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Wavelet Estimation of Asymmetric Hedge Ratios: Does Econometric Sophistication Boost Hedging Effectiveness?

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Abstract

This paper utilises wavelet analysis, which is becoming popular in economics and finance, to estimate the hedge ratios for spot positions on the West Texas Intermediate crude oil, soybeans and the S&P500 index. This technique is combined with a two-stage regime switching threshold model to estimate asymmetric hedge ratios corresponding to positive and negative returns on futures contracts. Other simple and sophisticated techniques are also used as a benchmark for the purpose of comparison, including the naïve model and the asymmetric error correction GJR-GARCH model. On the basis of the variance ratio test and variance reduction, it is revealed that econometric sophistication does not boost hedging effectiveness.

Key words: asymmetric hedge ratios; variance ratio; variance reduction; wavelets

JEL classification: G30; C22; C53

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1. Introduction

Financial hedging is the covering risk resulting from price changes by taking an opposite position on a hedging instrument, which is typically a derivative (such as a future or option). The hedging process consists of steps that are implied by the

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following questions. (i) Should we hedge a particular position or leave it uncovered? (ii) If the decision to hedge is taken, what hedging instrument should be used? (iii) Having selected the hedging instruments, how much of the position should be hedged? The third question, which is addressed in this paper, pertains to the determination of the hedge ratio.

A significant strand of literature is concerned with the methods used to estimate the hedge ratio and whether or not the estimation technique and/or the specification of the underlying model have implications for hedging effectiveness. Examples of differences in model specification are linear versus nonlinear models, static versus dynamic models, symmetric versus asymmetric models, first difference versus level models, and first difference versus error correction models. Estimation methods refer to procedures such as ordinary least squares (OLS), maximum likelihood, Kalman filters, and methods for models with ARCH/GARCH errors. The same model specification may produce different estimates for the hedge ratio when different estimation methods are used.

This paper extends the literature on the estimation of the hedge ratio by using wavelet analysis and a two-stage regime switching process to estimate the optimal hedge ratio using futures contracts. Wavelet analysis is becoming popular in economics and finance, as it has been used to study a number of issues. The technique has been used, for example, to study stock returns and economic activity (Gallegati, 2008), the relation between financial variables and real economic activity (Kim and In, 2003), and the decomposition of the relation between money and income (Ramsey and Lampart, 1998). Other applications can be found in Gencay et al. (2001) who describe the use of wavelets and other filtering methods in finance and economics.

Wavelet analysis allows the use of long time intervals when more precise low-frequency information is needed and short time intervals when more precise high-frequency information is needed. Using a particular filter, a time series is transformed into multiple series consisting of wavelet coefficients and to a single series consisting of scaling coefficients. The scaling coefficients reflect long-term variation, which would exhibit a similar trend to the original series. To our knowledge, this is the first paper in which wavelet analysis is employed to estimate asymmetric hedge ratios using daily data. Both symmetric and asymmetric models are used to estimate the hedge ratio by wavelet analysis. The results obtained by applying wavelet analysis are then compared with those obtained from other estimation methods and models, such as the naïve model and the EC-GJR-GARCH method. Another advantage of wavelet decomposition is that high-frequency data can be used to create the wavelet detail that broadly matches the hedge horizon. This procedure solves the problem of small sample size resulting from attempts to match data frequency with the hedge horizon.

More specifically, this paper investigates three issues. The first issue is whether or not the specification of the model and/or the estimation method used to calculate the hedge ratio has a significant effect on risk reduction and hence hedging effectiveness. The second issue is whether or not the calculation of separate hedge ratios for rising and falling markets leads to improvement in the performance of the hedge. As part of this exercise we conduct formal testing of the equality of the separate hedge ratios. The third issue is an investigation into the relation between hedge horizon and data frequency. Chen et al. (2003, 2004) suggest that the data frequency should be matched with the hedge horizon. When hedging over long horizons, this procedure results in a substantial decrease in sample size. Wavelet analysis does not suffer from this limitation, because high-frequency (e.g., daily) data can be used to uncover long-run dynamics. Shedding some light on these issues should be rewarding, not only in terms of the academic value added to the literature but also in terms of the practical implications of the results, given that risk management commands increasing importance as a corporate activity.

The paper is organised as follows. Following the presentation of a brief literature survey, a description of wavelet analysis is presented in general terms. This is followed by an exposition of the two-stage regime switching threshold model and how wavelet analysis is applied to the measurement of the hedge ratio. Then we will describe a dataset and present empirical results. The paper ends with concluding remarks.

2. A Look at the Literature

The literature on estimating the hedge ratio is extensive (e.g., Moosa, 2003a, Ch. 5; Chen et al., 2003). The early literature focused on the estimation of the hedge ratio by applying OLS to a first difference model in which the response variable is the rate of return on the unhedged position and the predictor variable is the rate of return on the hedging instrument (e.g., Ederington, 1979). This model has been criticised as being misspecified, consequently leading to erroneous hedging decisions. For example, Lien (1996) has shown analytically that if the prices of the asset underlying the hedged position and that of the hedging instrument are cointegrated, the optimal hedge ratio must be estimated from an error correction model rather than a straight first difference model. Lien concludes that a hedger acting on the hedge rate estimated from an error correction model will be underhedged. However, Moosa (2003b) has shown empirically that the estimated hedge ratio does not appear to be sensitive to model specification. Specifically, he points out that while Lien's analytical results are sound, the quantitative difference between the hedge ratios derived from a first difference model and an error correction model is negligible to the extent that there are hardly any practical ramifications.

Another issue that often arises in the hedging literature pertains to the assumption of a constant hedge ratio, as some economists argue for time-varying hedge ratios derived from ARCH/GARCH models or state space models (e.g., Kroner and Sultan, 1993). Yet another modification to the basic model used to calculate the hedge ratio is the introduction of nonlinearity, as suggested by Broll et al. (2001). Chen et al. (2003) provide a review of different theoretical approaches to the estimation of the optimal hedge ratio, based on minimum variance, mean-variance, expected utility, mean extended-Gini coefficient, and semivariance. They also discuss various ways of estimating the hedge ratio, ranging from OLS to complicated heteroscedastic cointegration methods.

An important issue that is related to nonlinearity (and also to time-varying hedge ratios), which is not dealt with extensively in the literature, is that of asymmetry. If the restrictive assumption of risk aversion is relaxed, then hedgers would aim at maximising utility (rather than minimising risk), in which case both risk and return will be taken into account. In a bull market, hedgers may reduce the hedge ratio in a rising market to benefit from any rise in prices. Conversely, they tend to use a higher hedge ratio in a declining market to protect themselves from adverse market moves. This proposition is supported by the notion of "one-sided risk." For example, Adams and Montesi (1995) found that corporate managers are mostly concerned with downside risk. Petty and Scott (1981) suggested that many Fortune 500 firms identify risk as the possibility of falling below a target return.

But even if a risk-minimising hedge ratio is chosen, there is no reason to assume a priori that this ratio has the same value in a rising market as in a falling market. Symmetric hedge ratios would be obtained only if the rate of return on the unhedged position reacts in a similar manner to a rise as to a fall in the rate of return on the hedging instrument. Given that the optimal hedge ratio depends on correlation between the rates of return on the unhedged position and that on the hedging instrument, correlation asymmetry would necessarily imply hedge ratio asymmetry. Correlation asymmetry means that correlation between two rates of return in a bull market is different from what is found in a bear market. This is important for the hedging decision because unstable correlation makes it difficult to hedge exposure by taking an offsetting position on other assets. From the risk manager's point of view, knowledge of the particular pattern of correlation under different market conditions is a major concern. This is because the effectiveness of hedging based on estimated correlation depends on how accurate the estimate is, and how representative the estimation period is, of the period when hedging is most needed. Failure to take correlation asymmetry into consideration may lead to suboptimal positions for the hedger. Demirer and Charnes (2003) argue that analysis of correlation asymmetry is even more important for risk managers who are concerned with downside risk. For a hedger who aims to avoid falling beyond a target rate of return, failing to take higher downside correlation into account leads to lower than optimal hedge positions, thus reducing the effectiveness of the hedge.

Recent research shows that correlation asymmetry does exist in financial markets, specifically that correlation is stronger in bear markets than in bull markets (e.g., Bookstaber, 1997; Loretan and English, 2000; Bekaert and Wu, 2000; Ang and Bekaert, 2000; Ang and Chen; 2002). These studies invariably deal with asymmetric correlation between individual stocks and the market as a whole. Only a few studies have extended this literature to the derivatives market and hence to the hedging problem. Brooks et al. (2002) analysed the impact of asymmetry on time-varying hedge ratios involving daily FTSE 100 stock index and stock index futures contracts over the period 1985 to 1999. They concluded that a model allowing for time variation and asymmetry gives superior hedging performance. Demirer and Charnes (2003) studied the correlation structure of spot and futures returns across 10 markets and analysed the implications of asymmetric correlation for optimal hedge positions. For stock index and oil futures contracts, they found correlation between spot and futures returns to be much greater on the downside than on the upside, particularly for extreme moves. Their analysis also indicates that hedge ratios that minimise downside risk yield generally better results than the traditional minimum variance hedge ratios.

Chen et al. (2003, 2004) argue that data frequency should be matched with the hedge horizon. For example a hedge horizon of one week should employ weekly data to estimate hedge ratios. The matching of data frequency with the hedge horizon clearly becomes problematical when hedging over (say) a three-month horizon, because of the substantial reduction in sample size. One approach to dealing with this problem is to create overlapping datasets while using sophisticated modelling approaches to correct for the effect of overlapping data on the estimators. Chen et al. (2004) employ an alternative approach whereby short-run and long-run hedge ratios are estimated. However, the estimation of short-run hedge ratios (hedging over a period ranging between one week and eight weeks) still suffers from the need to match data frequency with the hedge horizon, resulting in smaller sample size. This problem motivates the use of wavelet decomposition, which is employed in this paper. High-frequency (e.g., daily) data can be used to create the wavelet detail that broadly matches the hedge horizon.

Having gone through a brief literature review, it seems that there is some room for extending the literature by using the technique of wavelet analysis to estimate the hedge ratio. This is particularly so in view of the importance of the issue of the sensitivity of the results, and therefore hedging effectiveness, to the underlying econometrics. The next step is to present a description of wavelet analysis.

3. Wavelet Analysis

Wavelet analysis is a technique that can be applied to any (e.g., stationary or nonstationary, linear or nonlinear) time series. It is a windowing technique with variable size regions that allows the use of long time intervals when more precise low-frequency information is needed, and short time intervals when more precise high-frequency information is needed. If, for example, we want to examine the behaviour of daily time series over different time periods (such as weeks or months), temporal aggregation would result in the loss of useful information. With wavelet analysis, this can be done without aggregation and hence without any loss of information (Gencay et al., 2003). In what follows, a brief description of wavelet analysis is presented (see Percival and Walden (2000) for more details).

Given a signal represented by $\{x(t): -\infty < t < \infty\}$, the collection of coefficients $\{W(\lambda,t): \lambda > 0, -\infty < t < \infty\}$ is known as the continuous wavelet transform of $x(\cdot)$ such that:

$$W(\lambda,t)=\int_{-\infty}^{\infty}\psi_{\lambda,t}(u)\;x(u)\;du \tag{1}$$

and

$$\psi_{\lambda,t}(u)\equiv\frac{1}{\sqrt{\lambda}}\psi\left(\frac{u-t}{\lambda}\right), \tag{2}$$

where λ is the scale associated with the transformation and t is its location. The function $\psi(\cdot)$ is a wavelet filter that satisfies the properties:

$$\int_{-\infty}^{\infty}\psi(u)\;du=0 \tag{3}$$

and

$$\int_{-\infty}^{\infty}\psi^2(u)\;du=1\,. \tag{4}$$

It is admissible if its Fourier transform:

$$\Psi(\omega)\equiv\int_{-\infty}^{\infty}\psi(u)\;e^{-i\omega u}\;d\omega \tag{5}$$

is such that

$$0\;<\int_{-\infty}^{\infty}\frac{\left|\Psi(\omega)\right|^2}{\omega}\;d\omega<\infty\,. \tag{6}$$

Many types of wavelet filters are available, the most commonly used of which are the Haar and Daubechies filters (Percival and Walden, 2000, Ch. 4).

By applying wavelet analysis to time series observed over discrete points in time ($t=1,\ldots,T$), we would be interested in the discrete wavelet transform (DWT), which can be thought of as a sensible sub-sampling of $W(\lambda,t)$, in which we deal with "dyadic" scales (Percival and Walden, 2000, Ch. 1). This means that we need to pick λ_j to be of the form 2^{j-1}, $j=1,\ldots,J$ and then within a given dyadic scale 2^{j-1}, we pick points in time that are separated by multiples of 2^j. For the scale $\lambda_j=2^{j-1}$, there are $T_j=T/2^j$ wavelet coefficients that can be defined. For example, if $T=256=2^8$, eight dyadic scales $2^0,\ldots,2^7$ will be available at which there are 128, 64, 32, 16, 8, 4, 2 and 1 wavelet coefficients respectively. Hence, the wavelet coefficients for the eight scales account for the DWT coefficients, the number of which is equal to one less the length of the time series. The single remaining coefficient (which, together with the total number of DWT coefficients, is equal to the length of the time series) is known as the scaling coefficient. In practice, we may choose to decompose a time series using a fewer number of scales, depending on the length of the series. For example, if we pick 5 scales $2^0,\ldots,2^4$, then the wavelet coefficients corresponding to these scales would be 128, 64, 32, 16 and 8 respectively, whereas the eight remaining coefficients would be the scaling coefficients.

The wavelet coefficients are proportional to the differences in averages of the time series observations at each scale, whereas the scaling coefficients are proportional to the averages of the original series over the largest scale. The scaling coefficients reflect long-term variations, which would exhibit a similar trend to the original series. Long time scales give more low-frequency information about the series, whereas short time scales give more high frequency information about the time series. The DWT re-expresses a time series in terms of coefficients that are associated with a particular time and particular dyadic scale. These coefficients are fully equivalent to the information contained in the original series in that a time series can be perfectly reconstructed from its DWT coefficients.

The reconstruction of a time series X from its wavelet and scaling coefficients defines what is known as a multi-resolution analysis (MRA) of X (Percival and Walden, 2000, Ch. 4). The non-decimated wavelet transformation, or the stationary wavelet transform (MODWT), is a modification of the DWT in that it deals with all points in time and not just multiples of 2^j. For example, if a series of length $T = 256$ is considered, there would be 256 wavelet coefficients at each scale. Retaining all possible times at each scale of the MODWT decomposition has the advantage of retaining the time invariant property of the original series.

In general, the wavelet coefficients at scale $\lambda_{i\neg}$ are associated with frequencies in the interval $[2^{-(j+1)}, 2^{-j}]$. Hence, the wavelet coefficients at the first scale, $\lambda_{1\neg}$, are associated with frequencies in the interval $[2^{-2}, 2^{-1}]$, whereas the coefficients at the second scale λ_2 are associated with frequencies in the interval $[2^{-3}, 2^{-2}]$, and so on. If the time series under consideration are daily, then the first scale captures the dynamics of the time series within a 2- to 4-day period, the second scale captures the dynamics of the time series within a 4- to 8-day period, and so on. Here λ_1, which is the shortest time scale, contains the highest frequency information about the time series. The level of information of the time series conveyed from one scale to the next decreases as the time scale increases.

4. The Threshold Model and Hedge Ratios

In this section we present the specification of the models used to estimate symmetric and asymmetric hedge ratios, applying these models to total series and the wavelet transformation of the return series. The simplest model used to calculate the hedge ratio is the first difference model:

$$\Delta s_t = \alpha_0 + h\Delta f_t + u_t , \tag{7}$$

where s and f are the logs of the spot and futures prices and h is the risk-minimising hedge ratio. A limitation of (7) is that it does not capture the asymmetric nature of the response of s to f, which makes it invalid if the response of s to f depends on whether f is rising or falling. In order to capture these asymmetric effects, Δf is decomposed as follows:

$$\Delta f_t^+ = \begin{cases} \Delta f_t & \text{if } \Delta f_t \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

$$\Delta f_t^- = \begin{cases} \Delta f_t & \text{if } \Delta f_t \leq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Equation (7) can therefore be rewritten as:

$$\Delta s_t = \alpha_0 + h^+ \Delta f_t^+ + h^- \Delta f_t^- + u_t . \tag{10}$$

The asymmetric hedge ratio from t to $t+1$ requires the hedger to form expectation at time t of the direction of the futures price change from time t to $t+1$. An expectation of a positive (negative) futures price change implies the implementation of h^+ (h^-). Here, we make the assumption that the hedger is able to predict accurately the direction, but not the magnitude, of the futures price change each period. Given that returns are largely unpredictable, it is acknowledged that this assumption may be unrealistic and that its adoption produces overstatement of the risk reduction resulting from hedging. If the proposed approach (assuming perfect foresight) produces superior risk reduction to some of the alternative approaches used in the literature, this assumption is relaxed in the second stage of the analysis where a model is used to predict the direction of price changes. If, however, the proposed approach does not produce superior risk reduction, then it can be concluded that even the best case scenario fails to deliver improvement in hedging effectiveness relative to existing approaches.

Hedging effectiveness is measured by the variance ratio (VR) and variance reduction (VD). In the absence of asymmetry, the variance ratio is calculated as

$$VR = \frac{\sigma^2(\Delta s_t)}{\sigma^2(\Delta s_t - h\Delta f_t)}, \tag{11}$$

in which case the null hypothesis $\sigma^2(\Delta s_t) = \sigma^2(\Delta s_t - h\Delta f_t)$ is rejected if $VR > F(n-1, n-1)$, where n is the number of observations. If there are asymmetric ratios, the VR is calculated as:

$$VR = \frac{\sigma^2(\Delta s_t)}{\sigma^2(\Delta s_t - I^+ h^+ \Delta f_t^+ - I^- h^- \Delta f_t^-)}, \tag{12}$$

where $I^+ = 1$ if $\Delta f_t \geq 0$, $I^+ = 0$ if $\Delta f_t \leq 0$, $I^- = 1$ if $\Delta f_t \leq 0$, and $I^- = 0$ if $\Delta f_t \geq 0$. Variance reduction can be calculated from the variance ratio as:

$$VD = 1 - \frac{1}{VR}\ . \qquad (13)$$

In this study we decompose spot and futures returns into wavelet detail series at different resolutions using the MODWT. The symmetric model represented by (7) is estimated for returns and wavelet detail series. As such the estimated slope coefficient is $h = \mathrm{cov}(\Delta s^*, \Delta f^*)/\mathrm{var}(\Delta f^*)$, where Δs^* and Δf^* are the appropriate wavelet detail series. To obtain the corresponding asymmetric model (10), positive and negative futures returns and positive and negative wavelet detail values are separated at each level. Likewise, the asymmetric h^+ and h^- coefficients are obtained for each wavelet scale. A test for symmetry will then be conducted.

5. Data and Empirical Results

The empirical results are derived from the analysis of three sets of daily data.

1. The NYMEX futures price for the near-month contract on light, sweet crude oil and the spot price of the West Texas Intermediate (WTI) crude, which is the primary deliverable crude oil against the NYMEX contract. Futures prices were obtained from NYMEX, whereas spot prices were obtained from the US EIA database. Daily data are used in this analysis covering the period 7 June 1989 to 12 October 2005 (a total of 4096 observations).
2. A dataset on soybeans spot and futures prices that consists of 2048 daily observations from 26 April 1999 to 28 February 2007. Both series were obtained from Datastream.
3. A dataset that consists of 4096 daily observations on the S&P500 index and futures prices, covering the period from 5 January 1988 to 19 March 2004. The index data were obtained from IRESS, whereas the floor settlement price for futures contracts was obtained from the Chicago Mercantile Exchange.

The splicing of futures contracts to form a continuous time series is an issue that arises in all studies of this kind. When modelling spot and futures dynamics, any jump on rollover is commonly ignored (e.g., Koutmos and Tucker, 1996; Baillie and Myers, 1991; Bhar, 2001). Furthermore, visual examination of the data reveals no significant jumps in the futures return series on rollover. The splicing of crude oil data is based on the procedure used by Ripple and Moosa (2007), which is based on the use of price and associated trading activity. As a contract approaches maturity, the market shifts attention away from the near-month contract to the next-to-near-month contract before the near-month contract reaches its last trading day. When both the daily trading volume and open interest for the next-to-near month contract exceed those for the near-month contract, this is taken as evidence that the market's attention has shifted away from the near-month contract. At this point, the series is shifted to the next-to-near month contract. For the S&P500 series, a rollover of 10 trading days prior to expiry is used, whereas a 20-trading day rollover is employed for the soybeans data.

Table 1 shows descriptive statistics of returns measured as the first log differences of prices. For each dataset, the sample means of the series are close to 0. The return distributions are non-normal, exhibiting excess kurtosis and negative skewness. Non-normality is implied by the high values of the Jarque-Bera statistic.

Table 1. Descriptive Statistics of Daily Futures and Spot Return Series

Return Series	Mean	Standard Deviation	Excess Kurtosis	Skewness	Jarque-Bera Statistic
Panel A: Crude Oil					
Δf_t	0.0003	0.0240	13.9807	−0.9510	33967.51
Δs_t	0.0003	0.0251	13.9622	−0.8818	33792.84
Panel B: Soybeans					
Return Series	Mean	Standard Deviation	Excess Kurtosis	Skewness	Jarque-Bera Statistic
Δf_t	0.0002	0.0149	3.4206	−0.2672	1022.84
Δs_t	0.0002	0.0159	9.7320	−0.9196	8370.78
Panel C: S&P500					
Return Series	Mean	Standard Deviation	Excess Kurtosis	Skewness	Jarque-Bera Statistic
Δf_t	0.0004	0.0110	5.5051	−0.3855	5273.73
Δs_t	0.0004	0.0104	4.1557	−0.2268	2982.49

Notes: Δf_t and Δs_t are futures and spot returns respectively.

For crude oil and S&P500 datasets, wavelet decomposition results in an 11-scale MRA because the length of the daily series is equal to $4096 = 2^{12}$. For the soybean dataset, wavelet decomposition produces a 10-scale MRA because the length of the daily series is equal to $2048 = 2^{11}$. In practice, however, we do not need to consider all possible scales in the decomposition because larger scales result in very smooth detail series, thus failing to provide much useful information about the original time series. Consequently, we use an 8-scale decomposition for each dataset and the least asymmetric wavelet filter of length 8 to decompose the series. The least asymmetric filter is a variation of the Daubechies filter, which has been chosen because it has good alignment properties (see Percival and Walden, 2000). The results presented in Tables 2 and 3 cover the return series (without wavelet decomposition) and 8 wavelet detail series where wavelet decomposition is performed on both the spot and future return series of each dataset.

Table 2 displays the results of estimating asymmetric and symmetric models over the whole sample period for daily returns and for the wavelet detail series. The hedge coefficients for returns are obtained by estimating (7) for the symmetric model and (10) for the asymmetric model. The results include the estimated hedge ratios (two in the case of the asymmetric model) and the associated p-values for judging their statistical significance, as well as the p-value for testing the null of symmetry (i.e., $h^{+} = h^{-}$). Also reported are the variance ratio (VR) and variance reduction (VD) to assess the effectiveness of the hedge, using the hedge ratios derived from asymmetric and symmetric models. Starting with the asymmetric model, the estimated hedge ratios are significant and very close to 1. The null hypothesis of symmetry is rejected only in the cases of detail 7 for the crude oil data, detail 5 for the soybeans data, and for the daily return series of the S&P500 data. The variance ratio is significant in all cases, producing variance reduction of over 75% for the crude oil data, over 70% for the soybeans data, and over 90% for the S&P500 data. For all three datasets, very close results are produced by the symmetric model for the estimated hedge ratios, variance ratios and variance reductions. One would tend to think that if there is no evidence for asymmetry, then the asymmetric and symmetric models would produce similar results in terms of hedging effectiveness. What is more important perhaps is that the hedging effectiveness produced by a simple OLS estimation of the hedge ratio (daily return series) is not significantly different from what is produced by using wavelet decomposition.

Table 2. Hedge Ratio Estimates and Other Measures for Daily Return Series and Wavelet Decomposition: Total Sample

Panel A: Crude Oil									
Scale of Wavelet Decomposition (j)		1	2	3	4	5	6	7	8
Dynamics in days		2–4	4–8	8–16	16–32	32–64	64–128	128–256	256–512
	Daily Returns Series	Detail 1	Detail 2	Detail 3	Detail 4	Detail 5	Detail 6	Detail 7	Detail 8
Asymmetric Model									
Hedge Ratio h^+	0.939	0.911	0.886	0.956	0.990	1.007	1.004	1.014	1.013
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Hedge Ratio h^-	0.902	0.898	0.891	0.962	0.980	1.005	1.001	1.011	1.013
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Test for Symmetry									
p-value	0.095	0.612	0.826	0.715	0.437	0.601	0.169	0.016	0.610
VR	4.323	4.320	4.302	4.273	4.274	4.202	4.216	4.182	4.179
VD	0.769	0.768	0.768	0.766	0.766	0.762	0.763	0.761	0.761
Symmetric Model									
Hedge Ratio	0.919	0.897	0.882	0.975	0.977	1.007	1.001	1.013	1.014
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
VR	4.321	4.32	4.305	4.275	4.271	4.2	4.216	4.182	4.179
VD	0.769	0.768	0.768	0.766	0.766	0.762	0.763	0.761	0.761

Notes: *VR* and *VD* are the variance ratio and variance reduction respectively. h^+ and h^- are hedge ratios corresponding to positive and negative returns as in (10).

Table 2. (Continued)

Panel B: Soybeans									
Scale of Wavelet Decomposition (j)		1	2	3	4	5	6	7	8
Dynamics in days		2–4	4–8	8–16	16–32	32–64	64–128	128–256	256–512
	Daily Return Series	Detail 1	Detail 2	Detail 3	Detail 4	Detail 5	Detail 6	Detail 7	Detail 8
Asymmetric Model									
Hedge Ratio h^+	0.929	0.924	0.944	0.899	0.873	1.074	0.940	0.978	1.036
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Hedge Ratio h^-	0.902	0.906	0.882	0.913	0.898	1.116	0.952	1.009	1.033
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Test for Symmetry									
p-value	0.442	0.662	0.132	0.653	0.456	0.027	0.565	0.169	0.586
VR	3.664	3.637	3.666	3.637	3.656	3.272	3.662	3.662	3.555
VD	0.727	0.725	0.727	0.725	0.726	0.694	0.727	0.727	0.719
Symmetric Model									
Hedge Ratio	0.915	0.915	0.913	0.906	0.885	1.094	0.946	0.995	1.034
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
VR	3.664	3.644	3.664	3.638	3.637	3.246	3.664	3.664	3.566
VD	0.727	0.726	0.727	0.725	0.725	0.692	0.727	0.727	0.720

Table 2. (Continued)

Panel C: S&P500									
Scale of Wavelet Decomposition (j)		1	2	3	4	5	6	7	8
Dynamics in days		2–4	4–8	8–16	16–32	32–64	64–128	128–256	256–512
	Daily Return Series	Detail 1	Detail 2	Detail 3	Detail 4	Detail 5	Detail 6	Detail 7	Detail 8
Asymmetric Model									
Hedge Ratio h^+	0.997	0.999	1.036	1.022	1.017	1.002	1.007	1.006	1.003
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Hedge Ratio h^-	1.051	1.026	1.042	1.025	1.027	1.009	1.011	1.007	1.003
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Test for Symmetry									
p-value	0.000	0.098	0.599	0.744	0.147	0.130	0.430	0.719	0.713
VR	10.478	9.720	11.105	12.083	12.096	12.635	12.322	12.750	13.079
VD	0.905	0.897	0.910	0.917	0.917	0.921	0.919	0.922	0.924
Symmetric Model									
Hedge Ratio	1.024	1.012	1.039	1.023	1.022	1.006	1.009	1.006	1.003
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
VR	12.146	11.058	11.357	12.663	11.739	12.612	11.747	12.463	12.986
VD	0.918	0.910	0.912	0.921	0.915	0.921	0.915	0.920	0.923

Table 3. In-Sample and Out-of-Sample Hedge Ratio Estimates and Other Measures for Daily Return Series and Wavelet Decomposition

Panel A: Crude Oil									
Scale of Wavelet Decomposition (j)		1	2	3	4	5	6	7	8
Dynamics in days		2–4	4–8	8–16	16–32	32–64	64–128	128–256	256–512
	Daily Returns Series	Detail 1	Detail 2	Detail 3	Detail 4	Detail 5	Detail 6	Detail 7	Detail 8
Asymmetric Model									
Hedge Ratio h^+	0.931	0.897	0.876	0.973	0.986	1.011	1.000	1.015	1.014
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Hedge Ratio h^-	0.897	0.897	0.887	0.978	0.969	1.003	1.002	1.012	1.014
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Test for Symmetry p-value	0.024	0.997	0.669	0.825	0.311	0.244	0.628	0.041	0.944
VR (in-sample)	4.354	4.351	4.337	4.287	4.287	4.208	4.224	4.186	4.184
VR (out-of-sample)	4.268	4.262	4.240	4.246	4.248	4.189	4.200	4.172	4.170
VD (in-sample)	0.770	0.770	0.769	0.767	0.767	0.762	0.763	0.761	0.761
VD (out-of-sample)	0.766	0.765	0.764	0.764	0.765	0.761	0.762	0.760	0.760
Symmetric Model									
Hedge Ratio	0.919	0.897	0.882	0.975	0.977	1.007	1.001	1.013	1.014
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
VR (in-sample)	4.352	4.351	4.339	4.289	4.285	4.206	4.224	4.187	4.184
VR (out-of-sample)	4.264	4.262	4.242	4.248	4.245	4.187	4.201	4.172	4.170
VD (in-sample)	0.770	0.770	0.770	0.767	0.767	0.762	0.763	0.761	0.761
VD (out-of-sample)	0.765	0.765	0.764	0.765	0.764	0.761	0.762	0.760	0.760

Notes: VR and VD are the variance ratio and variance reduction respectively. h^+ and h^- are hedge ratios corresponding to positive and negative returns as in (10).

It is arguable that hedging effectiveness should be assessed on an out-of-sample basis. For this reason, we split the samples into two thirds for the estimation period and one third for the testing period as follows.

1. Crude oil: the estimation period is 7 June 1989 to 25 April 2000 whereas the testing period is 26 April 2000 to 12 October 2005.
2. Soybeans: the estimation period is 26 April 1999 to 16 July 2004 whereas the testing period is 17 July 2004 to 28 February 2007.
3. S&P500: the estimation period is 5 January 1988 to 19 October 1998 whereas the testing period is 20 October 1998 to 19 March 2004.

Table 3. (Continued)

Panel B: Soybeans									
Scale of Wavelet Decomposition (j)		1	2	3	4	5	6	7	8
Dynamics in days		2–4	4–8	8–16	16–32	32–64	64–128	128–256	256–512
	Daily Return Series	Detail 1	Detail 2	Detail 3	Detail 4	Detail 5	Detail 6	Detail 7	Detail 8
Asymmetric Model									
Hedge Ratio h^+	0.912	0.881	0.914	0.964	0.848	1.042	0.916	0.925	1.019
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Hedge Ratio h^-	0.895	0.867	0.918	0.962	0.885	1.182	0.912	0.914	0.997
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Test for Symmetry p-value	0.593	0.754	0.922	0.938	0.368	0.000	0.889	0.569	0.007
VR (in-sample)	5.796	5.764	5.796	5.714	5.801	4.961	5.789	5.787	5.534
VR (out-of-sample)	3.066	3.038	3.070	3.065	3.050	2.870	3.067	3.068	3.027
VD (in-sample)	0.827	0.827	0.827	0.825	0.828	0.798	0.827	0.827	0.819
VD (out-of-sample)	0.674	0.671	0.674	0.674	0.672	0.652	0.674	0.674	0.670
Symmetric Model									
Hedge Ratio	0.915	0.874	0.916	0.963	0.866	1.116	0.914	0.919	1.008
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
VR (in-sample)	5.792	5.775	5.792	5.716	5.764	4.915	5.793	5.790	5.555
VR (out-of-sample)	3.069	3.044	3.069	3.066	3.037	2.851	3.068	3.070	3.034
VD (in-sample)	0.827	0.827	0.827	0.825	0.827	0.797	0.827	0.827	0.820
VD (out-of-sample)	0.674	0.671	0.674	0.674	0.671	0.649	0.674	0.674	0.670

This sample split enables us to measure the hedging effectiveness in-sample and out-of-sample. We implement out-of-sample hedging as follows.

1. Hedge ratios are estimated using the in-sample data. These hedge ratios then remain constant throughout the out-of-sample period (i.e., they are not updated each period by adding an additional observation to the estimation period).
2. At time t (the first out-of-sample period), the hedger determines the likely direction of the futures price change from time t to time $t+1$. As stated above, we assume the best case scenario in the first stage, where the hedger correctly predicts the direction each period.
3. The hedger takes the appropriate position in the futures contracts, then the portfolio return from t to $t+1$ is calculated.
4. At $t+1$, the hedger determines the direction of the futures price change and if necessary updates the futures position, then the portfolio return is calculated.

This one-step-ahead procedure is performed through the end of the out-of-sample period, where total portfolio returns can be calculated.

Table 3. (Continued)

Panel C: S&P500									
Scale of Wavelet Decomposition (j)		1	2	3	4	5	6	7	8
Dynamics in days		2–4	4–8	8–16	16–32	32–64	64–128	128–256	256–512
	Daily Return Series	Detail 1	Detail 2	Detail 3	Detail 4	Detail 5	Detail 6	Detail 7	Detail 8
Asymmetric Model									
Hedge Ratio h^+	1.032	1.079	1.044	1.027	1.024	1.003	1.022	1.010	0.997
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Hedge Ratio h^-	1.095	1.087	1.058	1.025	1.028	1.019	1.017	1.006	1.000
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Test for Symmetry									
p-value	0.000	0.695	0.339	0.866	0.627	0.044	0.514	0.003	0.220
VR (in-sample)	8.076	7.466	8.603	9.455	9.468	9.976	9.675	10.081	10.409
VR (out-of-sample)	14.238	13.267	14.953	16.021	16.032	16.530	16.256	16.647	16.915
VD (in-sample)	0.876	0.866	0.884	0.894	0.894	0.900	0.897	0.901	0.904
VD (out-of-sample)	0.930	0.925	0.933	0.938	0.938	0.940	0.938	0.940	0.941
Symmetric Model									
Hedge Ratio	1.024	1.052	1.045	1.010	1.035	1.012	1.035	1.016	1.001
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
VR (in-sample)	9.513	8.560	8.814	9.997	9.147	9.948	9.155	9.807	10.315
VR (out-of-sample)	16.083	14.909	15.248	16.570	15.665	16.524	15.674	16.388	16.843
VD (in-sample)	0.938	0.933	0.934	0.940	0.936	0.939	0.936	0.939	0.941
VD (out-of-sample)	0.918	0.910	0.912	0.921	0.915	0.921	0.915	0.920	0.923

The split-sample results are reported in Table 3 for the asymmetric and symmetric models and for the daily return series and the 8 wavelet details. Evidence for asymmetry was found in the daily return and detail 7 series for the crude oil data, in details 5 and 7 series for the soybean data, and in the daily return and details 5 and 7 series for the S&P500 data. But while there is some evidence of asymmetry, we find that hedging effectiveness is not affected by using the hedge ratios estimated from the symmetric or asymmetric models. Furthermore, hedging effectiveness is generally not improved by using wavelet decomposition irrespective of whether we use in-sample or out-of-sample analysis. The exception is the (in-sample) soybeans dataset, where the wavelet approach results in the best variance reduction. The results of Table 2 are confirmed by the results reported in Table 3.

The results also largely support the empirical evidence produced by Chen et al. (2004), which demonstrates that as data frequency is reduced (from, say, daily to quarterly), the hedge ratio approaches unity, which is the value implied by the naïve model. This result is consistent with the hedge ratios for crude oil and soybeans, which approach unity as the scale increases. Chen et al. (2004) also demonstrate that hedging effectiveness increases with hedge horizon. The results presented in this paper, however, are not supportive of this finding.

It seems, therefore, that the econometric sophistication introduced by using wavelet analysis does not lead to any improvement in hedging effectiveness. One possible reason for the poor performance of the wavelet hedge ratios is their inability to match the hedge horizon to a wavelet detail series. If implementing a hedge over the course of a week, should one employ detail 1 (2 to 4 days) or detail 2 (4 to 8 days)? The proposed approach, therefore, involves a trade-off between not having to reduce the sample (particularly for long-term hedges) and the inability of the procedure to match exactly the hedge horizon with an appropriate scale.

Table 4. In-Sample and Out-of-Sample Results for Other Methods

	Naive	OLS	EC-GARCH (Symmetric)	EC-GJR-GARCH (Asymmetric)
Panel A: Crude oil				
VR (in-sample)	4.2245	4.3539	4.9532	4.9097
VR (out-of-sample)	4.2079	4.2790	4.6192	4.5611
VD (in-sample)	0.7633	0.7703	0.7981	0.7963
VD (out-of sample)	0.7624	0.7663	0.7835	0.7808
Panel B: Soybeans				
VR (in-sample)	4.0787	4.2288	4.7591	4.7539
VR (out-of-sample)	3.0391	3.0619	3.3679	3.3673
VD (in-sample)	0.7548	0.7635	0.7899	0.7897
VD (out-of sample)	0.6709	0.6734	0.7031	0.7030
Panel C: S&P500				
VR (in-sample)	10.365	13.296	14.218	14.019
VR (out-of-sample)	16.880	15.456	18.421	17.883
VD (in-sample)	0.9035	0.9248	0.9297	0.9287
VD (out-of sample)	0.9408	0.9353	0.9457	0.9441

Notes: *VR* and *VD* are the variance ratio and variance reduction respectively.

The question that arises here is whether or not hedging effectiveness can be improved by introducing another kind of econometric sophistication. For this reason, we measure *VR* and *VD* based on hedge ratios estimated using symmetric EC-GARCH and asymmetric EC-GJR-GARCH models. The in-sample and out-of-sample results are reported in Table 4, which also includes the results of using the naïve model and OLS (corresponding to the total series in Tables 2 and 3). Again, there is no evidence showing that more sophisticated approaches boost hedging effectiveness significantly. Consider, for example, the out-of-sample variance reduction for crude oil: it is 0.76 for the naïve model, 0.77 for the OLS model, 0.77 for the wavelet asymmetric and symmetric models (detail 1), and 0.78 for the EC-GARCH symmetric model and for the EC-GJR-GARCH asymmetric model. It is clear that model specification and econometric sophistication do not contribute to the improvement of hedging effectiveness.

6. Conclusion

This paper utilises wavelet analysis to estimate hedge ratios. This technique allows the use of long time intervals when more precise low-frequency information is needed and short time intervals when more precise high-frequency information is needed. This makes it possible to examine the features of a daily time series in different time intervals, such as weekly or monthly, without the need for aggregation, which would result in loss of useful data. By using wavelet analysis in conjunction with symmetric and asymmetric models, we calculate various hedge ratios that are subsequently used to assess the hedging effectiveness of crude oil futures contracts, soybeans futures contracts and the S&P500 futures contract. For comparative purposes, other models and methods are used to estimate hedge ratios and corresponding hedging effectiveness.

Based on the variance ratio, we find that wavelet analysis does not lead to an improvement in hedging effectiveness, irrespective of whether symmetric or asymmetric hedge ratios are used for constructing the hedge and irrespective of the contract used (crude oil, soybeans or S&P500). The general conclusion reached by this study is that econometric sophistication does not boost hedging effectiveness.

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