd. To see how the notion of load contributes, imagine that n players are in a (Chinese) chess contest. Each player has to play exactly one game with every other player. Under the rules to be elaborated in Section 2, we know from the load exactly how many chess boards (and sets) the organizer must provide at each point in time. For another application, consider the scenario where n teams are playing "3 on 3" half-court basketball games. Under the assumption that any two teams only encounter once, via peak load computation we can figure out the demand for referees and the court facilities.

2. Organized Kissing

The teacher asks all n kids to be seated and form a circle. If kids have ID numbers, we can have them as kids $1, 2, \ldots, n-1$, and n sitting clockwise in a circle. If they have not got any numbers, the teacher can ask them to take numbered seats in a sequence. [As to the incentive for getting their cooperation in the spirit of self-selection, shortly we will see that kids with smaller numbers will be able to finish all $n-1$ kissing requirement earlier, hence being able to enjoy some free time before the class begins, as compared to kids with larger numbers.]

Once kids are seated, the teacher asks kid 1 to get up, walk toward kid 2, and kiss kid 2. We call it first round. After that, kid 1 will kiss kid $3, \ldots,$ and finally kid n in that order to complete her mission in these $n-1$ rounds. At the moment that kid 1 is about to walk toward kid 4, kid 2 should get up, walk toward kid 3, and kiss kid 3. So, at the third round, the kissing between kids 1 and 4 occurs when kid 2 is kissing kid 3. Similarly, kid 2 will move on to kiss kid $4, \ldots,$ all the way to kid n. When kid 2 is about to walk toward kid 5, kid 3 should get up, walk toward kid 4, and kiss kid 4. Those descriptions come out of our simple rule in designing: when kid i is making her first kiss (i.e., i starts kissing), the next kid (kid $i+1$) must wait for two rounds to make her first kiss. The idea of making two rounds as the lag is self-explanatory, as Figure 1 illustrates.

Figure 1. Illustrating round 1 thru round 10 for the case of having more than 10 kids

round	1	2	3	4	5	6	7	8	9	10
	1→2	1→3	1→4	1→5	1→6	1→7	1→8	1→9	1→10	1→11
			2→3	2→4	2→5	2→6	2→7	2→8	2→9	2→10
					3→4	3→5	3→6	3→7	3→8	3→9
							4→5	4→6	4→7	4→8
									5→6	5→7

In Figure 1, the expression "$1 \to 2$" stands for the action that kid 1 kisses kid 2. At round 1, kid 1 starts kissing; at round 3, kid 2 starts kissing; at round 5, kid 3 starts kissing; at round 7, kid 4 starts kissing, …and so on.

Note that $1 \to n$ takes places at round $n-1$. Hence $2 \to n$ takes places at round $n-1+(\mathbf{2}-1)=n$; $3 \to n$ takes places at round $n-1+(\mathbf{3}-1)=n+1$; …; $n-1 \to n$ takes places at round $n-1+(\mathbf{n-1}-1)=2n-3$. Cases of $n=6$ and $n=7$ are illustrated in Figure 2 and Figure 3 respectively.

Figure 2. Illustrating the case of having 6 kids

round	1	2	3	4	5	6	7	8	9
	1→2	1→3	1→4	1→5	1→6				
			2→3	2→4	2→5	2→6			
					3→4	3→5	3→6		
							4→5	4→6	
									5→6

Figure 3. Illustrating the case of having 7 kids

round	1	2	3	4	5	6	7	8	9	10	11
	1→2	1→3	1→4	1→5	1→6	1→7					
			2→3	2→4	2→5	2→6	2→7				
					3→4	3→5	3→6	3→7			
							4→5	4→6	4→7		
									5→6	5→7	
											6→7

In Figure 2, round 5 is highlighted because that three kissing activities take place. We shall call 3 the maximal load for $n=6$. For $n=7$, the maximal load is 3 occurring at rounds 5, 6, and 7 as illustrated in Figure 3. Obviously at round $2n-3$, the load is 1, i.e., only kid $n-1$ is kissing kid n, which completes the process. Hence, for $n=35$, the process takes $2(35)-3=67$ rounds. So, by assuming that each kiss takes 5 seconds *a la* Harbaugh, our organized kissing will take 335 seconds (about 5.6 minutes) for $n=35$. Even if each kiss is assumed to take slightly under 9 seconds, then our organized kissing can still get the job done within the 10-minute class break.

Having learned at which round each kid makes the last kiss, we would like to find out the load at each round. This can be done by asking at which round each kid starts kissing. [See numbers in bold face in Figures 2 and 3.]

First, recall that kids start kissing at rounds that are odd numbers, 1, 3, 5, ..., and $2n-3$.

At $t=1=2(\mathbf{1})-1$, kid **1** starts kissing (and the first kid kissed is kid 2); load is 1.
At $t=3=2(\mathbf{2})-1$, kid **2** starts kissing; load is 2.
At $t=5=2(\mathbf{3})-1$, kid **3** starts kissing; load is 3.
At $t=7=2(\mathbf{4})-1$, kid **4** starts kissing; load is 4.
……
For any odd number $t \le n-1$, we see that at round t, kid $(t+1)/2$ starts kissing; load is $(t+1)/2$.

Suppose that n is even. We have seen that at $t=n-1$, kid $\mathbf{n/2}$ starts kissing and the load is $n/2$.
At $t=n+1=2[\mathbf{(n+2)/2}]-1$, kid $\mathbf{(n+2)/2}$ starts kissing and the load is $n/2-1$.
……
At $t=2n-3=2(\mathbf{n-1})-1$, kid $\mathbf{n-1}$ kisses kid n and nobody else; load is 1.
We conclude that for even number n, the maximal load is $n/2$, taking place at $t=n-1$.

Figure 4. Illustrating the case of n being odd

round	**…**	**n -2**	**n - 1**	**n**	**n + 1**	**n + 2**
		1→n-1	1→n			
		2→n-2	2→n-1	2→n		
		.	.	.	3→n	
		.	.	.	.	4→n
		(n-1)/2→(n-1)/2 + 1	(n-1)/2→(n-1)/2 + 2	.	.	.
				(n+1)/2→(n+1)/2 + 1	(n+1)/2→(n+1)/2 + 2	.
						(n+3)/2→(n+3)/2 + 1

Now suppose that n is odd, as illustrated in Figure 4.
At $t=n-2=2[\mathbf{(n-1)/2}]-1$, kid $\mathbf{(n-1)/2}$ starts kissing; load is $(n-1)/2$. (Note that kid 1 is kissing kid $n-1$ at the same time.)
At $t=n=2[\mathbf{(n+1)/2}]-1$, kid $\mathbf{(n+1)/2}$ starts kissing; load is $(n-1)/2$.
At $t=n+2=2[\mathbf{(n+3)/2}]-1$, kid $\mathbf{(n+3)/2}$ starts kissing; load is $(n-1)/2-1$.
We see from Figure 4 that for odd number n, the maximal load is $(n-1)/2$, taking place at rounds $n-2$, $n-1$, and n.

We now wrap up our findings and present them as a Proposition.

Proposition. For the method presented in the beginning of this section, the process is completed in $2n-3$ rounds; the maximal load is $n/2$ (occurring at $t=n-1$) if n is even; the maximal load is $(n-1)/2$ (occurring at $t=n-2$, $n-1$, and n) if n is odd.

Notice that the number of kisses needed, $C(n,2)$, is fast increasing as n

becomes larger. Yet in the scheme proposed above, the time required for completing this task is only linear in n, which is not bad as compared to the strictly convex case.

3. Merits in Teaching

Here, by playing with dots and array, we first answer the question "How many kisses will it take", next we propose a systematic way of organizing kissing activities for the sake of efficiency. For any natural number n (no less than 2), we are able to calculate exactly how long the process takes. Furthermore, we know what the maximal load is and when it occurs. Students enrolled in courses such as Operations Research, Quantitative Methods for Business, Mathematics for Economists or Mathematics in Management might find this problem-solving demonstration useful. We might remind them that little mathematics is used, let alone calculus.

Students in Probability and Statistics might also enjoy reading this pedagogical note as it goes beyond the formula $C(n, 2)$ to design a simple process for solving a real-world problem.

While teaching Money and Banking or Macroeconomics, instructors often elaborate how the barter economy works and how many exchange ratios are needed for an economy consisting of n consumption goods. Our scenario can be easily modified so as to demonstrate how to organize all n vendors (with the assumption that each commodity has only one vendor) to first bilaterally negotiate and then post the agreed exchange ratios between two distinct commodities (e.g., five oranges for one apple or the relative price of apple in reference to orange is 5).

Some years ago an instructor teaching a course related to Personnel Management or Public Relation conducted a bizarre midterm exam as follows. She asked more than 60 students (who barely knew each other as it was an elective upper-division course) to chat with all others and memorize important data such as date of birth, full name, major, career plan, and hobbies. She then asked (through repeated random drawing) to find out how students performed in this task. As the clock was ticking, students were not organized, pointlessly running around in a crowded classroom, with only panic and anger shown on their faces. As it is clear to us now, asking 60 kids to kiss each other without offering them some systematic procedure will lead to time-consuming and even chaos.

Last but not least, we surely can supplement lectures on Principles of Economics with this episode. Ask students to imagine how likely it will be for kid i planning to kiss kid j yet kid j might be heading toward kid k, where i, j and k are all distinct. In the absence of some organization, kids' search endeavor could easily generate negative externality. As Mankiw (2007) puts, one principle of economics says that governments can sometimes improve market outcomes, here a systematic way of organizing kissing can enhance the efficiency in the society formed by all kids in that class. It might also be a good timing to introduce the notion of transaction cost. Furthermore, without our systematic procedure, on one hand, how would any kid clearly remember whom she has not yet kissed? On the other hand, how would their teacher know that this social custom is carried out as expected? Instructors can move on to talk about ideas behind information asymmetry and bounded rationality alike.

4. Concluding Remarks

We are not claiming that the method presented in Section 2 is the most efficient (or effective) one. Any other systematic process that can accomplish the task in less than $2n-3$ rounds definitely proves to be more efficient than ours. In fact, for a specific small number n, one can set up the rules of engagement for kids to follow and complete the task in a shorter period of time. But generalization will be a tough task left behind.

Exactly how bad can the kissing activities be should there be no systematic procedure implemented for them to follow? A simple experiment was conducted for a group of 35 college freshmen at Wenzao Ursuline College of Languages (Taiwan) who heard about the puzzle last term and all knew each other well. For obvious reasons we slightly modified the social custom as follows. Each student was given a sheet containing 35 names and asked to immediately sign his/her name (with a circle going around the name for the purpose of marking). Under no suggested route, students then made contacts to bilaterally exchange signatures. That is, when student i met with another student, say j, i would sign her name on j's sheet; j would sign his name on i's sheet. Signatures were carefully cross-examined afterwards to assure that no forgery happened. They turned in sheets to the instructor as soon as 35 signatures were collected. Students were told in advance that various extra credits would be given depending on how soon the task was completed. Halfway through the process, the instructor walked around the extraordinarily noisy classroom and foreclosed three sheets because they were left unattended, possibly due to the intention of passing around sheets for quickly collecting signatures. Those three students, mixed with astonishing and worried feeling, did stay to provide signatures to others altruistically. Out of 32 students in this contest, the fastest one spent 15 minutes while the slowest one finished it in 19 minutes with the average being 17 minutes and 23 seconds. We believe that the process might take far more than 20 minutes if all 35 students proceeded exactly as instructed. By contrast, the aforementioned systematic process, under the assumption that that each bilateral exchange of signatures takes 10 seconds, demands only 11 minutes and 10 seconds.

One possible avenue for future research along this line but with much more complexity is about speed dating that offers singles the opportunity to meet a large number of potential mates over a short pre-determined period of time, which is common in many western countries including the United States and United Kingdom. After registration, people sit at the assigned table, men move around (after each date that lasts for 3 minutes) while women usually stay seated at the same table. Men have very little time to move to the next table to begin a new date. Knowing that most events do not have equal numbers of women and men, if we have many participants, there could be some better way other than only asking men to move around as described above. Organizing switching dates for the sake of time-saving and exposure-maximizing apparently differs from our organizing kissing in light of the nature of matching taking place between women and men.

The writing of this note was partially inspired by Becker (2007), arguing that students need to learn that the very nature of a science is to have unresolved topics and an on-going scrutiny of theories no matter how steeped they are in tradition. Nowadays the internet provides not only students but also instructors with up-to-date data, commentary and academics' views on the economy and current events. Occasionally, instructors can easily sample some, let students play with the real problem however trivial it might be, remind them of concepts scattered in textbooks as their hands get dirty, and soon guide them to be away from passive digesting.

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