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Designing Central Bank Loss Functions

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Kydland and Prescott (1977) show that optimal policy proves inconsistent because of rational expectations. This paper shows that the inconsistency of optimal policy comes from the inconsistency of the social loss function with the economic structure. As a result, we delegate to the central bank a different loss function that is consistent with the economic structure. Under the delegated loss function, consistent policy proves optimal for social welfare. We interpret the delegated central bank loss function as an intermediate objective (an institutional mechanism) and the social loss function as the ultimate objective. The ultimate objective is optimized through the institutional mechanism.

We design the central bank loss function based on three observations. First, the social loss function—e.g., the representative household's utility, Arrow's (1951) social welfare function, or Rawls's (1971) maximin criterion—reflects a normative problem in philosophy. It provides a criterion for designing a public institution, not the direct loss function for that institution (here a central bank). Second, the economic structure partly determines the central bank loss function, since the optimal loss function must prove consistent with the economic structure. Third, the delegated targets must be attainable; without attainable targets, monetary policy lacks credibility and accountability.

The design of the central bank loss function proceeds as follows. First, compute the optimal policy, given the social loss function and the economic structure. The optimal policy provides a benchmark for designing the central bank loss function. Second, design the central bank loss function according to the three stated observations.

The model

We adopt the model in Kydland and Prescott (1977, pp. 477-480), assuming an

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expectations augmented Phillips curve as follows:

$$u_t = \lambda\left(x_t^e - x_t\right) + u^*, \tag{1}$$

where u_t is unemployment in period t, x_t is the inflation rate, x_t^e is the expected inflation rate, λ is the response of unemployment to unexpected inflation, and u^* is the natural rate of unemployment. They also assume rational expectations:

$$x_t^e = E(x_t). \tag{2}$$

To complete the model and rationalize policy choice, they assume some social objective function, $S(x_t, u_t)$, which they illustrate in Figure 1 (Kydland and Prescott, 1977, p. 479). According to Figure 1, $S(x_t, u_t)$ actually takes the form:

$$L_t^S = \alpha(x_t)^2 + 2(u_t - u^*) \quad \text{or equivalently} \quad L_t^S = \alpha(x_t)^2 + 2u_t, \tag{3}$$

where L_t^S represents the social loss and α the weight that society places on inflation relative to unemployment. Loss function (3) means a zero inflation rate target and an unbounded negative unemployment target (not the natural rate!).

The game between the central bank and the private sector proceeds sequentially as follows. First, the wage setter and the firm sign a wage contract, where the wage setter sets the nominal wage and the firm sets the quantity of labor that it hires. Since the contract fixes the nominal wage, the wage setter must form a rational expectation of the inflation rate, x_t^e, to keep a certain real wage level when setting the nominal wage. Second, a (negative) shock, ε_t, occurs. Third, the central bank chooses the inflation rate, x_t, to minimize the social loss function. Finally, the firm hires its labor.

Since supply shocks do not affect the inconsistency of optimal policy, we assume that the game plays under certainty, and, thus, the wage setter uses perfect foresight (i.e., $x_t^e = x_t$).

Optimal policy

The optimization problem is as follows:

$$\begin{aligned} &\min_{x_t} L_t^S = \alpha(x_t)^2 + 2(u_t - u^*) \\ &s.t. \begin{cases} u_t = \lambda(x_t^e - x_t) + u^* \\ x_t^e = x_t. \end{cases} \end{aligned} \tag{4}$$

The first-order condition for an optimum, assuming that the central bank takes the expected inflation rate as endogenously responding to its decision, yields the following:

$$\frac{\partial L_t^S}{\partial x_t} = 2\alpha x_t = 0 . \tag{5}$$

Thus $x_t = 0$, $x_t^e = 0$, $u_t = u^*$, and $L_t^S = 0$.

Consistent policy

We solve the game between the wage setter and the central bank, both of which are subject to the firm's behavioral equation, by backward induction. The optimization problem is:

$$\begin{cases} x_t^e = x_t \\ \min\limits_{x_t} L_t^S = \alpha (x_t)^2 + 2(u_t - u^*) \\ s.t.\ u_t = \lambda (x_t^e - x_t) + u^* . \end{cases} \tag{6}$$

Given the wage setter's expected inflation rate, x^e, the central bank minimizes the social loss by selecting the inflation rate, x_t. The first-order condition yields:

$$\frac{\partial L_t^S}{\partial x_t} = 2\alpha x_t - 2\lambda = 0 . \tag{7}$$

Thus, $x_t = \lambda/\alpha$, $x_t^e = \lambda/\alpha$, $u_t = u^*$, and $L_t^S = \lambda^2/\alpha$. Obviously, optimal policy is inconsistent in this simple model. Two inconsistencies exist. First, the social loss function is inconsistent with the economic structure. The central bank, with the social loss function, decreases unemployment to as low a level as possible, while unemployment always equals the natural rate given the economic structure. Second, the two targets of the social loss function conflict with each other, given the economic structure. The central bank, with an unbounded negative unemployment target, always possesses the incentive to inflate by making use of the Phillips curve, which conflicts with the zero inflation rate target.

Central bank loss function design

We adopt the same target variables and same functional form as those of the society but with possibly different parameters—target values and relative weight between targets—as follows:

$$L_t^b = \alpha^b (x_t - x^b)^2 + 2(u_t - u^b) \quad \text{or equivalently} \quad L_t^b = \alpha^b (x_t - x^b)^2 + 2u_t , \tag{8}$$

where the superscript b indicates the parameters that are delegated to the central bank.

The delegation consists of two stages. First, the government delegates the loss function (8) to the central bank by choosing parameters (x^b, α^b). Second, the central bank, which minimizes the delegated loss function, and the wage setter play

a game. That is,

$$\min_{\alpha^b,\ x^b} L_t^S = \alpha(x_t)^2 + 2(u_t - u^*) \tag{9}$$

$$s.t. \begin{cases} x_t^e = x_t \\ \min\limits_{x_t} L_t^b = \alpha^b (x_t - x^b)^2 + 2(u_t - u^b) \\ s.t.\ u_t = \lambda(x_t^e - x_t) + u^*. \end{cases} \tag{10}$$

We solve the delegation problem using backward induction in two steps. First, we solve game (10) between the wage setter and the central bank, producing equilibrium outcomes:

$$x_t = x^b + \lambda/\alpha^b\ ,\ \ x_t^e = x^b + \lambda/\alpha^b\ , \text{and}\ \ u_t = u^*\ , \tag{11}$$

which exactly mirrors the solution of consistent policy. Second, the government selects x^b and α^b to minimize social loss (9), subject to the equilibrium outcomes (i.e., $x_t = x^b + \lambda/\alpha^b$ and $u_t = u^*$). Thus:

$$x^b = -\lambda/\alpha^b\ , \tag{12}$$

where $0 < \alpha^b < \infty$ or

$$x^b = 0\ , \tag{13}$$

where $\alpha^b = \infty$. Therefore, the loss functions delegated to the central bank are:

$$L_t^b = \alpha^b (x_t + \lambda/\alpha^b)^2 + 2u_t\ , \tag{14}$$

or

$$L_t^b = (x_t)^2\ . \tag{15}$$

Either (12) or (13) causes the equilibrium outcomes (11) to equal the optimal outcomes in (5). That is, consistent policy is optimal for either loss function (14) or (15).

The delegated loss function in (14) is similar to the delegation in Svensson (1997), who explains the optimality of delegation as that "lower inflation is due to lower inflation targets rather than lower weights on employment stabilization!" (p. 109). The delegation in (15) works as follows. Given the economic structure, the private sector determines the unemployment level at the natural rate. As a result, the central bank concentrates on the inflation rate target (i.e., $\alpha^b = \infty$). As an alternative interpretation, the central bank cannot place any weight on unemployment, otherwise the central bank, with an unbounded negative unemployment target, always possesses an incentive to inflate by making use of the Phillips curve, resulting in an inflation bias. Therefore, the central bank must put

zero weight on unemployment (i.e., infinite weight on inflation). As to the inflation target itself, since the social target is zero, the government simply delegates a zero target (i.e., $x^b = 0$) to the central bank.

Either central bank loss function is consistent with the economic structure. We choose loss function (15), since the central bank can attain its target (i.e., $x_t^b = x_t = 0$). Using loss function (14), the central bank cannot attain its target (i.e., $x^b = -\lambda/\alpha^b$ but $x_t = 0$).

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