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Price vs. Quantity Regulation and Technological Modernization

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The impact of resource price and quotas on capital modernization of a firm is considered under embodied technological change. It is shown that a high price of resources (or capital) suppresses optimal growth and leads to no investment and no modernization. In contrast, limiting the quantity of available resources (such as energy quotas) stabilizes optimal growth and promotes capital modernization (e.g., scrapping obsolete capital in order to buy more efficient capital).

We will employ a partial equilibrium model of a profit-maximizing firm with exogenous exponential capital and resource prices, resource quota restrictions, and the technological change embodied in newer vintages of productive capital. Vintage models attract increasing interest for economic growth (see Boucekkine and Pommeret, 2004; Yatsenko and Hritonenko, 2007; and references therein). The optimization model under study is to find the unknown functions $m(t)$, $a(t)$, $c(t)$, $y(t)$, and $R(t)$, $t \in [t_0, \infty)$, that maximize the discounted net profit over $[t_0, \infty)$:

$$I = \int_{t_0}^{\infty} e^{-rt} c(t) dt, \quad r > 0, \tag{1}$$

in the vintage capital model with Leontief technology:

$$c(t) = y(t) - p(t)m(t) - q(t)R(t), \tag{2}$$

$$y(t) = \int_{a(t)}^{t} \beta(\tau, t) m(\tau) d\tau, \tag{3}$$

$$R(t) = \int_{a(t)}^{t} m(\tau) d\tau, \tag{4}$$

$$R(t) \le R_{\max}(t), \tag{5}$$

with initial conditions $a(t_0) = a_0 < t_0$ and $m(\tau) = m_0(\tau)$, $\tau \in [a_0, t_0]$. In (2)-(4), $c(t)$ is the net profit, $y(t)$ is the product output, $R(t)$ is the consumption of a limited resource (e.g., labor or energy), $p(t)$ is a given capital price, $q(t)$ is a given resource price, $\beta(\tau, t)$ is the efficiency at time t of the vintage τ, $m(t)$ is the

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investment into new capital, and $a(t)$ is the oldest capital vintage still in use at time t. The embodied technological change means that the efficiency $\beta(\tau, t)$ increases in τ. Inequality (5) describes a regulatory quota restriction on resource R. The decision variables are m and a, where $m \geq 0$, $a'(t) \geq 0$, $a(t) \leq t$, whereas unknown y, R, and $c \geq 0$ are expressed in terms of m and a.

For brevity and clarity, we restrict ourselves to exponential technology and prices:

$$\beta(\tau, t) = B\exp(c_\tau \tau + c_t t),\ p(t) = P\exp(c_p t),\ q(t) = Q\exp(c_q t), \tag{6}$$

where B, P, Q, c_τ, c_t, c_p, and c_q are constants. By (2) and (3), $c(t) \geq 0$ requires $Q < B\exp(-c_\tau(t_0 - a_0))$.

Lemma **(necessary extremum condition).** Let (m, a, y, c, R) be a solution of the problem (1)-(6). Then:

(A) If $R(t) < R_{\max}(t)$ at $t \in \Delta \subset [t_0, \infty)$ then

$$I_m'(t) \leq 0 \text{ at } m(t) = 0;\ I_m'(t) = 0 \text{ at } m(t) > 0,\ c(t) > 0;\ I_m'(t) \geq 0 \text{ at } c(t) = 0; \tag{7}$$

$$I_a'(t) \leq 0 \text{ at } \frac{da}{dt} = 0;\ I_a'(t) = 0 \text{ at } \frac{da}{dt} > 0,\ a(t) < t;\ I_a'(t) \geq 0 \text{ at } a(t) = t, \tag{8}$$

$$I_m'(t) = \int_t^{a^{-1}(t)} e^{-ru}\left[Be^{c_\tau t + c_t u} - Qe^{c_q u}\right]du - e^{-rt}Pe^{c_p t}, \tag{9}$$

$$I_a'(t) = Qe^{c_q t} - Be^{c_\tau a(t) + c_t t},\ t \in \Delta, \tag{10}$$

where $a^{-1}(t)$ is the inverse function of $a(t)$.
(B) If $R(t) = R_{\max}(t)$ at $t \in \Delta$, then

$$\hat{I}_m'(t) \leq 0 \text{ at } m(t) = \hat{m}(t);\ \hat{I}_m'(t) \geq 0 \text{ at } c(t) = 0;\ \hat{I}_m'(t) = 0 \text{ at } m(t) > \hat{m}(t),\ c^*(t) > 0, \tag{11}$$

$$\hat{I}_m'(t) = B\int_t^{a^{-1}(t)} e^{-ru}\left[e^{c_\tau t + c_t u} - e^{c_\tau a(u) + c_t u}\right]du - e^{-rt}Pe^{c_p t},\ t \in \Delta, \tag{12}$$

where $\hat{m}(t) = \max\{0, R'_{\max}(t)\}$, and the unknown a is determined from (4).

Case (B) was proved in more general settings by Hritonenko and Yatsenko (2005) and case (A) is similar. The unique inverse $a^{-1}(t)$ in (9) and (12) exists because $a'(t) \geq 0$.

As usual in similar economic problems, the structure of optimal trajectories involves a transition part (possibly a corner solution) and long-term interior regimes such that $I_a' \equiv 0$ and/or $I_m' \equiv 0$.

Theorem. Let $R(t_0) < R_{\max}(t_0)$ and $R_{\max}(t)$ be bounded on $[t_0, \infty)$. Then:

(A) If $c_p > c_\tau + c_t$ or $c_q > c_\tau + c_t$, then the optimal dynamics satisfy $R(t) < R_{\max}(t)$ on $[t_0, \infty)$ and no investment $m^* \equiv 0$ on (μ, ∞) starting at some $\mu > t_0$.

(B) If $c_p < c_\tau + c_t$ and $c_q \le c_\tau + c_t$, then the optimal dynamics satisfy $R(t) = R_{\max}(t)$ starting at some $\mu > t_0$, an interior long-term trajectory $a(t)$ found from $\hat{I}_m{}'(t) = 0$, and $m \not\equiv 0$ found from (4).

(C) If $c_p = c_q = c_\tau + c_t$, then the optimal dynamics is as in case (A) when $P > P_{cr}$ and as in case (B) when $P > P_{cr}$, $P_{cr} = (B/r - c_t)[1-(Q/B)^{r-c_t/c_\tau}] - (Q/r - c_t - c_\tau)[1-(Q/B)^{r-c_t-c_\tau/c_\tau}]$. If $P = P_{cr}$, then the optimal dynamics satisfy $a(t) = t - const$, $m \not\equiv 0$, and $R(t) < R_{\max}(t)$, at least at large $R_{\max}$ and small m_0.

Proof. First, let $c_q > c_\tau + c_t$. Then, starting at some $\mu > a_0$, $I_a{}'(t) > 0$ and $I_m{}'(t) < 0$ by (9) and (10) for any possible $0 < a(t) < t$. Then $m^*(t) = 0$ on (μ, ∞) by (7) and we have case (A) with $R(t) < R_{\max}(t)$ over $[t_0, \infty)$. Next, let $c_q < c_\tau + c_t$. If $c_q \le c_t$, then $I_a{}'(t) = Qe^{c_q t} - Be^{c_\tau a(t) + c_t t} < 0$ by (10) and $a^*(t) = a_0$, $t \in [t_0, \infty)$. If $c_q > c_t$, then $q(t) > \beta(a_0, t_0)$ starting at some $\mu > a_0$, and an interior regime

$$a(t) = \frac{c_q - c_t}{c_\tau} t - \ln \frac{B}{Q}, \tag{13}$$

is determined from $I_a{}'(t) = 0$ at $t \in [\mu, \infty)$. The corresponding $I_m{}'(t)$ is known at a given by (13) and may be positive, negative, or zero depending on c_p. If $c_p > c_\tau + c_t$, then $I_m{}'(t) < 0$ for large t, $m(t) = 0$ by (7), and we obtain case (A). If $c_p < c_\tau + c_t$, then $I_m{}'(t) > 0$ and optimal $m(t)$ is the maximum possible (such that $c(t) = 0$) and increases exponentially. By (4), $R(t)$ increases and, since $R_{\max}(t)$ is bounded, reaches $R_{\max}(t)$ in a finite time, which leads to case (B). In case (B), the described dynamics with $\hat{I}_m{}'(t) = 0$ over (μ, ∞) was proved in Hritonenko and Yatsenko (2005).

This theorem demonstrates a relative advantage of a quantity regulation versus price regulation (see also Kelly, 2005) in the firm's partial equilibrium under technological replacement. It reveals that, in general case, there are two qualitatively different scenarios (A) and (B) of long-term development, depending on the dynamics of prices and efficiency. If the given resource and capital prices $q(t)$ or $p(t)$ increase faster than the efficiency $\beta(\tau, t)$, then the optimal dynamics is no investment (and no growth). If $\beta(\tau, t)$ increases faster than prices $p(t)$ and $q(t)$, then the optimal dynamics in the long run uses all available (possibly limited by, for example, an energy quota) resource $R(t) = R_{\max}(t)$ and scraps the obsolete capital in order to buy new more efficient capital (i.e., capital-intensive growth).

Scenario (C) is possible at the special choice of $c_p = c_q = c_\tau + c_t$ and P, B, Q and is not typical. It shows that the long-term optimal growth that does not use the entire resource, $R(t) < R_{\max}(t)$, is possible only at specially chosen prices (e.g., from general equilibrium reasoning) and special relationships between the given quota and initial condition.

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